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New Fuzzy Performance Indices for Reliability Analysis of Water Supply Systems

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NEW FUZZY PERFORMANCE INDICES FOR RELAIBILITY ANALYSIS OF WATER SUPPLY SYSTEMS

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EXECUTIVE SUMMARY

Large and complex engineering systems are subject to wide range of possible future loads and conditions. Uncertainty associated with the quantification of these potential conditions is imposing a great challenge to systems' design, planning and management. Therefore, the assurance of satisfactory and reliable system performance cannot be simply achieved.

Water supply systems, as typical example of these engineering systems, include collections of different types of facilities. These facilities are connected in complicated networks that extend over and serve broad geographical regions. As a result, water supply systems are at risk of temporary disruption in service due to natural hazards or anthropogenic causes, whether unintentional (operational errors and mistakes) or intentional (terrorist act).

Quantification of risk is a pivotal step in the engineering risk and reliability analysis. In this analysis, uncertainty is measured using different system performance indices and figures of merit to evaluate its consequences for the safety of engineering systems

The probabilistic reliability analysis has been extensively used to deal with the problem of uncertainty in many engineering systems. However, application of probabilistic reliability analysis is invariably affected by the well-known engineering problem of data insufficiency. Bayesian approach and subjective probability estimation are used to evaluate, express, and communicate uncertainty that stems from lack of information or data unavailability. They introduce a formal procedure for incorporating subjective belief and engineering understanding together with the available data.

Fuzzy set theory, on the other hand, was developed to try to capture people judgmental believes, or as mentioned before, the uncertainty that is caused by the lack of knowledge. Fuzzy set theory and fuzzy logic contributed successfully to the technological development in different application in real-world problems of different kinds, (Zimmermann, 1996).

This study explores the utility of the fuzzy set theory in the field of engineering system reliability analysis. Three new fuzzy reliability measures are suggested: (i) reliability index, (ii) robustness index, and (iii) resiliency index. These measures are evaluated, together with fuzzy reliability measure developed by Shrestha and Duckstein (1998), using two simple hypothetical cases. The new suggested indices are proven to be able to handle different fuzzy representations. In addition, these reliability measures comply with the conceptual approach of the fuzzy sets.

1 UNCERTAINTY AND WATER SUPPLY SYSTEMS

1.1 Introduction

One of the main goals of engineering design is the assurance of the system performance under wide range of possible future loads and conditions. This is generally not a simple goal to achieve, especially for large and complex engineering systems.

Water supply systems are not an exception to that rule, as they include a collection of different types of facilities. They usually include conveyance facilities, such as pipes and pumps, treatment facilities, such as sedimentation tanks and filters, and storage facilities such as reservoirs and tanks. These elements are connected in complicated networks that extend over and serve broad geographical regions. Each element is vulnerable to temporary disruption in service due to natural hazards or anthropogenic causes, whether unintentional (operational errors and mistakes) or intentional (terrorist act). Water supply systems vary in terms of their scales, structures, and configurations and consequently their vulnerability to potential hazards.

Uncertain exogenous factors, i.e. uncontrolled external factors, affect the water supply capacity of each element and consequently its performance. As a result, risk of future system failure is often unavoidable, (Ang and Tang, 1984). Determination of demand pattern also is not a simple problem too, therefore estimation of both, supply and demand, is necessary for the system reliability analysis. Several approaches are available for quantification of uncertainty. They provide a basis for realistic measures of system reliability.

1.2 Types and Sources of Uncertainty

Uncertainty is associated with all engineering systems, as these systems rely on modelling of physical phenomena that are either inherently random or difficult to model precisely (Ang and Tang, 1984). The exact realization of random events is an arduous task that can only be described through non-deterministic models that incorporate any measure of variability as a way to express uncertainty.

Simonovic (1997) states that the two major sources of uncertainty are randomness and lack of knowledge. Randomness that he calls variability for water resources systems is further classified into: (i) temporal, (ii) spatial, and (iii) individual heterogeneity. Imprecision or ambiguity, some times called lack of knowledge, is the other type of uncertainty that stems from our inability to conceptualize the real-world processes in a mathematical form, especially for complex systems. Ang and Tang (1984) referred to the model prediction error as the other source of uncertainty. They mentioned two types of model prediction errors, (i) systematic error (bias), and (ii) random error.

The following is a summary of the taxonomy provided by Simonovic (1997) for the second source of uncertainty. A representation of this taxonomy is depicted in Figure (1.1). According to this classification, the lack of knowledge can be attributed to: (i) model formulation, (ii) parameter estimation, and (iii) decision making.

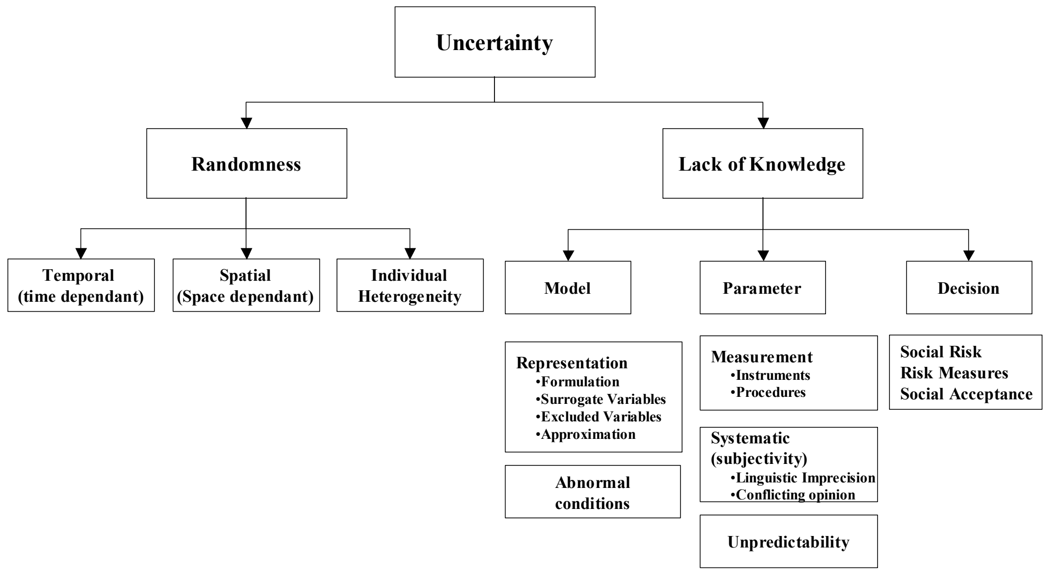


Figure (1.1) Sources of Uncertainty (after Simonovic, 1997).

The model uncertainty is the consequence of our inability to capture the real-life phenomenon in a well-defined form with the available tools. Therefore, it is the result of model representation scheme and abnormal conditions. The model representation involves scheme use of models to represent the real world physical phenomenon. It requires a set of variables, together with the approximations and assumptions. Models are usually calibrated and verified for limited number of conditions. Abnormal conditions (not captured by calibration and verification) represent another major source of the uncertainty.

The parameter uncertainty results from (i) measurement error that is related to the selected instruments and procedures, (ii) systematic error that is caused by the subjective judgment in capturing linguistic imprecision and conflicting expert opinions, and (iii) parameter unpredictability. The third source of uncertainty is the decision uncertainty. It is encountered by the decision maker when trying to incorporate social issues in the decision making process.

1.3 Quantification of Uncertainty

The diversity of uncertainty sources is imposing a great challenge to systems‘ design, planning and management, as it might need unattainable efforts to insure a satisfactory and reliable system performance. Adopting high safety factors is one of the means to avoid uncertainty by considering all unknown sources. However, high safety factor may result in an infeasible system solution. Therefore, it is necessary to quantify known uncertainty sources.

Engineering risk and reliability analysis is a general methodology for quantification of uncertainty and evaluation of its consequences for the safety of engineering systems (Ganoulis, 1994). Risk identification is the first step in any risk analysis, where all sources of uncertainty causing risk of failure are clearly detailed. Quantification of risk is the second step through which uncertainties are measured using different system performance indices and figures of merit. Stochastic (probabilistic) and fuzzy sets are the two main approaches for system reliability analysis.

1.4 Report Descriptions and Organization

Chapter (2) introduces a survey of the different approaches used in the framework of reliability analysis of the engineering systems. The chapter includes three subdivisions, (i) probabilistic approach, (ii) subjective probability and Bayesian approaches, and (iii) fuzzy set theory. The first part deals with the fundamentals of the probabilistic (Stochastic) approach to the problem of systems' reliability together with a brief description of the use of performance indices and figures of merit. The advantages and disadvantages of concept subjective probability and Bayesian approach are discussed in the second part. A relatively detailed review of the basics of fuzzy sets and its utility in the system reliability analysis is provided in the third part. The chapter concludes with a hypothetical case study evaluation of a fuzzy reliability measure suggested by Shrestha and Duckstein (1998).

Chapter (3) presents the development of new fuzzy reliability measures: (i) reliability index, (ii) robustness index, and (iii) resiliency index. The presentation is preceded by a detailed discussion of the basic notions involved in the development of the fuzzy performance indices. The utility of the suggested indices is examined using the previously used hypothetical case study in Chapter (2)

2 RELIABILITY ANALYSIS OF ENGINEERING SYSTEMS

2.1 Probabilistic Reliability Analysis

The problem of engineering system reliability has received considerable attention from the statisticians and probability scientists. The probabilistic (stochastic) reliability analysis has been extensively used to deal with the problem of uncertainty in many engineering systems. In the probabilistic approach, the analysis involves describing supply and demand as belonging to respective possible probability distributions. As a result, uncertainty in both, supply and demand, is introduced through the use of random variables. Therefore, the where system reliability may be realistically measured in terms of probability. The principle objective of the probabilistic reliability analysis is to insure that the demand does not exceed the supply throughout a specified time horizon in terms of probability

$$P_S = P(\hat{X} > \hat{Y}) \qquad \text{……….(2.1)}$$

where:

P_S is the probability of satisfactory performance;

$\hat{X}$ is the random supply capacity; and

$\hat{Y}$ is the random demand requirement.

The complementary event $(\hat{X} < \hat{Y})$ is the corresponding measure of unreliability (failure). Assuming that the probability distributions of $\hat{X}$ and $\hat{Y}$ are known, the probability of failure event can be calculated using

$$P_F = P(\hat{X} < \hat{Y}) = \sum_{\text{all } y} P(\hat{X} < \hat{Y} \mid \hat{Y} = y)\, P(\hat{Y} = y) \qquad \text{……….(2.2)}$$

where:

P_F is the probability of failure;

y is the value of the random demand requirement;

$P(X < Y \mid \hat{Y} = y)$ is the conditional probability that the demand exceeds the supply for a certain demand value y; and

$P(\hat{Y} = y)$ is the probability that the demand value is y.

Assuming statistical independence between $\hat{X}$ and $\hat{Y}$, that is

$$P(\hat{X} < \hat{Y} \mid \hat{Y} = y) = P(\hat{X} < y) \qquad \text{……….(2.3)}$$

where:

$P(\hat{X} < y)$ is the probability that the random supply $\hat{X}$ is less than the demand value y.

Therefore, Equation (2.1) can be re-written for continuous $\hat{X}$ and $\hat{Y}$

$$P_F = \int_0^{\infty} F_{\hat{X}}(y)\, f_{\hat{Y}}(y)\, dy \qquad \text{……….(2.4)}$$

where:

$F_{\hat{X}}(y)$ is the cumulative conditional probability distribution of failure; and

$f_{\hat{Y}}(y)$ is the probability density function of the random demand $\hat{Y}$.

The shaded area of the overlap region between $f_{\hat{X}}(x)$ and $f_{\hat{Y}}(y)$, in Figure (2.1), represents the conditional probability of failure with respect to y.

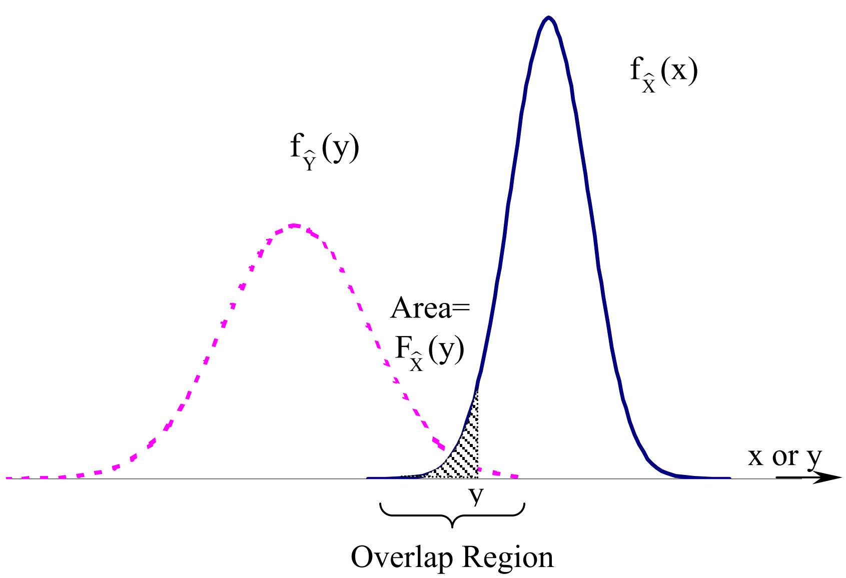


Figure (2.1) Schematic Presentation of the Probability of Failure $F_{\hat{X}}(y)$ (after Ang and Tang, 1984).

In case of statistical correlation between supply and demand, that is

$$P(\widehat{X} < \widehat{Y} \mid \widehat{Y} = y) \neq P(\widehat{X} < y)$$
and ……….(2.5)
$$P(\widehat{Y} < \widehat{X} \mid \widehat{X} = x) \neq P(\widehat{Y} < x)$$

Therefore, the probability of failure is expressed in terms of joint probability density function $f_{\widehat{X},\widehat{Y}}(x, y)$ as follows

$$P_F = \int_0^{\infty} \left[\int_0^{y} f_{\widehat{X},\widehat{Y}}(x, y)\, dx \right] dy \qquad \text{……….(2.6)}$$

2.1.1 Margin of Safety and Factor of Safety

The supply-demand problem is usually formulated in terms of safety margin or factor of safety, defined as follows

$$\widehat{M} = \widehat{X} - \widehat{Y}$$

and ……….(2.7)

$$\widehat{\Theta} = \frac{\widehat{X}}{\widehat{Y}}$$

where:

$\widehat{M}$ is the margin of safety; and

$\widehat{\Theta}$ is the factor of safety.

Both, $\widehat{\mathrm{M}}$ and $\widehat{\Theta}$ are random variables with corresponding probability density functions, $f_{\widehat{M}}(m)$ and $f_{\widehat{\Theta}}(\theta)$. The failure event is the event where $(\widehat{\mathrm{M}} < 0)$ or $(\widehat{\Theta} < 1)$, or in mathematical from

$$P_F = \int_{-\infty}^{0} f_{\widehat{M}}(m)\, dm = F_{\widehat{M}}(0)$$

or ……….(2.8)

$$P_F = \int_{0}^{1} f_{\widehat{\Theta}}(\theta)\, d\theta = F_{\widehat{\Theta}}(1)$$

where:

$f_{\widehat{M}}(m)$ is the probability density function of the margin of safety;

$F_{\widehat{M}}(0)$ is the cumulative distribution function at $\widehat{\mathrm{M}}$ =0;

$f_{\widehat{\Theta}}(\theta)$ is the probability density function of the factor of safety; and

$F_{\widehat{\Theta}}(1)$ is the cumulative distribution function at $\widehat{\Theta}$ =1.

Figures (2.2a) and (2.2b) depict the failure event for both cases by the area under the $f_{\widehat{M}}(m)$ curve below 0, in case of using margin of safety, and the area under the $f_{\widehat{\Theta}}(\theta)$ curve below 1, in the case of factor of safety.

Calculation of the above integrals requires the prior knowledge of the probability density functions of both, supply and demand, and/or their joint probability distribution functions. In practice, data is usually insufficient to provide such information, as there is a need for previous failure experience for different types of failure events and/or system behavior under severe conditions of loading. Even if data is available to estimate these distributions, approximations are almost always necessary to calculate system reliability, (Ang and Tang, 1984).

Several approximation methods are suggested in the literature to overcome these problems. For example, in some cases it is suggested to use the normal representation of non-normal distributions as a practical alternative. In this case, data has to be available to estimate the first two moments of the assumed normal distribution or to use the second moment formulation, which in turn limits the implementation of reliability concept.

Another approach to avoid the problem of data insufficiency is the use of subjective judgment of the decision maker to estimate the probability distribution of random event, i.e. subjective probability. The third, and final, approach is the integration of judgment with the observed information using Baye's theory (Ang and Tang, 1984). In either cases, i.e. subjective probability or Bayesian approach, the accuracy of the derived distributions is strongly dependent on the realistic estimation of the decision maker's judgment.

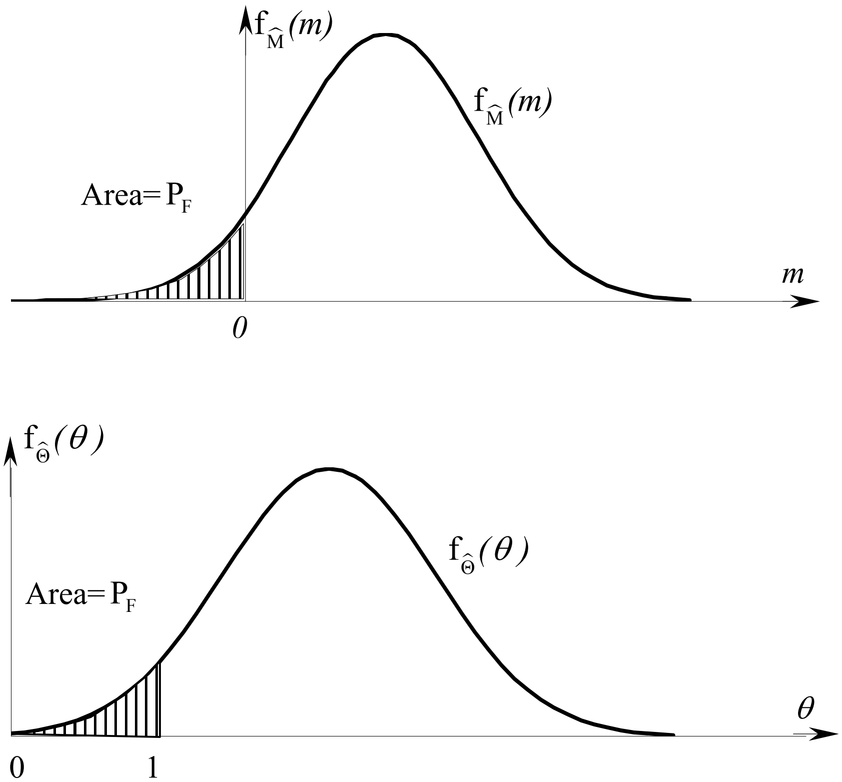


Figure (2.2) The Probability Density Functions for Margin of Safety and Safety Factor (after Ang and Tang, 1984).

2.1.2 System Performance Function

Engineering systems involve multiple components that control their performance. Supply capacity and demand requirement may be functions of other system variables. Therefore, it is more accurate to use system performance functions, i.e. functions of state variables, to identify the state of the system

$$g(\hat{\underline{\mathbf{X}}}) = g(\hat{X}_1, \hat{X}_2, ..., \hat{X}_n) \qquad ....(2.9)$$

where:

$\underline{\widehat{\mathbf{X}}}$ is the vector of state random variables $(\widehat{X}_1, \widehat{X}_2, ..., \widehat{X}_n)$;

n is the number of the state variables; and

$g(\underline{\widehat{\mathbf{X}}})$ is the function that determines the system performance or the system state.

As a result, the limiting performance requirement may be defined as system reliability, the probability of the system to perform its intended function. Consequently, the limiting-state of the system is defined as

$$
\begin{aligned}
&g(\underline{\widehat{\mathbf{X}}}) = 0.0 \\
&g(\underline{\widehat{\mathbf{X}}}) > 0.0 \quad \textit{Safe State} \qquad \ldots\ldots\ldots(2.10) \\
&g(\underline{\widehat{\mathbf{X}}}) < 0.0 \quad \textit{Failure State}
\end{aligned}
$$

The performance functions, expressed by the margin of safety or factor of safety, are written as follows

$$g(\underline{\widehat{\mathbf{X}}}) = \widehat{M} = 0$$

and ……….(2.11)

$$g(\underline{\widehat{\mathbf{X}}}) = \widehat{\Theta} - 1 = 0$$

The limit-state equation, i.e. Equation (2.9), is an n-dimensional surface that may be called the *failure surface*. Geometrically, one side of the failure surface is the safe state region, where $g(\hat{\underline{\mathbf{X}}}) > 0.0$, and the other side is the failure state region, where $g(\hat{\underline{\mathbf{X}}}) < 0.0$, as shown in Figure (2.3) for the case of two state-variables.

The probability of satisfactory performance, i.e. reliability, can be calculated using the joint probability density function for the design variables

$$P_S = \int \text{.........} \int_{g(\hat{\underline{X}})>0} f_{\hat{x}_1,\hat{x}_2.....,\hat{x}_n}(x_1, x_2.....,x_n)\, dx_1 dx_2.....dx_n = \int_{g(\hat{\underline{X}})>0} f_{\hat{\underline{X}}}(\hat{\underline{X}})dX \qquad \text{……….(2.12)}$$

where:

$f_{\hat{\underline{X}}}(\hat{\underline{X}})$ is the joint probability distribution function of the design variables; and

$(\hat{X}_1, \hat{X}_2, ..., \hat{X}_n)$ are the design variables.

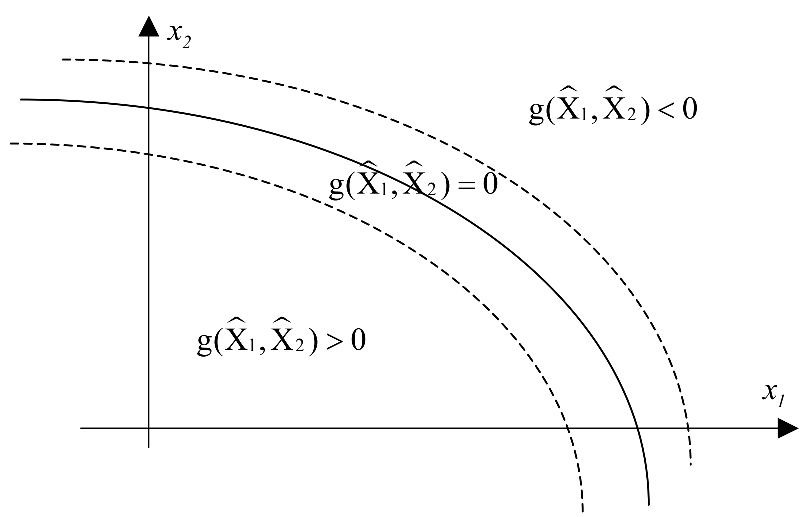


Figure (2.3) Safe and Failure States in Two State-Variables Space (after Ang and Tang, 1984).

Integration of this function, if known, is formidable task, which needs an approximation in order to evaluate P_S and P_F. Hence, different approximation methods are found in the literature for linear and non-linear performance functions, as well as for correlated and uncorrelated variables. The minimum distance from the origin to the failure surface, represented by the performance function $g(\hat{\underline{\mathbf{X}}})$, can be used as an equivalent measure of system reliability (Ang and Tang, 1984).

2.1.3 Multi-Component Systems

The previous reliability problem involves a single failure mode, i.e. a single component system that is represented by a single limit state function. Most of the engineering systems consist of collection of different components with different failure modes. As a result, the overall system failure involves a multiple modes of failure. The same probabilistic approach is extended to consider potential system modes of failure. Assuming that the system performance can be represented as

$$g_j(\hat{\underline{X}}) = g_j(X_1, X_2, ..., X_n); \quad \forall\ j = 1, 2, ..., k \qquad ....(2.13)$$

where:

k is the number of system potential failure modes, .i.e no of components; and

n is the number of state variables.

The individual failure event is defined as

$$E_{Fj} = [g_j(\hat{\underline{X}}) < 0] \quad \forall\ j \in k \qquad \text{……….(2.14)}$$

Then its compliment (safe event)

$$E_S = \overline{E_{Fj}} = [g_j(\hat{\underline{X}}) > 0] \quad \forall\ j \in k \qquad \text{……….(2.15)}$$

The failure and safe events are represented in Figure (2.4) for three-failure modes system with two state variables. The limit state equations are represented by the three equations $g_j(\hat{\underline{X}}) = 0$. The safety of the system is the event in which none of the k-potential failure modes occur

$$E_S = \overline{E_F} = \overline{E_{F1}} \cap \overline{E_{F2}} \cap ... \cap \overline{E_{Fk}} \qquad \text{……….(2.16)}$$

Therefore, the system reliability is calculated using the volume integral of the joint probability density function

$$P_S = \int ....\int_{(\overline{E_{F1}} \cap \overline{E_{F2}} \cap ... \cap \overline{E_{Fk}})} f_{\hat{x}_1, \hat{x}_2 ....., \hat{x}_n}(x_1, x_2 ....., x_n)\, dx_1 dx_2 ..... dx_n \qquad \text{……….(2.17)}$$

The use of the integral, Equation (2.17), to calculate the system reliability is generally difficult, therefore approximation methods are used to evaluate P_S or P_F. Lower and upper probability bounds of the corresponding probability are used to overcome the integration problems, (Ang and Tang, 1984).

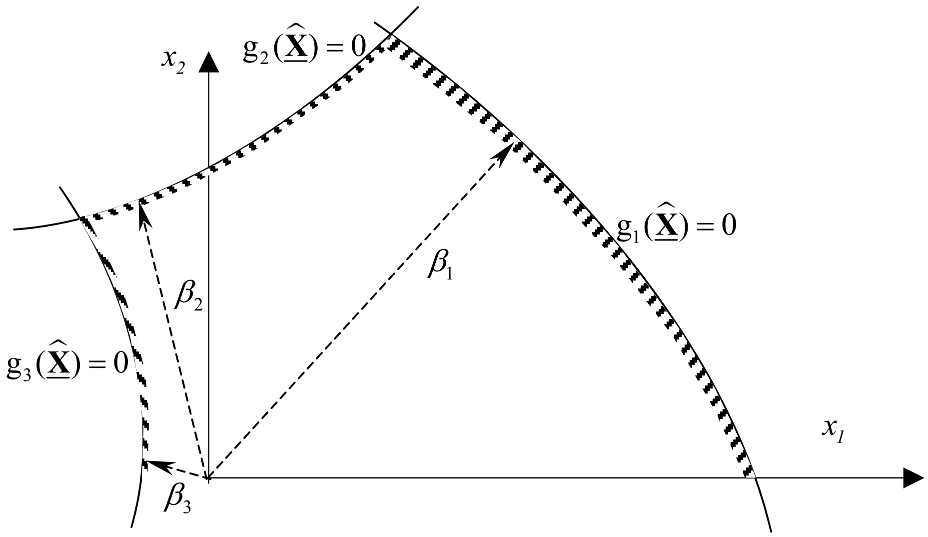


Figure (2.4) Multiple Modes of Failure (after Ang and Tang, 1984).

Redundancy

System redundancy affects the overall system reliability, i.e. the reliability of redundant system is higher than a non-redundant system where component failure is tantamount to the overall system failure. Probabilistic reliability analysis takes into account system redundancy for different types of system configurations, serial, parallel, or combined. Multiple failure mode systems approach is used to evaluate the reliability of multi-component system configurations.

Serial System

The overall system failure, in case of serial configuration, depends on the weakest component of the system. The system fails if any of its components fail. Therefore, the failure event of the system is represented by

$$E_F = E_{F1} \cup E_{F2} \cup ... \cup E_{Fm} \qquad \text{.........(2.18)}$$

where:

E_F is the system failure event;

E_{Fm} is the failure event of the m-th component; and

m is the number of the system components.

The system safety event is mathematically expressed as the complementary event of system failure, that is

$$E_S = \overline{E_F} = \overline{E_{F1}} \cap \overline{E_{F2}} \cap ... \cap \overline{E_{Fm}} \qquad \text{.........(2.19)}$$

where:

E_S is the system safe event;

$\overline{E_F}$ is the system complement of the failure event; and

m is the number of the system components.

<u>Parallel System</u>

The overall system failure, in the case of the parallel configuration, requires the failure of all the system's components

$$E_F = E_{F1} \cap E_{F2} \cap ... \cap E_{Fm} \qquad \text{.........(2.20)}$$

Combined System

Combined systems may be decomposed, if possible, into several serial and parallel systems, where the overall system failure or safety is the combination of these events based on the system decomposition.

2.1.4 Performance Indices

The early works of Hashimoto et al (1982a and 1982b) are the basis for the use of performance indices to evaluate the risk and reliability of water resources systems. They suggest *Reliability*, *Resiliency*, *Vulnerability*, and *Robustness* as criteria for evaluating the performance of water resources systems.

It is assumed that the performance of the water resources systems could be described by a stationary stochastic process, as an acceptable approximation (probability density functions that describe the system output time series do not change with time).

Reliability

System reliability is defined as the probability of no failure occurrence within a fixed time period

$$\alpha = \text{Prob}(X_t \in S) \qquad \text{……….(2.21)}$$

where:

α is the reliability index;

X_t is the system's output status at time t; and

S is the satisfactory state.

Risk is defined as the opposite of reliability and mathematically expressed as

$$\text{Risk} = 1 - \alpha \qquad \ldots\ldots\ldots(2.22)$$

Duckstein et al (1987) defined the reliability index as an estimate of the relative frequency that the system is not in a failure state

$$\text{Reliability Index} = \frac{t + 1 - \sum_{j=0}^{t} \delta(\mu, j)}{t + 1} \qquad \ldots\ldots(2.23)$$

where:

t is the time step; and

δ(μ,j) is the failure mode function in the j^{th} time period and defined as

$$\delta(\mu, j) = \begin{cases} 1 & \text{if system is in failure mode at time j} \\ 0 & \text{otherwise} \end{cases} \qquad \ldots\ldots\ldots(2.24)$$

Resiliency

Resiliency describes how quickly the system is likely to recover from failure once failure has occurred, (Hashimoto et al, 1982a). The mathematical representation is based on the definition of the average recovery rate of the system

$$\gamma = \frac{\rho}{1 - \alpha} = \frac{\text{Prob}(X_t \in S \text{ and } X_{t+1} \in F)}{\text{Prob}(X_t \in F)} \quad \ldots\ldots\ldots(2.25)$$

where:

γ is the system resiliency; and

ρ is the probability of the system being in the safe state S in the time period t and going to the failure state F in the period, t+1.

This index is also named 'repairability' by Duckstein et al (1987). They define repairability as the average length of time that a system stays in the failure state

$$\text{Repairability Index} = \frac{\sum_{j=0}^{t} \delta(\mu, j)}{t+1} = \frac{\sum_{n=1}^{N} d(\mu, n)}{N} \quad \ldots\ldots\ldots(2.26)$$

where:

$d(\mu, n)$ is the duration of the n-th mode of failure; and

N is the total number of failure modes.

When $N \rightarrow \infty$, the repairability index becomes a resiliency index as defined by Hashimoto et al (1982a).

Vulnerability

It is a measure of failure severity, and is defined as the likely magnitude of failure (Hashimoto et al, 1982a)

$$\nu = \sum_{j \in F} s_j e_j \qquad \text{……….(2.27)}$$

where:

s_j is the numerical indicator of system severity of the failure state j; and

e_j is the probability that the system state X corresponds to s_j.

Duckstein et al (1987) define the vulnerability index as the average severity of an incident event

$$\text{Vulnerability Index} = E(\mu) \qquad \text{……….(2.28)}$$

where:

E( ) is the expected value of the failure mode μ.

Robustness

Robustness is a measure of system performance that is concerned with the ability of the system to adapt to a wide range of possible demand conditions, in the future, at little additional cost (Hashimoto et al, 1982b). They define the system robustness as a measure that describes the overall economic performance of a water resources system.

A cost function C(q|D) is defined to account for accommodating the demand condition q with the project design D. This cost includes construction, operation and maintenance costs, and the costs of measures taken to satisfy the actual demand conditions with the design D. Therefore, the main interest is the minimum cost of a design that can satisfy the assumed demand conditions, that is

$$L(q) = \min_{\text{all } D} \left[C(q|D) \right] \qquad \ldots\ldots\ldots(2.29)$$

where:

L(q) is the minimum cost function for the demand condition q;

C( ) is the cost function; and

D is a particular design.

The design robustness is defined as the likelihood, or probability, that the design cost will be less than (100.β (%)) of the cost effective design, i.e. L(q)

$$R_\beta = P\left(C(q|D) \le (1+\beta)\, L(q)\right) \qquad \ldots\ldots\ldots(2.30)$$

where

$P(\ )$ is the probability of an event;

R_β is the design D robustness; and

β is a fraction less than unity.

Other Performance Indices

Duckstein et al (1987) suggested other performance indices that could be used to assess system performance. These indices are:

- *Grade of service:* the relative frequency of providing a service when it is required.
- *Quality of service:* percentage of requirement satisfied.
- *Speed of response:* the elapsed time between demand of a service and the response to that demand.
- *Incident period:* the mean interarrival time between entries into the failure mode.
- *Mission reliability:* an estimate of the probability that the system will not fail between the time of demand and delivery of the service.
- *Availability:* the probability that the system is not in the failure mode when the demand for service occurs.
- *Economic index vector:* a vector whose components may include expected costs, losses and benefit, etc.

2.1.5 Figures of Merit

They are functions of the performance indices, where combinations of the selected performance indices are used to express super criteria, (Ganoulis, 1994). Sustainability and engineering risk are examples of most widely used figures of merit. Comparison of two systems is therefore reduced to a comparison of two vectors of figures of merit, where multi-criteria analysis can be used to evaluate different decision alternatives, (Duckstein et al 1987).

2.2 Subjective Probability and Bayes' Theory

2.2.1 Subjective Probability

Probabilistic reliability analysis relies on the representation of the demand and supply as random variables to mathematically express uncertainty. The determination of the appropriate probability distributions requires an extensive amount of data, as specified by sampling theory. In addition, subjective judgment is required to some degree to assume the sampling model, confidence coefficients, and used estimators, (Martz and Waller, 1982). As reliability analysis is concerned partially with data concerning failure events, it is not usually easy to find the necessary data, in terms of quantity and quality. Therefore, subjective probability theory is introduced to overcome the problem of insufficient data.

Subjective probability is the quantified expression of engineering judgment about the likelihood of occurrence of an uncertain event, the existence of unknown condition, or the confidence in the truth of a preposition, (Vick, 2002). As a measure of confidence or belief, subjective probability is an essential tool for evaluating, expressing, and communicating uncertainty that stems from lack of information or data unavailability. Assessment of subjective probably entails the use of the same techniques central to engineering judgment and common-sense engineering practices.

2.2.2 Bayes' Theory

An enhanced inference approach is developed formally through the use of Baye's theory. Bayes' theory introduces the use of subjective belief and engineering understanding together with the available data. In its simplest form

$$P\left\{B\middle|A\right\} = \frac{P\left\{A\middle|B\right\} P\left\{B\right\}}{P\left\{A\middle|B\right\} P\left\{B\right\} + P\left\{A\middle|\overline{B}\right\} P\left\{\overline{B}\right\}} \qquad \text{.........(2.31)}$$

where:

$P\left\{B\middle|A\right\}$ is the conditional probability of event B given A has occurred;

$P\left\{B\right\}$ is the probability of event B; and

$\overline{B}$ is the complementary of Event B.

$P\{B\}$ in Equation (2.31) denotes the prior belief about B, using subjective assessment or prior knowledge, $P\{B|A\}$ denotes the posterior belief about B knowing A has occurred, and denotes the model used to generate the event A based on B knowledge. In case of continuous unknown quantity z, Bayes' theory takes the form

$$g(\hat{z}|\hat{x}) = \frac{g(\hat{x}|\hat{z})\, f(\hat{z})}{\int_{\text{all}\,\hat{z}} g(\hat{x}|\hat{z})\, f(\hat{z}) d\hat{z}} \qquad \text{.........(2.32)}$$

where

$g(\hat{z}|\hat{x})$ is the conditional posterior probability distribution function of $\hat{z}$ given $\hat{x}$;

$f(\hat{z})$ is the prior probability distribution function of $\hat{z}$;

$g(\hat{x}|\hat{z})$ is the conditional likelihood function of $\hat{x}$ given $\hat{z}$; and

$\int_{\text{all}\,\hat{z}} g(\hat{x}|\hat{z})\, f(\hat{z}) d\hat{z}$ is the integration of the likelihood function over the admissible range of $\hat{z}$.

The likelihood is the function through which the sample data $\hat{x}$ modify the prior information about $\hat{z}$.

The main practical benefits of the Bayesian analysis are: (i) the increased quality of inferences, provided the prior information accurately reflects the true variation in the parameter(s), (ii) the reduction in data requirements, and (iii) correction of the assumption that the prior information is the reason for unacceptable outcome, not the method of inference. Manipulation of probability statements on components of any system into corresponding system reliability are well known, while the manipulation of confidence statements are not. Therefore, Bayesian system reliability analysis has more appeal as it embody probability notions rather than confidence.

The main criticism to Bayesian reliability analysis is the subjectivity in choosing the prior distribution. The biased choice of the prior distribution will result in a biased inference that does not reflect the true uncertainty inherent in the system. The other problem involved with the use of Bayesian approach is the lack of observations that must be incorporated to enhance the prior information.

Subjective probability choice of prior distribution is unavoidable solution in the Bayesian approach. The choice of any subjective probability distribution, in the case of subjective probability or Bayesian approach, is not always easy, as it is difficult to translate the prior knowledge into meaningful probability distribution, especially in multi-parameter problems, (Press, 2003).

2.3 Fuzzy Sets

The concept of fuzzy sets as described by its founder Zadeh, (1965), is a formal attempt to capture, represent and work with objects with unclear or ambiguous boundaries. This concept although is relatively new, has its origin in the early application of multi-logic notion to overcome the difficulties faced by the dual-logic representation in the set theory. Therefore, fuzzy set theory and fuzzy logic were used to overcome ambiguity or lack of knowledge in human conception of real life phenomena as a source of uncertainty.

2.3.1 Basic Notions

The collection of objects that have similar properties or general features is the basic notion of the set theory. Humans tend to organize objects into sets so as to generalize knowledge about objects through classification of information. The ordinary set classification imposes dual logic in classification. The object belongs to a set or does not belong to it, as sets boundaries are well defined. For example, considering a set *A* in a universe **X**, as shown in Figure (2.5). It is obvious that object x_1 belongs to the set A, while x_2 does not. Denoting the acceptance of belonging to a set by 1 and rejection of belonging by 0, the classification is expressed through a characteristic (membership) function $\mu_{\tilde{A}}(x)$, for $x \in \mathbf{X}$

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases} \qquad \text{.........(2.33)}$$

where:

A(x) is the characteristic function denoting the membership of *x* to set *A*.

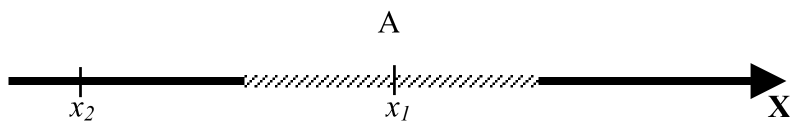


Figure (2.5) Ordinary set classification (after Pedrycz and Gomide, 1998).

The basic notion of the fuzzy sets is to relax this definition, and admit intermediate membership classes to sets. Therefore, the characteristic function can accept values between 1 and 0, expressing the grade of membership of an object to a certain set. According to this notion, the fuzzy set will be represented as a set of ordered pairs of elements, each present the element together with its membership value to the fuzzy set.

Assuming the existence of an ordinary set B with three values, 1, 2 and, 3 belonging to it. The set is mathematically represented as

$$\mathrm{B(x)} = \{1, 2, 3\} \qquad \text{……….(2.34)}$$

where

B(x) is the ordinary set; and

$1, 2, 3 \in \mathbf{X}$ are elements of universe belonging to set B(x)

If $\widetilde{B}(y)$ is a fuzzy set, with three objects belonging to, 4, 5, and 6, with membership values 0.6, 0.2, 1.0 respectively. This set can be represented as follows

$$\widetilde{B}(y) = \{(4, 0.6), (5, 0.2), (6, 1.0)\} \quad \text{.........(2.35)}$$

where:

$\widetilde{B}(y)$ is the fuzzy set; and

$4, 5, 6 \in \mathbf{X}$.

In both representation, the other elements in the universe **X** that does not belong to the ordinary set B(*x*), and the elements that have a membership values of 0 are not listed. Figure (2.6) depicts the difference in representation between ordinary set and fuzzy set, where horizontal axis represent the elements of the universe and the vertical axis represent the grade of membership of elements.

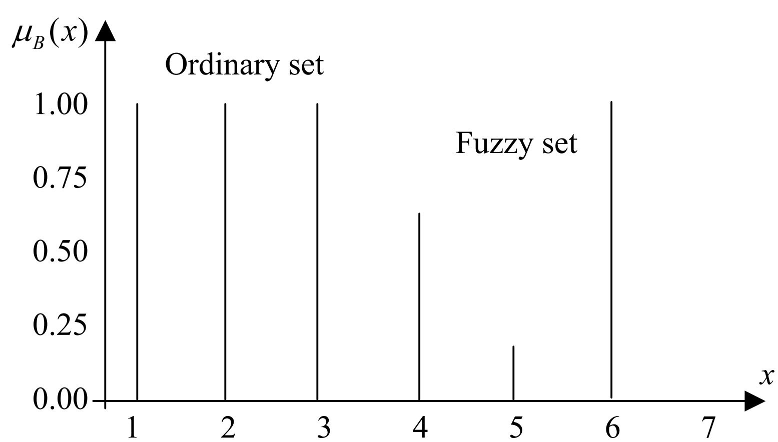


Figure (2.6) Ordinary and Fuzzy Set Representation

2.3.2 Uncertainty in Fuzzy Representation

2.3.3 Characteristics of Fuzzy Sets

The membership function is the crucial component of a fuzzy set, therefore all operations with fuzzy sets are defined through their membership functions, (Zimmermann, 1996). Following is a summarized introduction to the main characteristics of fuzzy sets, and the related definitions and operations.

The basic definition of a fuzzy set is that it is characterized by a membership function mapping the elements of a domain, space, or universe of discourse X to the unit interval [0,1], (Pedrycz and Gomide, 1998) that is

$$A : X \rightarrow [0,1] \qquad \text{……….(2.36)}$$

where:

A is the fuzzy set in universe of discourse **X**; and

X is the domain, or the universe of discourse.

The function in Equation (2.36) describes the membership function associated with a fuzzy set A. A fuzzy set is said to be normal *fuzzy set* if at least one of its elements has a membership value of one. A *convex fuzzy set* Z is the set in which for every real number a, b and c with a<b<c the following holds

$$Z(b) \geq \min(Z(a), Z(c)) \qquad \text{.........(2.37)}$$

where

$Z(\ )$ is the membership value; and

$\min(\ \ )$ is the minimum function.

This function may have different shapes and may be continuous or discrete, depending on the context in which it is used. Figure (2.7) shows three different types of continuous membership functions. Families of parameterized function such as the following triangular membership function can represent most of the common membership functions explicitly

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } x \leq a \\ \dfrac{x-a}{m-a}, & \text{if } x \in [a, m] \\ \dfrac{b-x}{b-m}, & \text{if } x \in [m, b] \\ 0, & \text{if } x \geq b \end{cases} \qquad \text{.........(2.38)}$$

where

m is the modal value; and

a, b are the lower and upper bounds of the non-zero values of the membership.

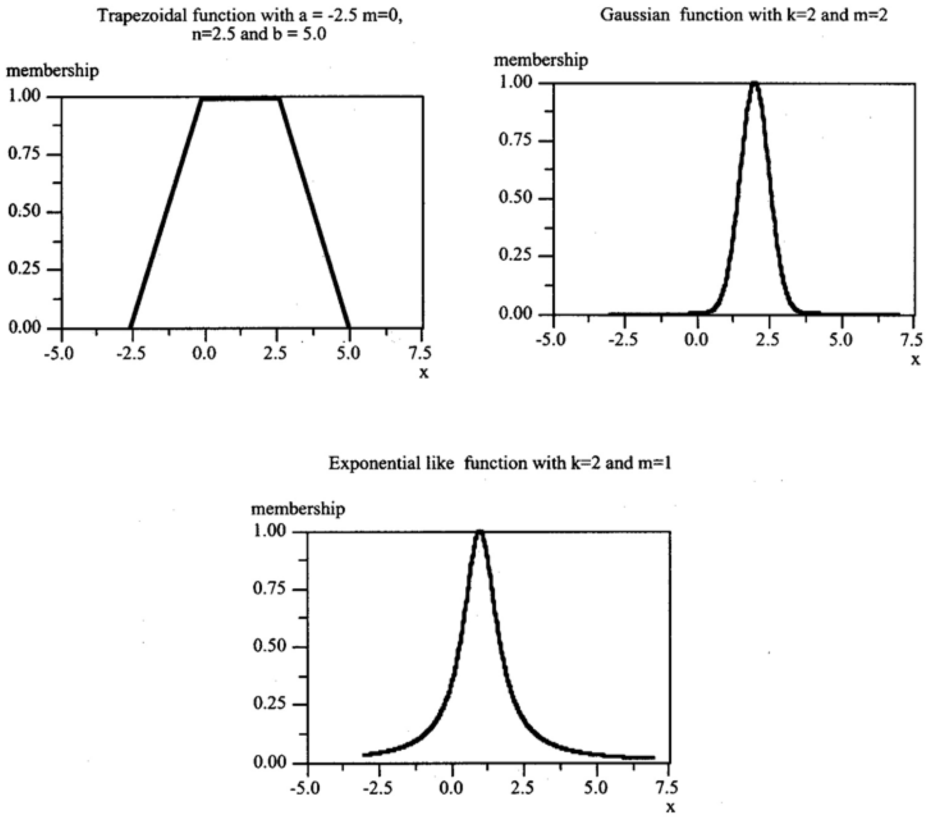


Figure (2.7) Trapezoidal, Gaussian, and Exponential Membership Functions (after Pedrycz and Gomide, 1998).

2.3.4 Fuzzy Numbers

Fuzzy numbers are special case of fuzzy sets, having the following properties, (Ganoulis, 1994):

(1) They are defined on the set of real numbers;

(2) Their membership functions reach maximum value, 1.0, i.e they are all normal fuzzy sets; and

(3) Their membership functions are unimodal consists of increasing and decreasing parts (convex fuzzy sets).

They are defined as follows

$$\widetilde{X} = \{(x, \mu_{\widetilde{X}}(x)) : x \in R; \mu_{\widetilde{X}}(x) \in [0,1]\} \qquad \text{.........(2.39)}$$

where:

$\widetilde{X}$ is the fuzzy number;

$\mu_{\widetilde{X}}(x)$ is the membership value of element *x* to the fuzzy number $\widetilde{X}$; and

R is the set of real numbers.

Credibility Level or α-Level Set

It is the ordinary set of all the elements belonging to the fuzzy number whose value of membership is α or higher, that is

$$X(\alpha) = \{x : \mu_{\widetilde{X}}(x) \geq \alpha; x \in R; \alpha \in [0,1]\} \qquad \text{.........(2.40)}$$

where

$X(\alpha)$ is the ordinary set at the α-level set; and

α is the credibility level.

Support of a Fuzzy Number

It is the ordinary set that is defined as follows

$$S(\widetilde{X}) = \widetilde{X}(0) = \{x : \mu_{\widetilde{X}}(x) > 0\} \qquad \text{.........}(2.41)$$

The fuzzy number support is the 0-level set and includes all the elements with the credibility level higher than 0. Figure (2.8) illustrates these definitions.

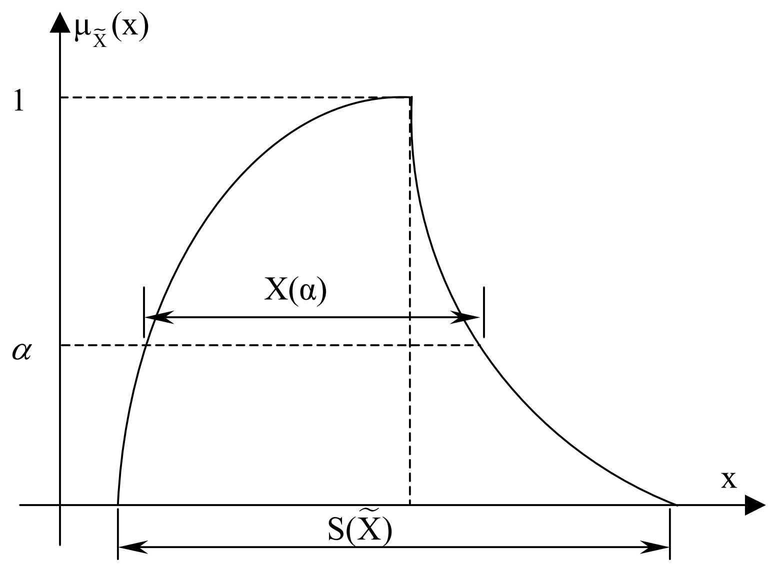


Figure (2.8) Credibility Level and Support of Fuzzy Set (after Ganoulis, 1994).

Set-Theoretic Operations for Fuzzy Sets

(i) Intersection

The membership function $\mu_{\widetilde{C}}(x)$ of the intersection $\widetilde{C} = \widetilde{A} \cap \widetilde{B}$ is defined by

$$\mu_{\tilde{C}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \quad x \in X \qquad \text{.........(2.42)}$$

where

$\mu_{\tilde{C}}(x)$ is the membership of the fuzzy intersection of $\tilde{A}$ and $\tilde{B}$;

$\min(\ )$ is the ordinary minimum operator;

$\mu_{\tilde{A}}(x)$ is the membership of fuzzy set $\tilde{A}$; and

$\mu_{\tilde{B}}(x)$ is the membership of fuzzy set $\tilde{B}$.

(ii) Union

The membership function $\mu_{\tilde{C}}(x)$ of the union $\tilde{C} = \tilde{A} \cup \tilde{B}$ is defined by

$$\mu_{\tilde{C}}(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \quad x \in X \qquad \text{.........(2.43)}$$

where

$\mu_{\tilde{C}}(x)$ is the membership of the fuzzy union of $\tilde{A}$ and $\tilde{B}$;

$\max(\ )$ is the ordinary maximum operator;

$\mu_{\tilde{A}}(x)$ is the membership of fuzzy set $\tilde{A}$; and

$\mu_{\tilde{B}}(x)$ is the membership of fuzzy set $\tilde{B}$.

(iii) Complement

The membership function $\mu_{\bar{\tilde{C}}}(x)$ of the complement of fuzzy set $\tilde{C}$ is defined by

$$\mu_{\bar{\tilde{C}}}(x) = 1 - \mu_{\tilde{C}}(x), \quad x \in X \qquad \ldots\ldots\ldots(2.44)$$

where

$\mu_{\bar{\tilde{C}}}(x)$ is the membership of the complement of fuzzy set $\tilde{C}$; and

$\mu_{\tilde{C}}(x)$ is the membership of fuzzy set $\tilde{C}$.

Figures (2.9a) and (2.9b), show the union and fuzzy union and intersection operators on fuzzy sets.

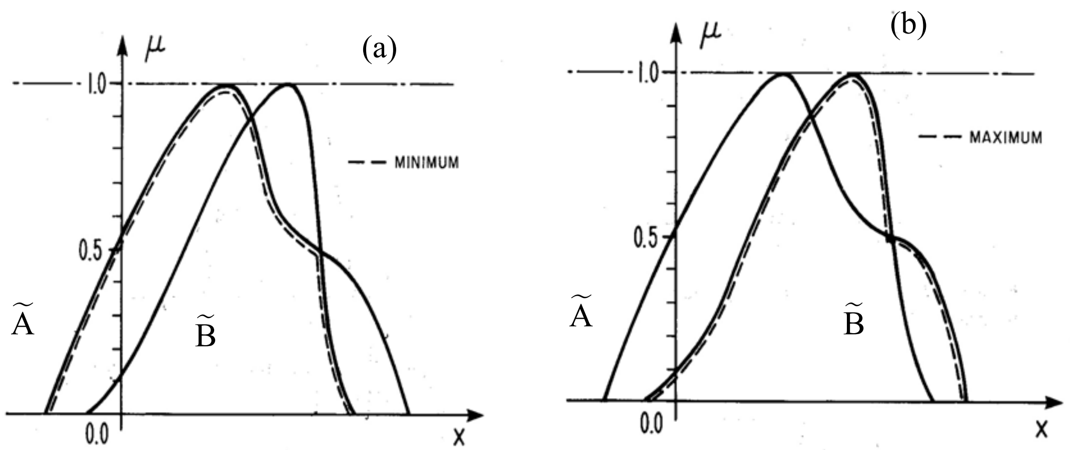


Figure (2.9) Fuzzy Intersection, Union and Complement (after Kaufmann and Gupta, 1985).

<u>(iv) AND –OR Operators</u>

Assuming that $\wedge$ denotes the fuzzy AND operation and $\vee$ denotes the fuzzy OR operation, the definitions for both operators are as follows, (Zimmermann, 1996)

$$\mu_{\tilde{A}\wedge\tilde{B}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \quad x \in X \qquad \ldots\ldots\ldots(2.45)$$

and

$$\mu_{\tilde{A}\vee\tilde{B}}(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \quad x \in X \qquad \text{..........(2.46)}$$

Fuzzy Arithmetic Operations on Fuzzy Numbers

At any α-level, the fuzzy number $\tilde{A}$ can be represented as follows

$$\tilde{A}(\alpha) = [a_1(\alpha), a_2(\alpha)] \qquad \text{..........(2.47)}$$

where

$\tilde{A}(\alpha)$ is the fuzzy number at α-level;

$a_1(\alpha)$ is the lower bound of the α-level interval; and

$a_2(\alpha)$ is the upper bound of the α-level interval.

As a result, the arithmetic operations on intervals of real numbers can be extended to the four main arithmetic operations for fuzzy numbers, i.e. addition (+), subtraction (-), multiplication (.), and division (/). The fuzzy operations of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are defined at any α-level cut as follows (Kaufmann and Gupta, 1985)

$$\tilde{A}(\alpha)\ (+)\ \tilde{B}(\alpha) = [a_1(\alpha) + b_1(\alpha)\ , a_2(\alpha) + b_2(\alpha)] \qquad \text{..........(2.48)}$$

$$\tilde{A}(\alpha)\ (-)\ \tilde{B}(\alpha) = [a_1(\alpha) - b_2(\alpha), a_2(\alpha) - b_1(\alpha)] \qquad \text{..........(2.49)}$$

$$\tilde{A}(\alpha)\ (.)\ \tilde{B}(\alpha) = [a_1(\alpha).b_1(\alpha)\ , a_2(\alpha).b_2(\alpha)] \qquad \text{..........(2.50)}$$

$$\widetilde{A}(\alpha)\ (/)\ \widetilde{B}(\alpha) = [a_1(\alpha)/b_1(\alpha), a_2(\alpha)/b_2(\alpha)] \quad \text{.........(2.51)}$$

Comparison Operations on Fuzzy Sets

The comparison of fuzzy sets can be performed using different methods. In this research, we will introduce the compatibility measure that might be useful in comparing notions represented by fuzzy numbers.

Possibility and Necessity Measures

The *possibility* measure quantifies the extent to which two fuzzy numbers overlap. It is defined as, (Pedrycz and Gomide, 1998)

$$\text{Poss}(\widetilde{A},\widetilde{B}) = \sup[\min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)\}], \quad x \in X \quad \text{.........(2.52)}$$

where:

$\text{Poss}(\widetilde{A},\widetilde{B})$ is the possibility measure of fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$;

sup[ ] is the least upper bound value, i.e. *supremum*; and

$\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)$ are the membership functions of the fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ respectively;

By virtue of the definition the possibility measure is a symmetrical measure, that is

$$\text{Poss}(\tilde{A},\tilde{B}) = \text{Poss}(\tilde{B},\tilde{A}) \qquad \text{……….(2.53)}$$

The n*ecessity* measure describes the degree to which certain fuzzy number is included in another fuzzy number. It is defined as, (Pedrycz and Gomide, 1998)

$$\text{Nec}(\tilde{A},\tilde{B}) = \inf[\max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}], \quad x \in X \qquad \text{……….(2.54)}$$

where:

$\text{Nec}(\tilde{A},\tilde{B})$ is the necessity measure of fuzzy numbers $\tilde{A}$ and $\tilde{B}$;

inf[ ] is the greatest lower bound value, i.e. *infimum*; and

$\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)$ are the membership functions of the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively;

The necessity measure is asymmetrical measure, that is

$$\text{Nec}(\tilde{A},\tilde{B}) \neq \text{Nec}(\tilde{B},\tilde{A}) \qquad \text{……….(2.55)}$$

Both measures hold the following relation

$$\text{Nec}(\tilde{A},\tilde{B}) + \text{Poss}(\overline{\tilde{A}},\tilde{B}) = 1 \qquad \text{……….(2.56)}$$

where:

$\mathrm{Nec}(\widetilde{A},\widetilde{B})$ is the necessity measure of fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$;

$\mathrm{Poss}(\overline{\widetilde{A}},\widetilde{B})$ is the possibility measure of fuzzy numbers $\overline{\widetilde{A}}$ and $\widetilde{B}$; and

$\overline{\widetilde{A}}$ is the fuzzy complement of fuzzy number $\widetilde{A}$.

2.3.5 Fuzzy Reliability Measure by Shrestha and Duckstein

Most of engineering reliability analyses rely on the use of the probabilistic approach. Both, supply and demand, are considered as random variables. The characteristics of supply and/or demand cannot always be measured precisely or treated as random variables. Therefore, the fuzzy representation of either one is examined. The reliability analysis is performed through the transformation of fuzzy imprecision into random uncertainty or use of the hybrid fuzzy-random representation. The case of both fuzzy supply and fuzzy demand is rarely addressed in the literature, (Shrestha and Duckstein, 1998).

Shrestha and Duckstein (1998) were the first to suggest a fuzzy reliability measure that can be used in the case of both, supply and demand, being fuzzy. The suggested measure uses margin of safety, as a criterion for the system failure, that is

$$\widetilde{M}(\alpha) = \widetilde{X}(\alpha) - \widetilde{Y}(\alpha); \qquad \forall \alpha \in [0,1] \qquad \text{.........}(2.57)$$

where;

$\widetilde{M}(\alpha)$ is the fuzzy margin of safety at α-level cut;

$\widetilde{X}(\alpha)$ is the fuzzy supply at α-level cut; and

$\widetilde{Y}(\alpha)$ is the fuzzy demand at α-level cut.

The fuzzy membership $\widetilde{M}(\alpha)$, from Equation (2.57), is considered the system failure surface. Failure is defined as the condition when demand $\widetilde{Y}(\alpha)$ exceeds supply $\widetilde{X}(\alpha)$ and consequently $\tilde{M} < 0$. Accordingly, they define the fuzzy reliability index, FR_e as

$$FR_e = \frac{\int_{\widetilde{M}>0} \mu_{\widetilde{M}}(m)\, dm}{\int_{\widetilde{M}} \mu_{\widetilde{M}}(m)\, dm} \qquad \text{.........(2.58)}$$

where:

$\mu_{\widetilde{M}}(m)$ is the membership function of the fuzzy failure surface.

The suggested fuzzy reliability index treats the membership function of the margin of safety as a probability density function. Figure (2.10) shows, the area under the $\mu_{\widetilde{M}}(m)$ membership function below 0, represented by $\int_{\widetilde{M}>0} \mu_{\widetilde{M}}(m)\, dm$, as the possible failure area. The complete failure event is the case when the whole area of the $\mu_{\widetilde{M}}(m)$ membership function falls below 0. The fuzzy reliability measure in Equation (2.58) is assumed to satisfy the following assumptions:

- Its maximum value is unity, which is the case when the supply exceeds the demand and both membership functions do not overlap at any α level.
- Its minimum value is zero, which is the case when the demand exceeds the supply and their membership functions do not overlap at any α level.
- It provides a consistent ranking of the system safety, monotonically increasing towards 1 with the increase in system safety.

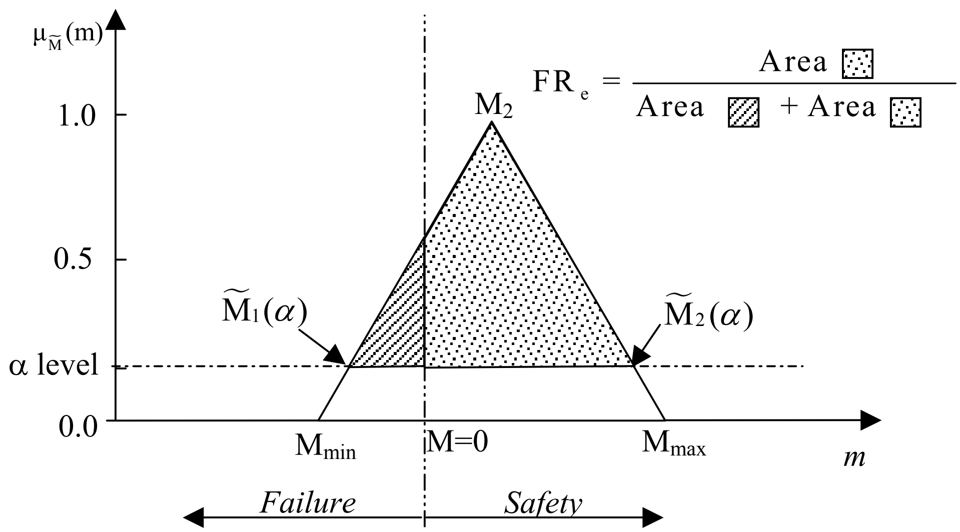


Figure (2.10) α Level Fuzzy Reliability Measure (after Shrestha and Duckstein, 1998).

Multi Component Systems

An overall system fuzzy reliability index is also suggested for different system configurations: serial; parallel; and combined. For a serial system, the minimum fuzzy function is used, where the failure of the system occurs if any of its elements fails, that is

$$\widetilde{M}_S = \min_n(\widetilde{M}_1, \widetilde{M}_2, \ldots \widetilde{M}_n) \qquad \ldots\ldots\ldots(2.59)$$

where;

$\widetilde{M}_S$ is the fuzzy margin of safety of the system;

$\widetilde{M}_1, \widetilde{M}_2, ... \widetilde{M}_n$ are the fuzzy margin of safety for the serial components; and

n is the total number of components.

System failure, for a parallel system, occurs at the failure of all the components. In this case, the maximum fuzzy function is used to calculate the system margin of safety as follows

$$\widetilde{M}_S = \max_{n}(\widetilde{M}_1, \widetilde{M}_2, ... \widetilde{M}_n) \qquad \text{.........(2.60)}$$

where;

$\widetilde{M}_S$ is the fuzzy margin of safety of the system;

$\widetilde{M}_1, \widetilde{M}_2, ... \widetilde{M}_n$ are the fuzzy margin of safety for the parallel components; and

n is the total number of components.

Other combinations of system configurations are dealt with as different combinations of serial or parallel subsystems. Therefore, the reliability index for each subsystem is calculated, using either Equations (2.59) or (2.60), independently and the overall system reliability index is obtained, according to the connection configuration.

2.3.6 Utility of Fuzzy Reliability Measure by Shrestha and Duckstein

The suggested fuzzy reliability measure by Shrestha and Duckstein (1998) is evaluated using two simple hypothetical cases. As shown in Figure (2.11), system A consists of a pump, single pipeline and a reservoir, while system B consists of a pump, two parallel pipelines and a reservoir. Introduction of the two parallel pipelines in system B increases system redundancy that should result in higher system reliability. Therefore, the reliability measure value should reflect the difference between the two systems. It has to be noted that both systems are exposed to the same demand requirement and have the same supply capacity.

Triangular and trapezoidal membership functions are used to investigate the sensitivity of the reliability measure to the shape of the membership function. Different elements of each system, i.e. pump, pipes, and reservoir, are serially connected. Therefore the overall system reliability depends on the reliability of the weakest element.

Assuming that the pipes reliability in both systems controls the overall system reliability, two different scenarios are suggested for system B: (i) both pipes have the same supply capacity, and (ii) one of the pipes has a supply capacity two times larger than the other pipe. The sum of the two pipes supply capacities, in both scenarios, is equal to the supply capacity of the pipe in system A.

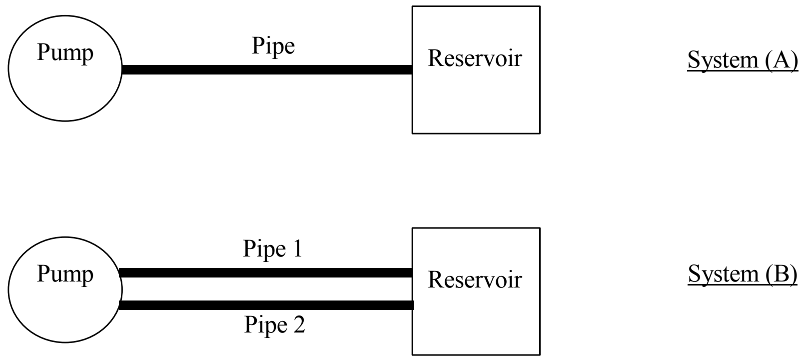


Figure (2.11) Schematic Representation of the Hypothetical Case Study.

Table (2.1) summarizes the four different cases tested for both systems:

- Case (I) triangular fuzzy membership representing fuzzy supply and fuzzy demand for both systems. System supply and demand are distributed between the two pipes in system B with the ratio 1:1 (equal distribution).
- Case (II) triangular fuzzy membership representing fuzzy supply and fuzzy demand for both systems. System supply and demand are distributed between the two pipes in system B with the ratio 1:2 (non-equal distribution).
- Case (III) trapezoidal fuzzy membership representing fuzzy supply and fuzzy demand for both systems. System supply and demand are equally distributed between the two pipes in system B.
- Case (IV) trapezoidal fuzzy membership representing fuzzy supply and fuzzy demand for both systems. System supply and demand are non-equally distributed between the two pipes in system B.

The results in Table (2.2) show the discrepancy in reliability of the two systems. For example, cases (I) and (III), yielded the same reliability index values of 0.644 and 0.571, respectively, for both systems A and B.

The effect of the shape of the membership function is shown in case (II) where system B reliability index value is 1.25 higher than the reliability of system A, 0.803 and 0.644, respectively. In case (IV) system B reliability index value is 1.27 higher than system A, 0.726 and 0.571, respectively.

Table (2.1) Summary of Test Cases

Case	**Case Description**	**System**	**Supply Capacity (m^3/Sec.)**	**Demand Requirement (m^3/Sec.)**
(I)	Triangular fuzzy membership with equal distribution between pipes in system B	A	(0.0,3.0,6.0)	(1.0,2.0,4.0)
		B	(0.0,1.5,3.0) (0.0.1.5,3.0)	(0.5,1.0,2.0) (0.5,1.0,2.0)
(II)	Triangular fuzzy membership with non-equal distribution between pipes in system B	A	(0.0,3.0,6.0)	(1.0,2.0,4.0)
		B	(0.0,1.0,2.0) (0.0,2.0,4.0)	(0.3,0.7,1.3) (0.7,1.3,2.7)
(III)	Trapezoidal fuzzy membership with equal distribution between pipes in system B	A	(0.0,1.0,5.0,6.0)	(1.0,2.0,3.0,4.0)
		B	(0.0,0.5,2.5,3.0) (0.0,0.5,2.5,3.0)	(0.5,1.0,1.5,2.0) (0.5,1.0,1.5,2.0)
(IV)	Trapezoidal fuzzy membership with non-equal distribution between pipes in system B	A	(0.0,1.0,5.0,6.0)	(1.0,2.0,3.0,4.0)
		B	(0.0,0.3,1.7,2.0) (0.0,0.7,3.3,4.0)	(0.3,0.7,1.0,1.3) (0.7,1.3,2.0,2.7)

The inability of the reliability index to reflect the difference in system reliability in some cases and inconsistency in others may be contributed to the following:

- Using the maximum operator to combine membership functions of parallel configuration overlooks the increase in reliability introduced by redundancy. This is apparent from the case of pipelines with equal loads. The membership function of system's margin of safety is identical to the membership function of individual pipeline, as shown in Figure (2.12)
- Using the membership function of the margin of safety as a failure surface is the other source of inconsistency. Representing the margin of safety by a fuzzy membership function implies that each value of the universe of discourse has a different grade of membership. This approach is different from the probabilistic approach, which uses the probability density function as a failure surface, where all values of the universe of discourse have the same membership value (value of 1).

Results of the simple analyses presented her show that the fuzzy reliability measure of Shrestha and Duckstein is producing inconsistent results. Its utility is therefore limited for the application in the water supply reliability analysis. There is a need for the new fuzzy reliability measure formulation that will be able to resolve inconsistencies observed in this research.

Table (2.2) Computed Fuzzy Reliability

Case	Case Description	System	Fuzzy Reliability	Result
(I)	Triangular fuzzy membership with equal distribution between pipes in system B	A	0.644	The measure *Failed* to indicate difference in reliability
		B	0.644	
(II)	Triangular fuzzy membership with non-equal distribution between pipes in system B	A	0.644	The measure *Indicated* the difference in reliability
		B	0.803	
(III)	Trapezoidal fuzzy membership with equal distribution between pipes in system B	A	0.571	The measure *Failed* to indicate difference in reliability
		B	0.571	
(IV)	Trapezoidal fuzzy membership with non-equal distribution between pipes in system B	A	0.571	The measure *Indicated* the difference in reliability
		B	0.726	

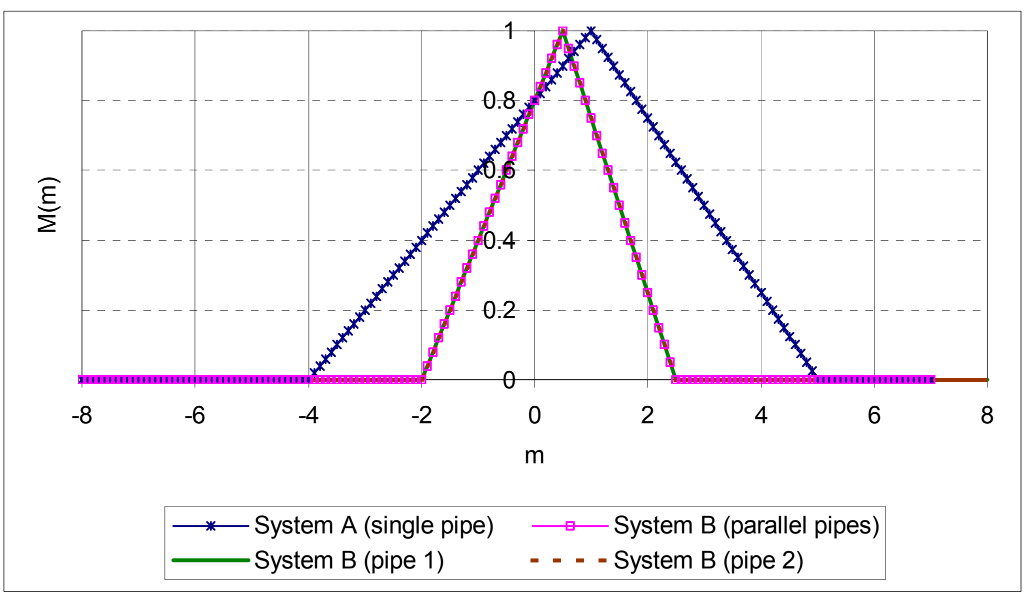


Figure (2.12) Membership Functions of Margin of Safety for System A and B in Case (I)

3 NEW FUZZY PERFORMANCE INDICES FOR ENGINEERING SYSTEMS

3.1 Introduction

Application of probabilistic reliability analysis is invariably related to the availability of data that can be used to determine probability distribution functions to be used, objectively or subjectively. Data insufficiency is a well-known problem in almost all engineering problems and is dealt within the probabilistic approach by using the Bayesian approach or the subjective probability estimation.

Bayesian method is one of the rigorous ways of dealing with uncertainty, especially when combined with multi-attribute utility theory to incorporate the variability in system performance and uncertainty in system parameters. The difficulty in the development of the utility function and its ability to capture the priorities of all interest groups in decision-making process are the main drawbacks of this method, (Hashimoto et al, 1982a).

Subjective probability, on the other hand, is a description of state of information (or state of uncertainty) where the degree of information is interpreted as a degree of belief, related to the personal state of information, (Spizzichino, 2001). To be valid, the subjective probability approach (i) should reflect the belief of the assessor of the uncertainty, and (ii) should be consistent with the basic probability axioms.

Decision-making processes involve multi-disciplinary teams from all fields and decision-makers might not be able to match these requirements. People's judgment and believes are rarely expressed using mathematical tools. They prefer to use what is known as heuristic, or simple mental strategies, to express uncertainty. These heuristic strategies are usually successful tools for dealing with the uncertainty. However, they may introduce bias or inconsistencies with the mathematical probability principles, (Vick, 2002).

Fuzzy set theory was intentionally developed to try to capture people judgmental believes, or as mentioned before, the uncertainty that is caused by the lack of knowledge. Relative to the probability theory, it has some degree of freedom with respect to aggregation operators, types of fuzzy sets (membership functions), etc, which enables the adaptability to different contexts. During the last twenty years, fuzzy set theory and fuzzy logic contributed successfully to the technological development in different application areas such as mathematics, algorithms, standard models, and real-world problems of different kinds, (Zimmermann, 1996). This study explores the utility of the fuzzy set theory in the field of engineering system reliability analysis.

3.2 New Fuzzy Performance Indices

3.2.1 Definitions

Failure

The main concern of any engineering system planner, designer, and manager is the assurance of system performance within the limitations of all exogenous factors, such as economy, environment, and society's tradeoffs between competing systems. Therefore, many system performance indices and figures of merit are developed to enable the integration of different aspects of system performance into a multi-objective framework, (Hashimoto et al, 1982a).

The calculation of performance indices depends on the exact definition of unsatisfactory system performance. Uncertainty in determining system supply (resistance), demand (load), and the accepted unsatisfactory performance threshold, makes it hard to sharply define the failure event. Figure (3.1) depicts a typical system performance (supply time series), with the constant demand during the operation horizon. According to the classical definition, the failure state is the state when supply falls below the demand, margin of safety $M<0.0$ or safety factor $\Theta<1.0$, shown in Figure (3.1) by the dashed horizontal line.

Engineering systems occasionally fail to perform their intended function to certain extent. For example, the available supply from different sources in the case of water supply system is highly variable. The actual demand may also fluctuate significantly.

Consequently, in the design of water supply systems, certain periods of water shortage have to be accepted. Hence, this crisp identification of failure is neither realistic nor practical. Acceptance of partial failure is a more realistic approach. A region of acceptable system failure can be introduced using the solid horizontal line, as shown in Figure (3.1)

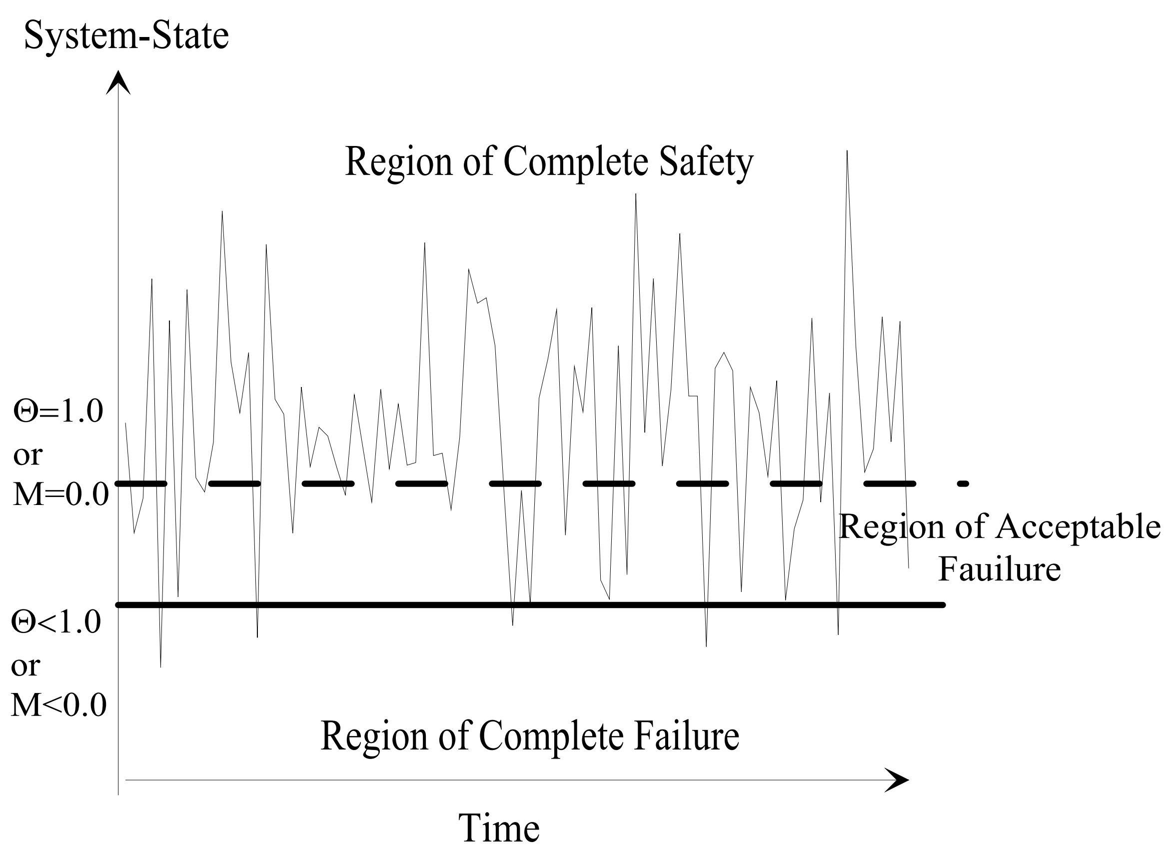


Figure (3.1) Variable System Performance

The boundary of the acceptable failure region is ambiguous and varies from one decision maker to the other depending on the personal perception of risk. Therefore, this boundary cannot be determined precisely. Fuzzy sets, by definition, are capable of representing the notion of imprecision better than the ordinary sets, used in the probabilistic approach. As a result, the acceptable level of performance can be represented as a fuzzy membership function, that is

$$\widetilde{M}(m) = \begin{cases} 0, & \text{if } m \le m_1 \\ \varphi(m), & \text{if } m \in [m_1, m_2] \\ 1, & \text{if } m \ge m_2 \end{cases}$$

or ………(3.1)

$$\widetilde{\Theta}(\theta) = \begin{cases} 0, & \text{if } \theta \le \theta_1 \\ \varphi(\theta), & \text{if } \theta \in [\theta_1, \theta_2] \\ 1, & \text{if } \theta \ge \theta_2 \end{cases}$$

where:

$\widetilde{M}$ is the fuzzy membership function of margin of safety;

$\varphi(m)$ and $\varphi(\theta)$ are functional relationships representing the subjective view of the acceptable risk;

m_1, m_2 are the lower and upper bounds of the acceptable failure region, respectively;

$\widetilde{\Theta}$ is the fuzzy membership function of factor of safety; and

θ_1, θ_2 are the lower and upper bounds of the acceptable failure region, respectively.

Figure (3.2) is a graphical representation of the notion presented in Equation (3.1). The lower and upper bounds of the acceptable failure region are introduced in Equation (3.1) by m_1 (or θ_1) and m_2 (or θ_2). The value of the margin of safety (or factor of safety) below m_1 (or θ_1) is definitely unacceptable. Therefore, its membership function value is zero.

On the other hand, value of the margin of safety (or factor of safety) above m_2 (or θ_2) is definitely acceptable. It certainly belongs to the acceptable failure region. Consequently, its membership value is one. The in-between values have varying membership values depending on the subjective opinion of the decision maker. Applying different functional forms for $\varphi(m)$ (or $\varphi(\theta)$) reflects this subjective view.

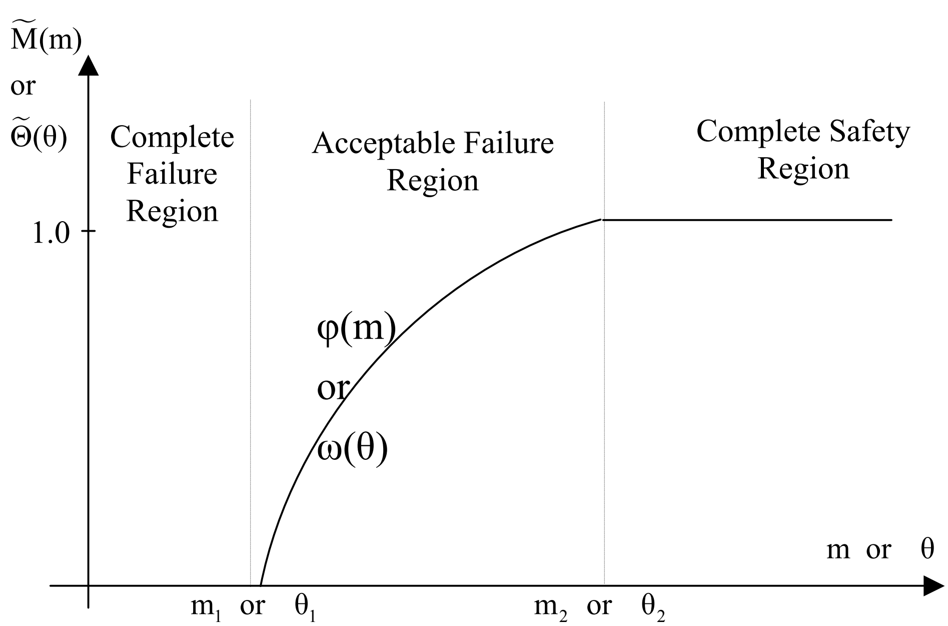


Figure (3.2) Fuzzy Representation of Acceptable Failure Region.

High system reliability is reflected through the use of high values of margin of safety (or factor of safety), i.e. high values for both m_1 and m_2 (or θ_1 and θ_2). The difference between m_1 and m_2 (or θ_1 and θ_2) inversely affects the system reliability, that is the higher the difference the lower the reliability. Therefore, the reliability reflected by the defined acceptable level of performance could be quantified in the following way

$$LR = \frac{m_1 \times m_2}{m_2 - m_1}$$

or ……….(3.2)

$$LR = \frac{\theta_1 \times \theta_2}{\theta_2 - \theta_1}$$

where:

LR is the reliability measure of the acceptable level of performance.

The freedom given by this definition of failure, through the choice of the lower bound, upper bound, and the function $\varphi(m)$ (or $\varphi(\theta)$) facilitates the introduction of the ambiguity of risk acceptance exhibited by different decision-makers. This approach, also, provides an easy and comprehensive tool for risk communication. That has been acknowledged as the major problem in the application of probabilistic approach.

Fuzzy System-State

Complexity of water supply system networks and variability in supply and demand require performing planning and design under conditions of uncertainty. In addition, system supply and demand are affected by many uncontrollable factors, such as the exposure of buried water supply mains to highly variable temperature, pressure and stress.

System supply and demand can be represented in fuzzy form to capture the uncertainty involved in the system performance. Determination of the membership function of the supply and demand is a strait forward procedure that can be performed easily, even in the case of limited data availability. Fuzzy arithmetic can be used to calculate the resulting margin of safety (or factor of safety) membership function as a representation of the system state at any time

$$\widetilde{M} = \widetilde{X}(-)\widetilde{Y}$$

and ……….(3.3)

$$\widetilde{\Theta} = \widetilde{X}(/)\widetilde{Y}$$

where;

$\widetilde{M}$ is the fuzzy margin of safety;

$\widetilde{X}$ is the fuzzy supply capacity;

$\widetilde{Y}$ is the fuzzy demand requirement;

$(-)$ is the fuzzy subtraction operator;

$(/)$ is the fuzzy division operator; and

$\widetilde{\Theta}$ is the fuzzy factor of safety.

Compatibility

The primary intent in comparing two fuzzy membership functions is to express the extent to which the two fuzzy sets match. Several classes of methods are available, none of which can be described as the best method, (Pedrycz and Gomide, 1998). The reliability assessment, presented in this study, involves a comparative analysis of the system-state membership function and the predefined acceptable level of performance membership function. Therefore, the compliance of two fuzzy membership functions can be quantified using the fuzzy compatibility measures.

Possibility and necessity quantify the compatibility of two fuzzy numbers. However, in some cases as in Figure (3.3), high possibility and necessity values do not reflect clearly the notion of compliance between the system-state membership function (margin of safety or factor of safety) and the acceptable level of performance membership function. As shown in Figure (3.3), two system-state functions, A and B, have the same possibility and necessity values. However, system-state A has larger overlap with the performance membership function than that the system-state B (shaded area in Figure (3.3)).

The overlap area between the two membership functions, as a fraction of the total area of the system-state expresses the compliance notion better than the possibility and necessity measures, that is

$$\text{Compliance} = \frac{\text{overlap area between system - state and performance level}}{\text{total area of system - state function}} \quad \text{.........(3.4)}$$

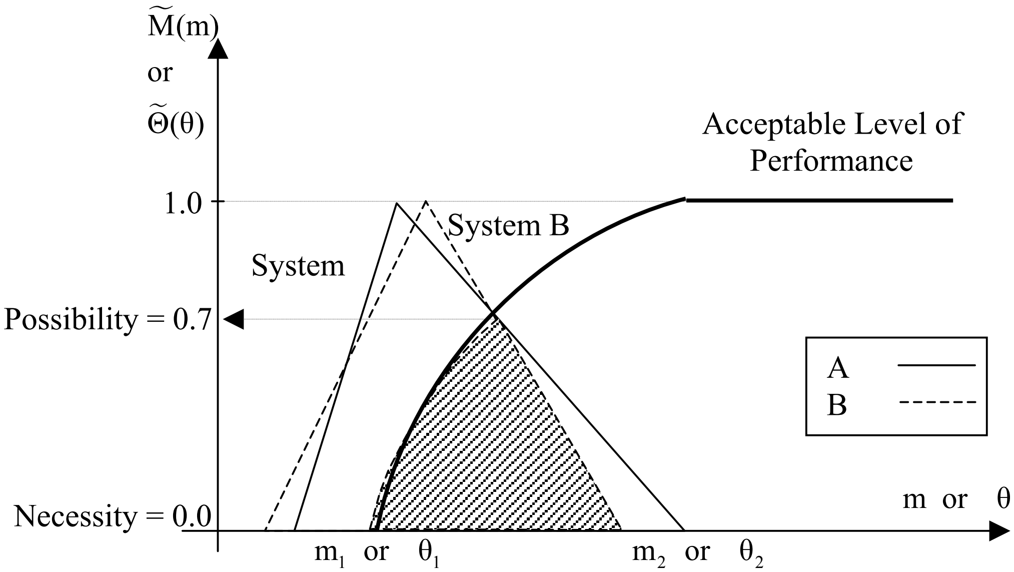


Figure (3.3) Compliance Between System-State and Acceptable Level of Performance.

Figure (3.4) depicts two different compliance cases. The first case represents the case of complete compliance, as accepted level of performance completely overlaps with the system-state. The second case is a case of partial compliance.

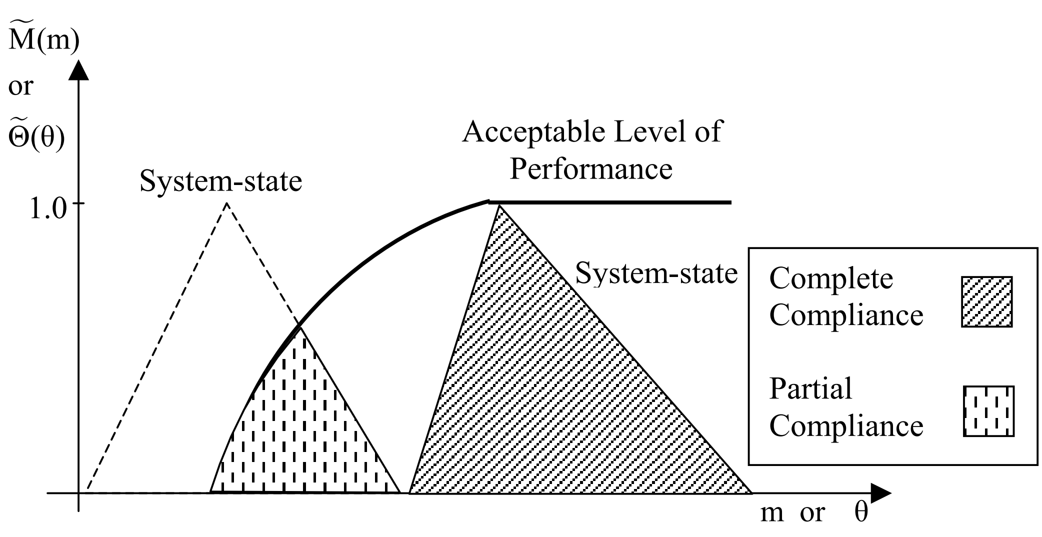


Figure (3.4) Two Compliance Cases.

Overlap of high significance area (area with high membership values) is preferable to overlap in low significance area, as shown in Figure (3.5). Therefore, the compliance measure should take into account the weighted area approach, (Verma and Knezevic, 1996).

Assume that a system-state is represented by the triangular membership function $\tilde{S}(u)$ defined on the universe of discourse U, as shown in Figure (3.6)

$$\tilde{S}(u) = \begin{cases} 0, & \text{if } u \le u_1 \\ \dfrac{u - u_1}{u_2 - u_1}, & \text{if } u \in [u_1, u_2] \\ \dfrac{u_3 - u}{u_3 - u_2}, & \text{if } u \in [u_2, u_3] \\ 0, & \text{if } u \ge u_3 \end{cases} \qquad \text{.........(3.5)}$$

where:

$\tilde{S}(u)$ is the system-state membership function;

u_2 is the modal value; and

u_1, u_3 are the lower and upper bounds of the non-zero values of the membership.

At any given α-level value of $\widetilde{S_\alpha}(u)$ the left and right values of the universe of discourse, U, variables are respectively

$$u_{l1} = u_1 + (u_2 - u_1)\widetilde{S_\alpha}(u)$$

and ……….(3.6)

$$u_{r1} = u_3 - (u_3 - u_2)\widetilde{S_\alpha}(u)$$

where:

$\widetilde{S_\alpha}(u)$ is the given system-state membership value;

u_{l1} is the first left (lower) universe of discourse variable value; and

u_{r1} is the first right (upper) universe of discourse variable value.

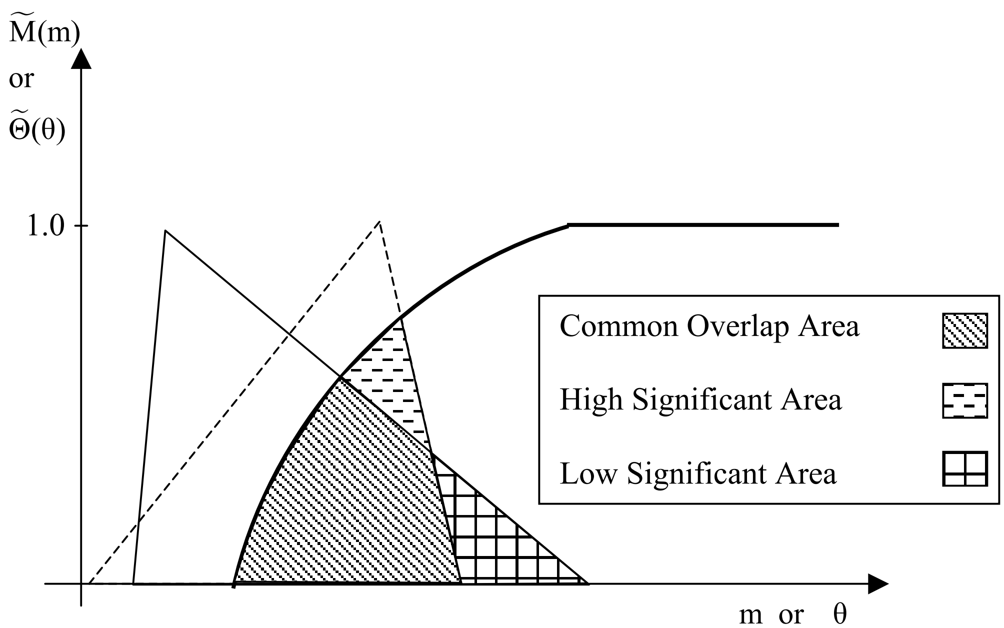


Figure (3.5) Overlap Analysis

At an incremental increase of ds, left and right values of the universe variables are respectively

$$u_{l2} = u_1 + (u_2 - u_1)(\widetilde{S_\alpha}(u) + ds)$$

and ……….(3.7)

$$u_{r2} = u_3 - (u_3 - u_2)(\widetilde{S_\alpha}(u) + ds)$$

where:

$\widetilde{S_\alpha}(u) + ds$ is the given system-state membership value;

u_{l2} is the second left (lower) universe variable value; and

u_{r2} is the second right (upper) universe variable value.

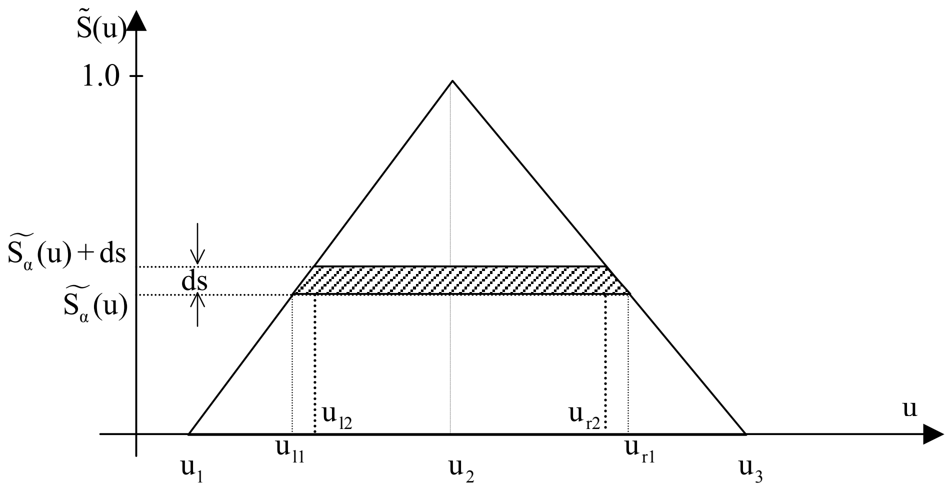


Figure (3.6) System-State Membership Function

The incremental area can be calculated as follows

$$dA = \frac{(u_{r1} - u_{l1}) + (u_{r2} - u_{l2})}{2} ds$$
$$= (u_3 - u_1)(1 - \widetilde{S_\alpha}(u) - \frac{ds}{2})ds$$
……….(3.8)

The weight of this area is the average value of the membership function, that is

$$\text{weight} = \frac{\widetilde{S}_\alpha(u) + \left(\widetilde{S}_\alpha(u) + ds\right)}{2} = \widetilde{S}_\alpha(u) + \frac{ds}{2} \qquad \text{.........(3.9)}$$

As a result, the weighted area equals

$$dA_w = \left[(u_3 - u_1)(1 - \widetilde{S}_\alpha(u) - \frac{ds}{2})ds\right]\left[\widetilde{S}_\alpha(u) + \frac{ds}{2}\right] \qquad \text{.........(3.10)}$$

Integration of equation (3.10) over the values of the membership function, from 0 to unity, results in the weighted area of the system-state.

$$\text{Weighted area of system-state function} = \int_S dA_w = \int_0^1 \left[(u_3 - u_1)(1 - \widetilde{S}_o(u) - \frac{ds}{2})ds\right]\left[\widetilde{S}_o(u) + \frac{ds}{2}\right] \text{.......(3.11)}$$

Performing a similar approach the weighted area of overlap can be calculated. Hence, the compatibility measure can be calculated using

$$\text{Compatibility Measure (CM)} = \frac{\text{Weighted overlap area}}{\text{Weighted area of system-state function}} \qquad \text{.........(3.12)}$$

3.2.2 Reliability Index

Reliability and vulnerability indices are used to provide a complete description of the system performance in case of failure and the magnitude of failure event, respectively. Determination of an acceptable level of performance in a fuzzy form implicitly specifies the anticipated performance in case of failure and the expected severity of failure.

Introduction of the lower and upper bounds (m_1 and m_2 (or θ_1 and θ_2) in Equation (3.1)) to the predefined acceptable level of performance limits the amount of anticipated deficit. Systems that are highly compatible with this acceptable level of performance would yield a similar performance. The magnitude of failure event is expressed by its maximum value (m_2 or θ_2) and range ($[m_1, m_2]$ or $[\theta_1, \theta_2]$). Therefore, Defining several acceptable levels of performance could be used to introduce the different views of decision-makers to the system reliability problem.

The comparison between fuzzy system-state membership function and predefined fuzzy acceptable level of performance membership function provides the information about the system reliability and vulnerability in the same time. The comparison is based on the closeness of the system-state to the predefined acceptable level of performance. The measure of closeness is expressed by the compatibility measure suggested in Equation (3.12).

For example, lets define three different levels of acceptable performance to be; (i) highly satisfactory level, (ii) satisfactory level, and (iii) risky level, as in Figure (3.7). Assume that a fuzzy triangular number represents a system-state membership function. The comparison indicates that the actual system state is to large extent contained in the risky level of acceptable performance (shaded area in Figure (3.6)). As a result, the system is considered risky and has a low reliability and high vulnerability. Thus, the suggested reliability index

$$\text{Reliability Index} = \frac{\max\limits_{i \in K}\left\{CM_1, CM_2, ...CM_i\right\} \times LR_{max}}{\max\limits_{i \in K}\left\{LR_1, LR_2, ...LR_i\right\}} \qquad \text{……….(3.13)}$$

where:

LR_{max} is the reliability measure of acceptable level of performance with which the system-state has the maximum compatibility value(*CM*);

LR_i is the reliability measure of the i-th acceptable level of performance;

CM_i is the compatibility measure for system-state with the i-th acceptable level of performance; and

K is the total number of defined acceptable levels of performance.

The reliability index is normalized to attain a maximum value of 1.0, by the introduction of the value $\max\limits_{i \in K}\left\{LR_1, LR_2, ...LR_i\right\}$ as the maximum achievable reliability.

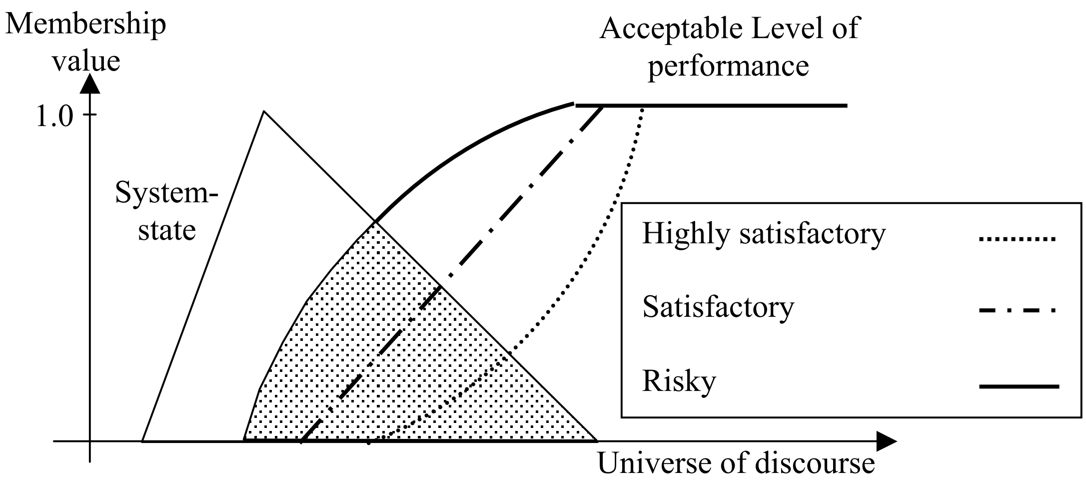


Figure (3.7) Reliability Index Based on the Compatibility Measure.

3.2.3 Robustness Index

Robustness is a measure of system performance that is concerned with the ability of the system to adapt to a wide range of possible demand conditions, in the future, at little additional cost (Hashimoto et al, 1982b). The fuzzy form of change in future conditions can be reflected through the redefinition of the acceptable level of performance and, also, in the change of the system-state membership function. As a result, the change in the compatibility measure *(CM)* provides an indication on the system robustness, that is

$$\text{Robustness Index} = \frac{1}{CM_1 - CM_2} \qquad \text{.........(3.14)}$$

where:

CM_1 is the compatibility measure before the change in conditions; and

CM_2 is the reliability after the change in conditions.

From Equation (3.14), the higher the change in the reliability the lower the value of the robustness index. Therefore, the high robustness index values reflect the better system adaptability to new conditions.

3.2.4 Resilience

Time of recovery from the failure state can be represented by a fuzzy set. For each type of failure the system might have a different recovery time, as shown in Figure (3.8). Therefore, a series of fuzzy sets, each for certain type of failure, can be developed for the system. Then the maximum recovery time can be used as representation of the system recovery time as follows, (Kaufmann and Gupta, 1985)

$$\tilde{T}(\alpha) = \left(\max_{j \in J}[t_{1_1}(\alpha), t_{1_2}(\alpha), \ldots\ldots., t_{1_J}(\alpha)], \max_{j \in J}[t_{2_1}(\alpha), t_{2_2}(\alpha), \ldots\ldots., t_{2_J}(\alpha)] \right) \qquad \ldots\ldots\ldots(3.15)$$

where:

$\tilde{T}(\alpha)$ is the system fuzzy maximum recovery time at α -level;

$t_{1_J}(\alpha)$ is the lower bound of the j-th recovery time α -level;

$t_{2_J}(\alpha)$ is the upper bound of the j-th recovery time α -level; and

J is total number of fuzzy recovery times.

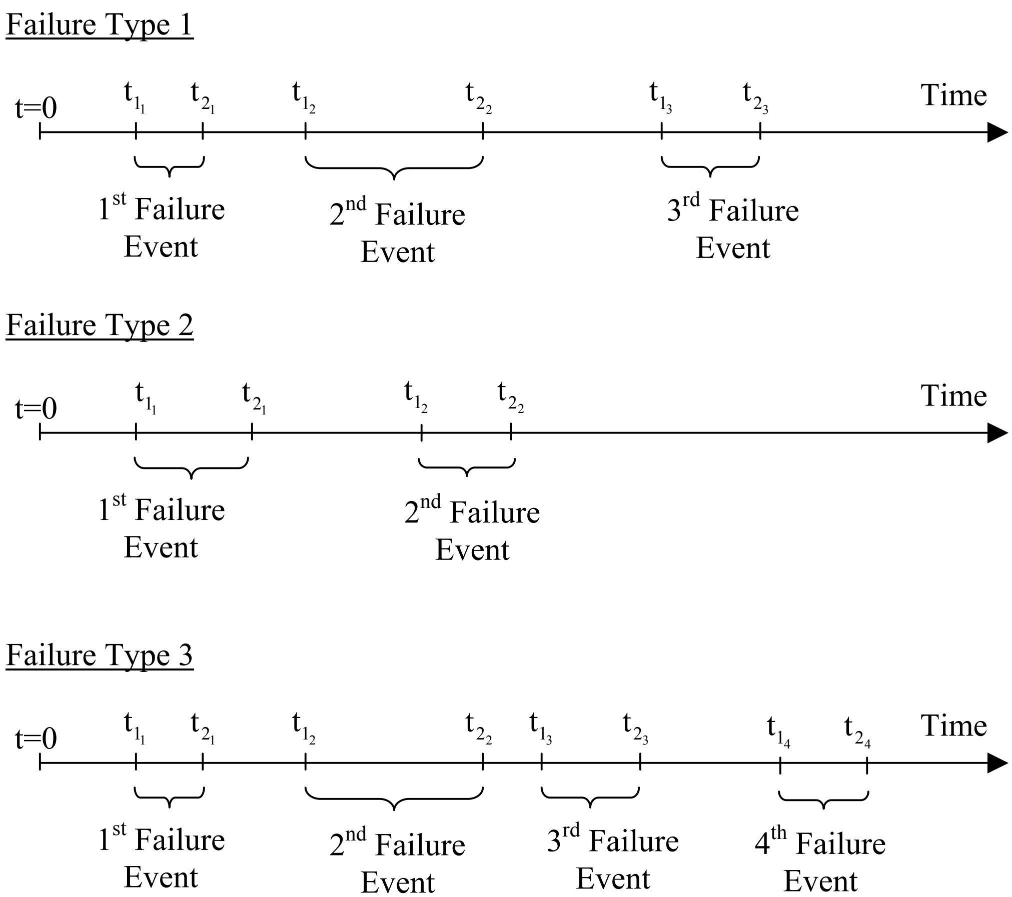


Figure (3.8) Recovery Times for Different Types of Failure.

The center of gravity of the maximum fuzzy set can be used as a real number representation. Therefore, the system resilience can be obtained as the inverse of the value of the center of gravity, (Klir et al, 1997)

$$\text{Resilience Index} = \left[\frac{\int_{t_1}^{t_2} t\, \tilde{T}(t)\, dt}{\int_{t_1}^{t_2} \tilde{T}(t)\, dt} \right]^{-1} \qquad \text{……….(3.16)}$$

where;

$\widetilde{T}(t)$ is the system fuzzy maximum recovery time;

t_1 is the lower bound of the support of the system recovery time (as defined by Equation (2.41)); and

t_2 is the upper bound of the support of the system recovery time (as defined by Equation (2.41)).

The inverse operation is useful to reflect the relation between the value of the recovery time and the resilience. The higher the recovery time the lower system's ability to recover fast from the failure and consequently the lower resilience.

3.2.5 Multi-Component Systems

System reliability assessment relies on the comparison between a system-state membership function and the predefined acceptable level(s) of performance. Multi-component systems have several system-state memberships representing the system-state of each component. Aggregation of these memberships will result in a system-state membership function for the whole-system. The resulting membership is a representative of the whole system-state membership that can be used in the comparison.

(i) Aggregation of System-State Functions

The main configurations of multi-component systems are; (i) serial, (ii) parallel, and (iii) combined. For each component, a fuzzy membership function representing the component's state can be calculated based on the component's demand and supply. The overall system state will be calculated depending on the system configuration.

(a) Serial Configuration

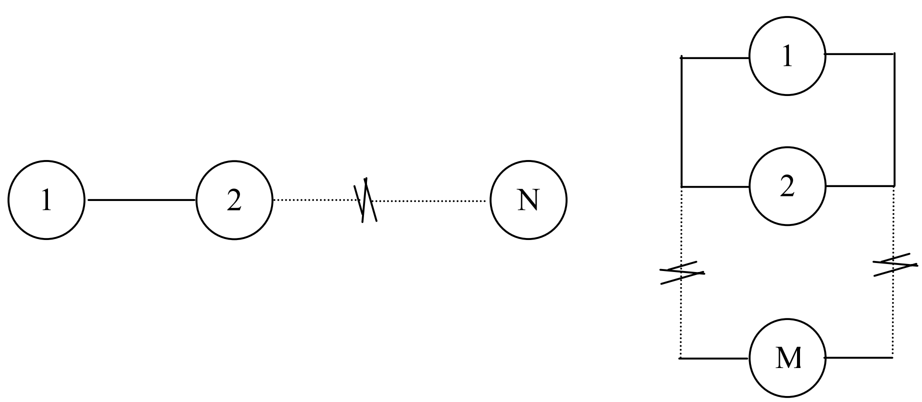
Assume that a serial configuration system is composed of N of components, as shown in Figure (3.9a). The *n*-th component has a state membership function $\tilde{S}_n(u)$, defined on the universe of discourse *U*. The weakest component, in terms of system-state, controls the whole system-state or causes the failure of the whole system. Therefore, the system-state can be calculated as follows

$$\tilde{S}(u) = \min_{N}\left(\tilde{S}_1, \tilde{S}_2, \ldots\ldots\ldots, \tilde{S}_N\right) \qquad \ldots\ldots\ldots(3.17)$$

where:

$\tilde{S}(u)$ is the whole system-state; and

$\left(\tilde{S}_1, \tilde{S}_2, \ldots\ldots\ldots, \tilde{S}_N\right)$ component system-states.



(a) (b)

Figure (3.9) Serial and Parallel System Configurations

(B) Parallel Configuration

A parallel system configuration is composed of M of components, as shown in Figure (3.9b). The m-th component has a state membership function $\tilde{S}_m(u)$, defined on the universe of discourse U. All the components' states contribute to the system-state. Failure of the system occurs if all components of the system fail. Hence, the system-state can be calculated as follows

$$\tilde{S}(u) = \sum_{m=1}^{M} \tilde{S}_m(u) \qquad \text{.........(3.18)}$$

where:

$\tilde{S}_m(u)$ is the m-th component system-state; and

M is the total number of parallel components.

(c) Combined Configuration

Combined systems are systems with parallel and serial subsystems. The system-state in this case can be calculated by calculating subsystems-states according to Equations (3.17) and (3.18). The whole system-state is then calculated by combining the subsystems-states using either equation.

(ii) Aggregation of Recovery Time Membership Functions

Aggregation of recovery time membership functions is different from the aggregation of system-state membership functions. System-state membership function determines the performance (or state) of the system, in both fussy satisfactory and unsatisfactory. Therefore, aggregation is based on the contribution of each component to the whole system state. Recovery time function, on the other hand, represents the system failure. Hence, aggregation of these membership functions should be different from the aggregation of system-state membership functions.

For serial configuration system composed of N components, the n-th component has a maximum recovery time membership function $\tilde{T}_n(t)$, defined on the universe of discourse T. The component having the longest recovery time controls the whole system recovery time. Therefore, the system recovery time can be calculated as follows

$$\tilde{T}(t) = \tilde{T}_c(t) \qquad \text{……….(3.19)}$$

given

$$S(\tilde{T}_c) = \max_N \left(S(\tilde{T}_1), S(\tilde{T}_2), \ldots\ldots\ldots, S(\tilde{T}_N) \right)$$

and ……….(3.20)

$$\tilde{T}_c(1) = \max_N \left(\tilde{T}_1(1), \tilde{T}_2(1), \ldots\ldots\ldots, \tilde{T}_N(1) \right)$$

where:

$\tilde{T}(t)$ is the whole system recovery time;

$\tilde{T}_c(t)$ is the controlling recovery time;

$S(\tilde{T}_c)$ is the support of the controlling recovery time fuzzy function (as defined by Equation (2.41)).;

$\left(S(\tilde{T}_1), S(\tilde{T}_2), ..., S(\tilde{T}_N)\right)$ are the support sets of the N components (as defined by Equation (2.41)).;

$\tilde{T}_c(1)$ is the controlling recovery time set at the credibility level=1 (as defined by Equation (2.40)).; and

$\left(\tilde{T}_1(1), \tilde{T}_2(1), ..., \tilde{T}_N(1)\right)$ are the recovery time sets at credibility level=1 of the N components (as defined by Equation (2.40))..

For parallel system configuration composed of M number of components, the m-th component has a maximum recovery time membership function $\tilde{T}_m(t)$, defined on the universe of discourse T. The total failure event equals the failure of every component in the system. As a result, the membership function of system recovery time can be calculated as follows

$$\tilde{T}(t) = \max_{M}\left(\tilde{T}_1, \tilde{T}_2, ..., \tilde{T}_M\right) \qquad \text{……….(3.21)}$$

where:

$\widetilde{T}(t)$ is the whole system recovery time; and

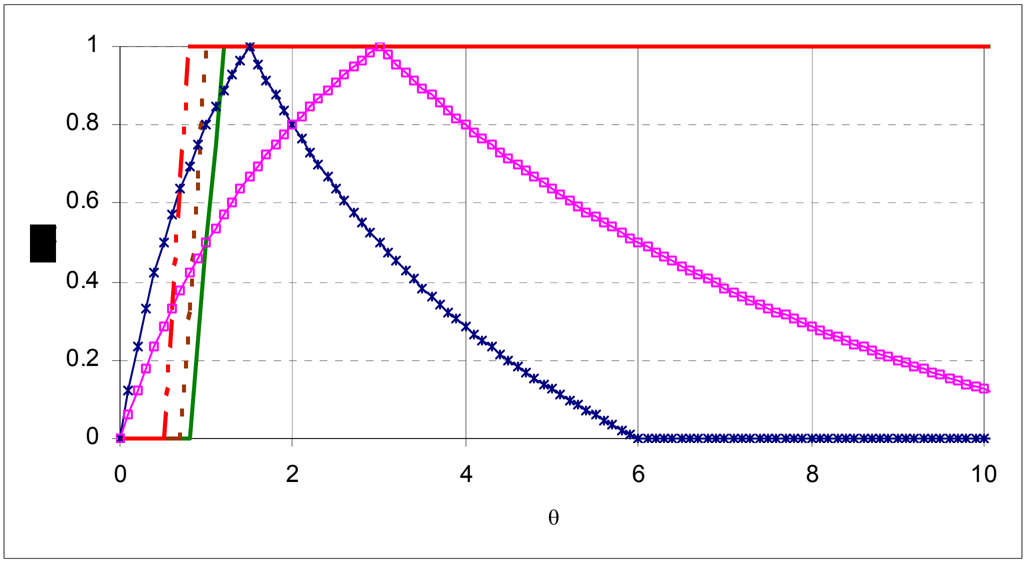
$\left(\widetilde{T}_1, \widetilde{T}_2, ..., \widetilde{T}_M\right)$ component system recovery times.

The combined system recovery time membership function can be calculated by calculating subsystems recovery time membership functions according to either Equation (3.19) or (3.21). The whole system recovery time membership is then calculated by combining the subsystems recovery times using either equation.

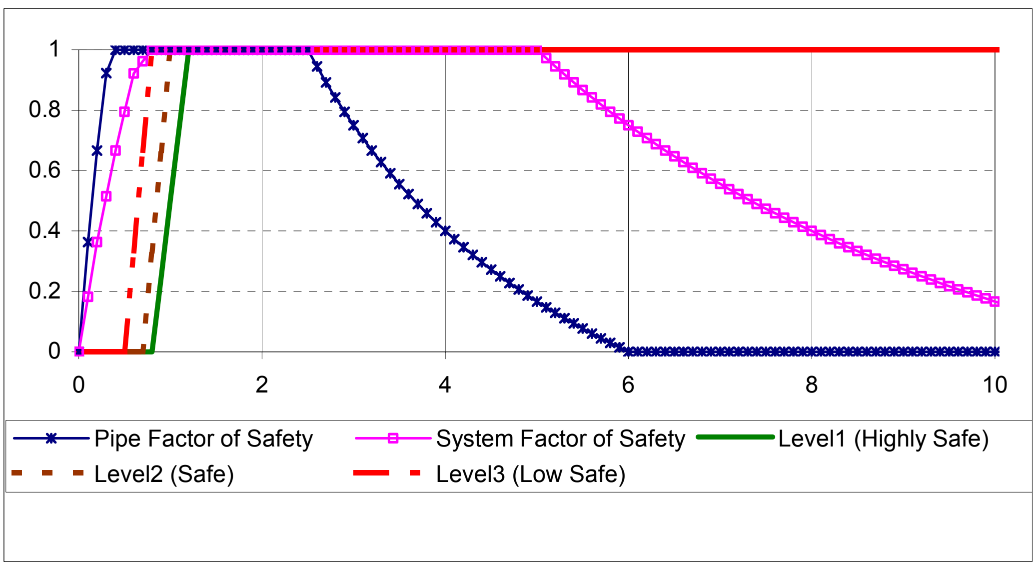
3.3 Utility of the New Fuzzy Performance Indices

The same hypothetical case study from Chapter (2) is used to evaluate the utility of the new fuzzy performance indices. Identical system supplies, capacities, and scenarios are used in the verification procedure. The factor of safety membership function is used as a performance membership function for both systems. The first two indices are calculated in each case for both systems and compared. The results of comparison are shown in Table (3.1).

Three acceptable levels of performance are defined on the universe of the safety factor. These levels are referred to as High-Safety Level, Safe Level, and Low-Safety Level. These levels are represented by three trapezoidal fuzzy numbers, (0.8,1.2,15,15), (0.7,1.0,15,15), and (0.5,0.8,15,15) respectively. The reliability measures (LR) of these levels are 2.40, 2.33, and 1.33 respectively. Figures (3.10a) and (3.10b) illustrate the system-state memberships for the case (I) and case (III) together with the memberships of the predefined acceptable levels of performance.



(a) Case I



(b) Case III

Figure (3.10) System-States for the Case (I) And (III) With the Predefined Acceptable Levels of Performance.

Table (3.1) Summary Results

Case	Case Description	System	Reliability Index	Robustness Index	Result
(I)	Triangular fuzzy membership with equal distribution between pipes in system B	A	0.53	16.9	The measure *Indicated* difference in reliability
		B	0.55	114.5	
(II)	Triangular fuzzy membership with non-equal distribution between pipes in system B	A	0.53	16.9	The measure *Indicated* difference in reliability
		B	0.55	120.8	
(III)	Trapezoidal fuzzy membership with equal distribution between pipes in system B	A	0.47	15.4	The measure *Indicated* difference in reliability
		B	0.53	30.9	
(IV)	Trapezoidal fuzzy membership with non-equal distribution between pipes in system B	A	0.47	15.4	The measure *Indicated* difference in reliability
		B	0.53	30.7	

From Table (3.1), it can be observed that the reliability of system B is higher than the reliability if system A, in cases (I) and (II) it increased from 0.53 to 0.55 and in cases (III) and (IV) from 0.47 to 0.53, respectively. These results agree with the main hypothesis on the reliability of both systems. In addition, the shape of the membership function does not affect the main conclusion about system reliability, which in turn reduces the effect of subjectivity in the decision making process.

The reliability index indicates that the use of pipes with equal capacity is as reliable as the use of unequal capacity, 0.55 in cases (I) and (II) and 0.53 in cases (III) and (IV).

Let us assume that the level of acceptable performance membership has changed from the low-safety level to the safe level in order to calculate the system robustness. As it can be seen in Table (3.1), the use of two parallel pipes increases the robustness of the system as the value of the fuzzy robustness index increases from 16.9 to 114.5 in case (I) for system A and B, respectively.

The increase in the case of triangular membership function is three times the increase in the case of trapezoidal function. The system robustness depends on the shape of the membership functions that represent the supply and demand and their position relative to the universe of discourse.

The ratio of load distribution between the parallel pipes affects the robustness of the system, as it is reflected in the increase from 114.5 to 120.8 for case (I) and case (II). No significant change for case (III) and case (IV) is recorded in this example.

As a final conclusion, the new suggested reliability index and robustness index demonstrated performance consistent with expectations. They are also able to handle different fuzzy representations. In addition, these measures comply with the conceptual approach of the fuzzy sets.

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