Uncertainty Assessment of the Impacts of Climate Change on Extreme Precipitation Events

A report prepared for the Canadian Foundation for Climate and Atmospheric Sciences project:

Quantifying the uncertainty in modelled estimates of future extreme precipitation events

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Executive summary

The main intent of this proposed research is quantifying the uncertainties associated with the frequency analysis of extreme precipitation events. It is a part of a research project which is intended to quantify the uncertainty in modelled estimates of future extreme precipitation events. The uncertainties in extreme precipitation events associated with the anticipated climate change and the inherent uncertainties pertinent to the statistical frequency analysis of extreme precipitation events will be investigated. The expected outcomes of this research can be used as inputs to develop improved procedures and guidelines for risk-based decision making for the design and management of urban water infrastructure with the main intent of minimizing risks and losses due to urban flood events.

The main task of this research will be assessing the uncertainty in extreme precipitation events in the form of intensity-duration-frequency (IDF) curves. Future scenarios of annual extreme precipitation series, which will be generated using the results from climate change scenario projections of different combinations of General circulation models (GCMs) and emission scenarios downscaled to scales relevant for urban water infrastructure, will be used to develop the estimated intensity-duration-frequency (IDF) curves. Both at-site and pooled (regional) frequency analysis methods will be employed. The obtained results will be compared and analyzed. Then the resulting IDF curves will be represented in a probabilistic manner to address the associated uncertainties. These probabilistic IDF curves will provide valuable information for setting policy related to design and management of urban water infrastructure.

The methodologies developed can be used elsewhere but in this particular research will be applied to the Upper Thames River Watershed in Ontario, particularly to the City of London, as a case study to demonstrate the techniques developed in this research.
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### Acronyms

- **AEM**: Arithmetic Ensemble Mean
- **AMS**: Annual Maximum Series
- **BBRS**: Balanced Bootstrap ReSampling
- **BMA**: Bayesian Model Averaging
- **CDF**: Cumulative Distribution Function
- **CRCM**: Canadian Regional Climate Model
- **CS**: Climate Station
- **DDF**: Depth-Duration-Frequency
- **EM**: Expectation-Maximization
- **GCM**: General Circulation Model
- **GHG**: Green House Gases
- **IDF**: Intensity-Duration-Frequency
- **IID**: Independent and Identically Distributed
- **K-S**: Kolmogorov-Smirnov
- **LCL**: Lower Confidence Interval
- **LM**: L-Moment
- **MAP**: Mean Annual Precipitation
- **MAR**: Mean Annual Rainfall
- **ML**: Maximum Likelihood
- **MOM**: Method Of Moments
- **PDF**: Probability Density Function
- **PDS**: Partial Duration Series
- **PWM**: Probability Weighted Moment
- **RCM**: Regional Climate Model
- **REA**: Reliability Ensemble Averaging
- **ROI**: Region Of Influence
- **SRES**: Standard Reference Emission Scenario
- **TS**: Time Series
- **UCL**: Upper Confidence Interval
- **UTRB**: Upper Thames River Basin
1. INTRODUCTION

1.1. Background

Urban environments are characterized by encroachments on the natural drainage systems, impervious watersheds, dense settlement and hence important developments vulnerable to the risks of flooding due to extreme hydrological events. Therefore, reliable estimation of design storms is indispensable for use in the design and management of urban water infrastructure such as storm sewers, culverts, etc. that are intended to serve their purpose safely during their design life. A proactive and sustainable approach towards design and management of urban water infrastructure needs to be devised to minimize risks of flooding and costs that may be required for periodic capacity expansions or the extra costs of over sizing.

Traditionally, the magnitudes of design storms corresponding to a certain return period are estimated from frequency analysis by fitting the observed (historical) extreme precipitation events to a particular “best-fit” statistical distribution (i.e. inferences about the future extreme events are made solely based on the past observations). The estimated quantiles are then summarized in the form of Intensity-Duration-Frequency (IDF) curves. However, statistical frequency analysis of extreme values is developed based on the assumption that the sample data are independent, identically distributed and stationary. Random variables are said to be independent if their joint distribution is equal to the product of their marginal density functions. Independence implies that there is no correlation of data. Identically distributed implies that the data should be from the same population. Stationarity implies that, given any subset of variables, the joint distribution of the same subset viewed m time points later remains unchanged i.e. trends, seasonality and other deterministic cycles are excluded by an assumption of stationarity (Coles, 2001). Therefore, under the assumptions of stationarity, the magnitude and frequency or
return period of extreme precipitation events are not changing with time. But, it is clear that this approach entails uncertainties due to:

1. The magnitude and frequency of future extreme precipitation events may be different from the past (historical) events and hence the above stationarity assumption in statistical frequency analysis of extreme values may not be valid during the intended design life mainly due to the impacts of anticipated climate change;

2. The statistical distribution of extreme precipitation events and/or parameters of the distribution may change during the design life time considered;

3. There is usually sampling uncertainty since the at-site record length of the sample data may not be sufficient to represent the underlying population and hence lead to “undue extrapolation” especially for longer return periods and pooling (regional) approach for the purpose of data augmentation may also introduce uncertainty due to the heterogeneity in extreme precipitation events among the stations considered; and

4. Inherent uncertainties in the frequency analysis of extreme values such as due to the choice of frequency distribution and parameter estimation methods.

In stationary approaches for extreme precipitation analysis, the magnitude, frequency, statistical distribution and estimated parameters are assumed to remain unchanged throughout the design life time of urban water infrastructure. Design and management of urban water infrastructure based on these assumptions may serve the intended purpose safely and reliably only if these assumptions remain valid throughout their design life. However, during the usually long design life time, changes in the magnitudes and frequency of future extreme precipitation events, statistical distributions and parameters can be expected mainly due to anthropogenic induced impacts such as climate change.

Global climate change is expected to have serious implications on the earth’s environment; the impacts may be especially severe for water resources. The Fourth
Assessment Report by the Intergovernmental Panel on Climate Change (IPCC, 2007) documents the likely impacts on water resources associated with climate change. Climate change projections by numerical models (Frei et al., 1998), trend detection from historical observations (Karl et al., 1998; Folland et al., 2001) revealed that warming may lead to an intensification of the hydrologic cycle and increases in mean and heavy precipitation. The Fourth IPCC Assessment Report North American Region (including Canada) shows an overall net negative impact of climate change on water resources and freshwater ecosystems. The beneficial impacts of increased annual runoff in some areas will be tempered by the negative effects of increased precipitation variability and seasonal runoff shifts on water supply, water quality, and flood risks. Although changes in long-term climate means are important, extremes usually have the greatest and most direct impact on our every day lives, community and environment (Vincent et al., 2005). Barrow et al. (2004) reported that by the middle of 21st century, the 20-year return values of annual maximum 24-hr precipitation over Canada are likely to increase by an average of approximately 14% as compared to present values and as a result, an extreme precipitation event that occurs once in 20 years on average in the current climate would occur once in less than 10 years. Changes in extremes may not be directly related to changes in mean precipitation. Fowler et al. (2007), Beniston et al. (2007) and Frei et al. (2006) noted in their work on estimating change in extreme European precipitation that in some areas, increases in extremes are associated with decreases in mean precipitation. Also the uncertainty in projections of rare extreme precipitation events is expected to be higher than the uncertainty in mean precipitation projections.

1.2. Description of uncertainties

The main uncertainties which are identified in section 1.1 are due to the impacts of climate change on future extreme precipitation events, the information content of the observed data used and the inherent uncertainties in statistical frequency analysis for extreme values.
To assess the changes in future extreme precipitation events, climate change scenario projections using global or regional climate models are widely employed. General circulation models (GCMs) are currently the most credible tools available for simulating the response of the global climate system to increasing greenhouse gas concentrations, and provide estimates of climate variables such as air temperature, precipitation, incoming radiation, vapor pressure, wind speed, etc. (Prodhomme et al., 2003). Extreme precipitation scenarios for future conditions are usually projected using different combinations of GCMs and emission scenarios, then the degree of the impacts on the magnitude and frequency of extreme precipitation events can be assessed through impact assessment models (statistical frequency analysis models in this case). However, these GCM models (which are from different sources, have different atmospheric resolutions and developed at different time periods, etc.) are simplified representations of the complex physical processes occurring in the earth-atmosphere-ocean system and also the emission scenarios are set based on uncertain future socio-economic, technological, etc. conditions that will prevail. Hence, projected outcomes of scenarios for extreme precipitation events (which are discrete and spatially variable in nature) from different climate models and emission scenarios may be significantly different. Therefore, climate change impact assessment models developed based on GCM output are subject to uncertainties. Hulme and Carter (1999) described uncertainties that arise from “incomplete” knowledge, reflected by the model designs (climate models and impact models) and “unknowable” knowledge/information pertinent to the inherent unpredictability of the future earth and socio-economic systems (key parameters in GCM modelling such as emission scenarios or the quantity of greenhouse gases expected to be released into the atmosphere). Planton et al. (2008) indicated that the two main causes of uncertainties associated with the models and the scenarios dominate the uncertainty of climate change projections and also noted that for extreme climate events the uncertainty due to internal climate variability also needs to be addressed.

The other source of uncertainty is the information content of the sample data series, which are of short length. Regional or pooled frequency analysis has been used to
augment the existing short historical records. In a regional analysis, homogeneous regions are identified and data from several sites are combined to yield a regional frequency curve. The use of regional information may reduce the sampling uncertainty and result in more robust estimates by introducing more data as long as homogeneous regions can be formed. The main challenge is the identification of similar stations in terms of extreme precipitation events from a number of stations available in the region. Several homogeneous pooling groups may be identified based on criteria for similarity and homogeneity measures yet the estimated quantiles from these pooling groups may be different. The inherent spatial and temporal variable nature of extreme precipitation events may further exacerbate this problem. Stations which are homogeneous for longer durations of extreme precipitation events may not appear homogeneous for shorter durations and vice versa. Also the delineated pooling groups may not remain homogeneous throughout the design life of the urban water infrastructure considered (i.e. homogeneous regions which are delineated based on historical observations may not remain homogeneous under future climate conditions). Therefore, these are additional sources of uncertainties which are pertinent to sampling in terms of the delineation of homogeneous pooling groups and need to be addressed in frequency analysis of extreme precipitation events.

There are also the inherent uncertainties associated with the impact assessment model (i.e. the frequency analysis of extreme precipitation events) namely due to the choice of frequency distribution and parameter estimation methods. For instance, although there may be several statistical distributions that can reasonably fit the observed or projected series of extreme precipitation data, there may be differences in the estimated quantiles since estimation of higher extreme quantiles is based on the upper tail of the probability distribution which implies that there is associated uncertainty. In addition, use of different parameter estimation methods may result in different quantile estimates as each parameter estimation method has its own strengths and limitations.
Other possible sources of uncertainties pertinent to the design and management of urban water infrastructure which will not be studied in this research are uncertainties in the transformation of design storm to design flood using simulation models in which considering the effects of antecedent soil moisture and snow melt conditions may be important, uncertainties related to hydraulic capacity of the urban water infrastructure and uncertainties due to land use changes (further urbanization) that has potential to further modify the components of urban hydrological cycle.

The existence of the above mentioned uncertainties pertinent to the frequency analysis of extreme precipitation events which is relevant for the estimation of design storm quantiles and IDF curves and the prevalent research gaps in the area substantiate the need for further research work that quantifies and represents these uncertainties. The expected outcome of this research will be utilized to set procedures and guidelines for risk based design and management of urban water infrastructure as an adaptation measure towards the impacts of climate change on extreme precipitation events associated with urban flooding.

1.3. Objectives of the research

The proposed work is motivated by observed and modelled increases in critical design precipitation events and seeks to better quantify the magnitude and uncertainty of extreme precipitation events. It is the intent of this proposed research to increase our understanding of extreme precipitation events and the uncertainties as a result of the impacts of climate change, delineation of homogeneous pooling groups and the inherent uncertainties in statistical frequency analysis of extreme precipitation events. Therefore, the main objectives of the proposed research are:

i. Quantifying the uncertainties in frequency analysis of extreme precipitation events which are associated with:
   - The impacts of climate change;
• Delineation of homogeneous pooling groups; and
• The inherent uncertainties in frequency analysis of extreme values

ii. Representation of these uncertainties in a probabilistic manner for use as input in developing procedures and guidelines for risk based design and management of urban water infrastructure.

To demonstrate this research work it will be applied to the Upper Thames River watershed in southwestern Ontario, particularly to the City of London, as a case study. It is expected that different stakeholders such as conservation authorities, municipalities, insurance companies, public, etc. would benefit from the outcome of this research.

The remainder of this proposal is organized in the following manner. Literature review is given in chapter 2. The third chapter describes the research proposal (research plan) and research progress to date. Conclusions and plans for future work are given in the fourth chapter. Finally, the report ends with the schedule of future work.
2. LITERATURE REVIEW

This section consists of literature reviews of regional (pooled) frequency analysis method, background on climate change and climate change scenario projections, impacts of climate change on extreme precipitation events and its associated uncertainties, uncertainty assessment methods and risk-based approaches to design and management of urban water infrastructure. Pooled frequency analysis based on region of influence similarity measure, L-moments parameter estimation method and the “index storm” procedures of quantile estimation are reviewed in detail.

Research pertinent to the impacts of climate change on extreme precipitation events based on numerical simulation models and trend analysis from historical observations (non-parametric and parametric methods), uncertainties associated with the impacts of climate change and climate change impact assessment, and adaptation strategies for the design and management of urban water infrastructure have been reviewed. The main impact assessment model commonly used to assess the impacts of climate change on extreme precipitation events relevant to the design of urban water infrastructure is the statistical frequency analysis model in which the degree of changes in the magnitude and frequency of extreme precipitation quantiles can be assessed.

The main sources of uncertainties pertaining to the impacts of climate change on extreme precipitation events are identified as GCMs, emission scenarios, downscaling and impact assessment models. Several GCMs, emission scenarios, downscaling techniques (both spatial and temporal) and impact assessment models are available for use yet may result in different results and hence entail associated uncertainties. Different uncertainty assessment methods for climate change scenario projections and impact assessment models such as Monte Carlo methods, reliability based ensemble averaging, Bayesian based model averaging and fuzzy based uncertainty analysis are available. Also non-parametric balanced bootstrap resampling is suitable for the assessment of uncertainties in terms of the confidence intervals of quantile estimates for instance the uncertainties
due to the delineation of homogeneous pooling groups, choice of statistical distributions and parameter estimation methods.

Risk based design and management of urban water infrastructure has been discussed. In stochastic risk based approaches, uncertainties in random variables of interest are expressed in terms of probability distributions and different methods for stochastic simulation, stochastic optimization and risk analysis/assessment are employed. But it is assumed that these probability distributions are perfectly known and stationary (i.e. not changing with time). However, under the anticipated climate change it is necessary to incorporate the impacts of climate change (with associated uncertainties) on the frequency, magnitude, statistical distribution and its parameters of extreme precipitation events and hence these uncertainties can be propagated to estimation of runoff in the risk based approaches to set procedures and guidelines for setting policies for better decision making to minimize the risks associated with urban flooding.

The main research gaps in the areas of the impacts of climate change on extreme precipitation events and that need further study are identified as:

i. Quantifying the uncertainties associated with the impacts of climate change, sampling in terms of delineation of homogeneous pooling groups and the inherent uncertainties in the frequency analysis of extreme precipitation events for the estimation of quantiles and IDF curves relevant to urban water infrastructure; and

ii. Representation of these uncertainties in a way that can be used as input for setting procedures and guidelines for risk based management of urban water infrastructure.

2.1. Regional or pooled frequency analysis

Frequency analysis of extreme precipitation events requires the availability of sufficient extreme precipitation data at sites of interest especially for reliable estimation of rare events (i.e. quantiles with large return periods). But in some regions there may be no
gauging sites or the length of observed record is short. Regional or pooled frequency analysis which is based on “index storm” approach has been widely employed for quantile estimations of extreme precipitation events through data augmentation for sites with short records. Hence additional information from sites within the region is utilized to improve the at-site estimates and also to obtain estimates for sites without observations. Hosking and Wallis (1990; 1993; 1997), Burn (1988; 1990; 1997; 2003), Martins and Stedinger (2002) demonstrated the importance of using regional information for frequency analysis of extreme hydrological events.

2.1.1. L-moments for pooled frequency analysis

If X is a real-valued random variable with cumulative distribution F (.), its probability weighted moments were defined by Greenwood et al. (1979) to be the quantities:

\[ M_{p,r,s} = E\left\{ X^p \left[ F(X)\right]^r \left[ 1-F(X)\right]^s \right\} \]

\[ \beta_r = M_{1,r,0} = E\left\{ X\left[ F(X)\right]^r \right\} \]

where p, r and s are non-negative integers. \( M_{p,0,0} \) represents the conventional moment of order p about the origin. The above probability weighted moments are difficult to interpret directly as a measure of the scale and shape of a probability distribution and this information is carried in certain linear combinations of the probability weighted moments and hence L-moments arose as modifications to PWMs based on the theory of order statistics (Hosking and Wallis, 1997).

2.1.1.1. L-moment representations and estimators of L-statistics

Certain linear combinations of the elements of an ordered observations \( (x_{r:n}) \) where n is the sample size and r denotes the ranks of observations (i.e. \( r = 1,2,\ldots,n \)) contain
information about the location, scale and shape of the distribution from which the sample is drawn. The L-moments of a probability distribution are defined by (Hosking and Wallis, 1997):

\[ \lambda_k = k^{-1} \sum_{j=0}^{k-1} (-1)^j \binom{k-1}{j} E \left( X_{k-j:k} \right) \]  

(2.2)

L-moments have been defined for a probability distribution, but their estimates are obtained from finite sample. Unbiased estimators of the probability weighted moment, \( \beta_r \) (Landwehr et al., 1979) are:

\[ b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-2)\ldots(j-r)}{(n-1)(n-2)\ldots(n-r)} x_{j:n} \]  

(2.3)

Hosking and Wallis (1997) and Serfling and Xiao (2007) have expressed L-moments and their sample estimators in terms of probability weighted moments and derived an expression for the sample L-moments (\( l_k \)) in terms of the ordered observations and their corresponding weights:

\[ l_k = n^{-1} \sum_{r=1}^{n} w_r^{(k)} x_{r:n} \]  

(2.4)

\[ w_r^{(k)} = \sum_{j=0}^{\min\{r-1,k-1\}} (-1)^{k-j} \binom{k-1}{j} \binom{k-1+j}{j} \binom{n-1}{j}^{-1} \binom{r-1}{j} \]  

(2.5)

Where \( w_r^{(k)} \) are the weights and \( x_{r:n} \) are the ordered observations.

The weights, which are the relative contributions of each observation to the first four L-moments for a sample size \( n \), are computed as:
\[
\begin{align*}
\tau_1 &= 1 \\
\tau_{2n} &= \frac{2r-n-1}{n-1} \\
\tau_{3n} &= \frac{(n-1)(n-2)+6(r-1)(r-2)-6(r-1)(n-2)}{(n-1)(n-2)} \\
\tau_{4n} &= \frac{-(n-1)(n-2)(n-3)+20(r-1)(r-2)(r-3)+12(r-1)(n-2)(n-3)-30(r-1)(r-2)(n-3)}{(n-1)(n-2)(n-3)} \\
\end{align*}
\]

...(2.6)

where \( r = 1, 2, 3, \ldots, n \) are ranks of observations in ascending order.

The sample L-moments, \( l_k \), are an unbiased estimator of \( \lambda_k \) (Hosking and Wallis, 1997). Some of the main advantages of L-moments are as given below:

i. L-moments being linear functions of the data are less sensitive than are conventional moments to sampling variability or measurement errors in the extreme data values, and may therefore be expected to yield more accurate and robust estimates of the parameters of an underlying probability distribution (Hosking, 1990);

ii. L-moment ratio estimators have small bias and variance in comparison with the conventional moments (Hosking, 1990); and

iii. Hosking et al. (1985) indicated that probability weighted moment estimators are superior estimates to maximum likelihood methods in providing the higher quantile for small sample size.

L-moment ratios are scale free (independent of units of measurement) higher moment descriptive measures and are defined as:

\[
\tau = \frac{\lambda_2}{\lambda_1}, \quad \tau_k = \frac{\lambda_k}{\lambda_2}
\]

............................................................................................................(2.7)
where \( \tau \) is the L-CV, \( \lambda_2 \) is the L-scale, \( \lambda_1 \) is the L-location or the mean, \( \tau_3 \) is the L-skewness and \( \tau_4 \) is the L-kurtosis, \( k \geq 3 \), \( \lambda_1 \) can take any value, \( \lambda_2 \geq 0 \) and \( |\tau_k| < 1 \) for all \( k \geq 3 \). For a distribution that takes only positive values, \( 0 \leq \tau < 1 \). Sample L-moment ratios \( t \) and \( t_k \) are natural estimators of \( \tau \) and \( \tau_k \) respectively and are not unbiased but their biases are very small in moderate or large samples and defined as (Hosking and Wallis, 1997):

\[
t = \frac{l_2}{l_1}, \quad t_k = \frac{l_k}{l_2}, \quad k \geq 3
\]

……………………………………………………………………………………………..(2.8)

Hosking and Wallis (1997) defined the regional average L-moments by:

\[
(l^R)^* = \frac{\sum_{i=1}^{N} n_i (l^R_i)^{(i)}}{\sum_{i=1}^{N} n_i}
\]

……………………………………………………………………………………………..(2.9)

where \( N \) is the total number of sites in a pooling group, \( n_i \) is the number of records for each site and \( R \) denotes the regional.

**2.1.2. Steps in regional frequency analysis**

The five main steps in pooled frequency analysis include (Hosking and Wallis, 1997):

i. Screening of the data

Data quality check such as for gross errors and inconsistencies need to be conducted before utilizing the data for analysis.
ii. Delineation of homogeneous regions

Sites are classified to their homogeneous pooling groups based on specified classification methods and homogeneity criterion.

iii. Choice of a frequency distribution

The choice of suitable regional parametric frequency distribution is usually done based on goodness-of-fit-test approach. The chosen distribution should not merely fit the data well but should also yield quantile estimates that are robust to physically plausible deviations of the true frequency distribution from the chosen frequency distribution (Hosking and Wallis, 1997).

iv. Parameter estimation of the frequency distribution

Parameters of a selected distribution can be estimated by using several methods such as method of moments (MOM), method of maximum likelihood (MLM), probability weighted moments (PWM) and L-moments (LM).

- Method of Moments parameter estimation (MOM)

In method of moments, the parameters of a distribution are estimated using the information about its moments through the relationships among the moments and the location, scale and shape parameters of a distribution (Rao et al., 2000).

- Maximum likelihood parameter estimation (ML)

The values of m parameters (θ’s) that maximize the likelihood that the particular sample is the one that would be obtained if n random observations (x’s) were selected from the distribution $f_X(x;\theta)$, are known as the maximum likelihood estimators. Parameter estimators are obtained from the solution to the set of simultaneous equations from partial differentiation of the likelihood function with respect to each parameters of the distribution (Rao et al., 2000).

- Probability weighted moments and L-moments (PWM and LM)

Hosking and Wallis (1997) developed a regional L-moment algorithm for parameter and quantile estimation through combining the at-site L-moment statistics to obtain the
regional weighted average and hence the regional frequency curve. PWM and LM are discussed in detail under section 2.1.1.

v. Quantile estimation

The main objective of frequency analysis is estimation of quantiles corresponding to a return period of interest. “Index storm” approach which is a similar approach to the index flood (Dalrymple, 1960) is commonly used for quantile estimation in regional analysis of extreme precipitation events.

2.1.2.1. Screening the data

Data quality for frequency analysis of extreme precipitation events is subject to two main sources of errors which are attributable to errors in measurements and errors due to the non-stationarity of the climate conditions under which data are collected. Therefore, quality checking for outliers and trends is indispensable.

2.1.2.2. Delineation of homogeneous pooling group

Pooled frequency analysis of extreme precipitation events involves the use of local observations at many gauged sites for better quantile estimation at a site of interest. One of the main tasks in regional frequency analysis is identification of homogeneous pooling group since proper delineation is a crucial factor for reliable quantile estimation.

In identification of homogeneous pooling group, first similar sites are identified in the region and ranked based on their proximity to the target site for which the frequency analysis is conducted. Several methods for the delineation of similar sites in a region
have been proposed among which region of influence approach or ROI (Burn, 1990; Zrinji and Burn, 1994; Gaál et al., 2008) for a target site of interest, which is based on focused pooling group, has the advantage of reducing the regional heterogeneity in choosing similar sites. The effective identification of a pooling group is governed by two fundamental principles, the homogeneity of the group and its target size (Burn et al., 2000). In ROI approach, the similarity of homogeneous pooling groups is defined based on climatic/meteorological and other characteristics (attributes) corresponding to the observation stations. Hence, the similarity among the sites is measured by a weighted and scaled Euclidian distance metric in P-dimensional space (Burn et al., 2000) defined by sets of attributes. The dissimilarity between two sites is defined as:

\[
    d_{i,j} = \sqrt{\sum_{k=1}^{P} w_k \left( \frac{x^i_k - x^j_k}{s_k} \right)^2}
\]

where \(d_{i,j}\) is the distance metric between site i and site j, \(w_k\) is the weight applied to attribute k reflecting the importance of the attribute with respect to the others, \(x^i_k\) and \(x^j_k\) are the values of the k\(^{th}\) attribute for sites i and j respectively, and \(s_k\) is the sample standard deviation of the k\(^{th}\) attribute.

### 2.1.2.1. Homogeneity evaluation

One of the most frequently used tests for regional homogeneity is based on the theory of L-moments by Hosking and Wallis (1997). This test compares the regional dispersion of L-moment ratios in a pooling group with the average dispersion of the L-moment ratios determined from NS simulations of homogeneous groups influenced only by sampling variability (Hosking and Wallis, 1997). They suggested sampling from a four parameter Kappa distribution. Three heterogeneity measures are used to test the variability of three different L-statistics: \(H_1\) for “coefficient of L-variation” (L-CV), \(H_2\) for the combination
of L-CV and L-skewness (L-SK) and H₃ for the combination of L-skewness (L-CS) and L-kurtosis (L-CK). For both real-world data and artificial simulated regions, the H₁ statistics has been shown to have much better discriminatory power than H₂ and H₃ statistics (Hosking and Wallis, 1997). Heterogeneity measures (H) statistics are calculated as:

\[ H_i = \frac{V_{obs,i} - \mu_{V_i}}{\sigma_{V_i}}, \quad i = 1, 2, 3 \]  

(2.11)

where \( \mu_{V_i} \) and \( \sigma_{V_i} \) are the means and standard deviations of the simulated values of dispersions (\( V_i \)) while \( V_{obs} \) is the regional dispersion calculated from the regional observations. V-statistics are defined as below:

\[
V_1 = \left\{ \frac{\sum_{i=1}^{N} n_i \left( \frac{t(i)}{t^R} \right)^2}{\sum_{i=1}^{N} n_i} \right\}^{\frac{1}{2}}
\]

\[
V_2 = \frac{\sum_{i=1}^{N} n_i \left\{ \left( t(i)^R \right)^2 + \left( t_3(i)^R \right)^2 \right\}^{\frac{1}{2}}}{\sum_{i=1}^{N} n_i}
\]

\[
V_3 = \frac{\sum_{i=1}^{N} n_i \left\{ \left( t_3(i)^R \right)^2 + \left( t_4(i)^R \right)^2 \right\}^{\frac{1}{2}}}{\sum_{i=1}^{N} n_i}
\]
where $V_1$ is the weighted standard deviation of the at-site sample L-CVs. $V_2$ and $V_3$ are the weighted average distance from the site to the group weighted mean on graphs of $t$ versus $t_3$ and of $t_3$ versus $t_4$ respectively and $t^R$, $t_3^R$ and $t_4^R$ are the regional average L-CV, L-SK, and L-CK, weighted proportionally to the sites’ record length respectively. Hosking and Wallis (1997) suggested that region can be regarded as “acceptably homogeneous” if $H < 1$, “possibly heterogeneous” if $1 \leq H < 2$, and “definitely heterogeneous” if $H \geq 2$. In the mean time they emphasized that valid use of $H$ requires that assignment of sites to regions be based on external site characteristics such as the physical characteristics or geographical location of the sites.

### 2.1.2.2.2. Discordancy measure

Hosking and Wallis (1997) proposed a measure of discordancy between the L-moment ratios of a site and the average L-moment ratios of a group of similar sites to identify those sites that are grossly discordant with the group as a whole and the method is explained in the procedures as follows. Suppose there are $N$ sites in the group, let $u_i = [t^{(i)} \ t_3^{(i)} \ t_4^{(i)}]^T$ be a vector containing the L-moment ratios $t$, $t_3$ and $t_4$ values for site $i$ and the superscript $T$ denotes transpose of a vector matrix, the group average $\bar{u}$ and sample covariance matrix $S$ are defined as:

$$
\bar{u} = \frac{1}{N} \sum_{i=1}^{N} u_i
$$

$$
S = \sum_{i=1}^{N} (u_i - \bar{u})(u_i - \bar{u})^T
$$

\[\text{(2.14)}\]
Then the discordancy measure $D_i$ for a site is given by equation 2.15. A site should be declared discordant if $D_i \geq 3.0$.

$$D_i = \frac{1}{3}N\left(u_i - \bar{u}\right)^T S^{-1}\left(u_i - \bar{u}\right) \tag{2.15}$$

2.1.2.3. A goodness-of-fit measure (selection of a regional frequency distribution)

The choice of statistical frequency distributions is determined mainly based on the goodness-of-fit measures which indicate how much the considered distributions fit the available data. When several distributions fit the data adequately, any of them is a reasonable choice for use in the final analysis, and the best choice from among them will be the distribution that is most robust, that is, most capable of giving good quantile estimates even though future values may come from distribution somewhat different from the fitted distribution (Hosking and Wallis, 1997). There are several methods available for testing the goodness-of-fit of a distribution to data. The goodness-of-fit criterion for each of various distributions defined in terms of L-moments and is termed the Z-statistic proposed by Hosking and Wallis (1997) is as follow: Fit a four parameter Kappa distribution to the regional average L-moment ratios $t_1^R$, $t_3^R$, and $t_4^R$. Simulate a large number, $N_{sim}$, of realizations of a region with $N$ sites, each from Kappa distribution. For the $m$th simulated region, calculate the regional average L-kurtosis $t_4^R[m]$, the bias and standard deviation of $t_4^R$:

$$\beta_4 = \frac{1}{N_{sim}} \sum_{m=1}^{N_{sim}} \left(t_4^R[m] - t_4^R \right)$$

$$\sigma_4 = \left\{ \frac{1}{N_{sim} - 1} \left[ \sum_{m=1}^{N_{sim}} \left(t_4^R[m] - t_4^R \right)^2 - N_{sim} \beta_4^2 \right] \right\}^{1/2} \tag{2.17}$$

And, for each distribution, the goodness-of-fit measure is:
\[ Z^{DIST} = \frac{\tau^4_DIST - \tau^R_4 + \beta_4}{\sigma_4} \]

where DIST refers to a particular distribution, \( \beta_4 \) and \( \sigma_4 \) are the bias and standard deviation of \( t^R_4 \) respectively. \( N_{\text{sim}} \) is the number of simulated regional data sets from a Kappa distribution in a similar way as for the heterogeneity statistic. The subscript \( m \) denotes the \( m^{th} \) simulated region obtained in this manner. Hosking and Wallis (1997) recommended that a fit is declared adequate if \( Z^{DIST} \) is sufficiently close to zero, a reasonable criterion being \( |Z^{DIST}| \leq 1.64 \).

### 2.1.2.4. Parameters and Quantile estimation

The ultimate objective of any pooled frequency analysis is the estimation of quantiles corresponding to certain return periods (T-years extreme event) from the regional observations for the site under consideration. Hence proper quantile estimation procedures should be followed to obtain reliable estimation. An “index-storm”, which is adopted from the index flood (Dalrymple, 1960), and L-moments homogeneity test (Hoskings and Wallis, 1997) are the commonly used quantile estimation approaches in the regional frequency analysis of extreme precipitation events. The key assumption of an “index storm” procedure is that the sites forming a homogeneous pooling group have identical frequency distribution (called the regional growth curve) apart from a site-specific scaling factor, the “index storm”. For gauged sites, location estimators such as sample mean or median are used as an “index storm”. For instance, for annual maximum series (AMS) of precipitation observations, quantile at a certain target site is estimated by multiplying the regional growth curve (the growth factor) obtained from AMS precipitation events by the mean of the AMS precipitation observations for the target site. Suppose pooled observations from \( N \) sites are considered, with site \( i \) having sample size \( n_i \). Let \( Q_i(F) \), \( 0 < F < 1 \), be the quantile function of the frequency distribution of extreme precipitation at site \( i \), for a homogeneous region we have:
\[ Q_i(F) = \mu_i q(F) \] .................................(2.19)

where \( i = 1, 2, \ldots, N \) and \( \mu_i \) is the site-dependent scale factor which is called the “index storm” and \( q(F) \) is the regional growth curve which is a dimensionless quantile function common to every site. Further detailed references on pooled frequency analysis based on L-moments can be obtained from Hosking and Wallis (1997).

2.2. Definitions and Background on Climate Change

Climate change is defined as (http://www.cccsn.ca/Scenarios/Scenarios_Introduction-e.html) a difference over a period of time (with respect to a baseline or a reference period) and corresponds to a statistically significant trend of mean climate or its variability, persistent over a long period of time e.g. decades or more. Climate change may occur due to both the internal natural variability of the climate system and external factors due to anthropogenic forcing in terms of increase in concentrations of greenhouse gases (GHG). The extreme events have greater impacts on the lives of the community and hence investigation of extreme events is crucial for impact assessment and adaptation studies hence data of fine temporal and spatial resolutions which can capture the wide ranges of climate regime and temporal-spatial variability must be taken into account. We need climate change scenarios to obtain data for impact and vulnerability assessment, for awareness development, for decision making in setting polices and adaptation strategies, etc. (http://www.cccsn.ca/Scenarios/Scenarios_Introduction-e.html).

For many climate change studies, scenarios derived directly from global climate model output may not be of sufficient spatial or temporal resolution to represent changes within a specific region hence downscaling is required. Spatial downscaling refers to the techniques used to derive finer spatial resolution climate information from coarser spatial resolution GCM or regional climate model (RCM) output while temporal downscaling refers to the derivation of highest temporal resolution data from the lowest temporal resolution output. Most climate change scenarios derived from GCM output are generally
based on changes in monthly or seasonal mean climate while extreme precipitation events of fine temporal resolutions are required for the design and management of urban water infrastructure. One way of obtaining daily weather data from monthly GCMs scenario information is to use a stochastic weather generator. Sharif and Burn (2006; 2007) demonstrated a K-nearest neighbor weather generating model for the simulation of extreme precipitation events. Another approach of temporal downscaling is direct downscaling (disaggregation) of extreme precipitation quantiles for shorter durations from quantiles corresponding to longer duration (Gupta and Waymire, 1990; Burlando and Rosso, 1996; Menabde et al., 1999; Pao-shan et al., 2004). Nguyen et al. (2002; 2007; 2009 in press) demonstrated a temporal downscaling (disaggregation) method for rainfall from low to high temporal resolution using the scaling GEV distribution based on “scale-invariance” or “scaling” concept. The scale invariance implies that statistical properties of extreme rainfall processes for different time scales are related to each other by an operator involving only the scale ratio (Nguyen et al., 2002).

2.3. Impacts of climate change and its associated uncertainties

Precipitation is one of the major components of the hydrologic cycle. An increase of GHGs produces increased surface heating with warmer surface temperatures, more evaporation, an increase in the ability of the atmosphere to hold moisture, and thus an increase in atmospheric moisture content with enhanced precipitation rates (Trenberth, 1999). Air at 30 °C may contain about seven times the quantity of water vapor that it is able to contain at 0 °C (Sumner, 1988). Therefore, changes in the meteorological variables that drive the hydrologic cycle can be expected to affect the spatial and temporal distribution of water, which can affect the capability of the impacted population to cope with natural hazards related to water excess or shortage. The projected impacts of climate change have potentially serious implications for the planning and operation of water resources infrastructure. Prodhomme et al. (2003) have noted that it is time for hydrologists to concentrate their efforts on designing techniques that lead to an optimization of the use of GCMs (or RCMs) outputs for flood studies impact such as the
creation of realistic climate time series for future time horizons and on implementing appropriate methods that incorporate climate change uncertainty in flood risk assessment planning. They described the weakness of Global Climate modeling of short-duration, localized heavy rainfall as there is a large difference in spatial and temporal scales when comparing Global Climate Models and convective storm events as convective events are usually short-lived and localized intense mechanisms, where as most GCMs offer changes at a monthly time-step, and at a $\approx 300$km grid scale. Uncertainties in these future estimates should also be assessed simultaneously to incorporate the uncertainty in the climate change scenario projections and climate change impact assessment studies. Climate change impact assessment models developed based on GCM output are subject to a lot of uncertainties. The main sources of uncertainties in assessing the impacts of climate change on extreme precipitation events can be from:

i. General circulation models (GCMs);

ii. Greenhouse gas emission scenarios (GHG);

iii. Downscaling techniques; and

iv. Impact assessment uncertainties:

   o Choice of impact assessment model (i.e. frequency analysis distributions in this case);

   o Model parameters: estimation of model parameters; and

   o Data: data quality and sampling uncertainties.

Much research has been pursued based on numerical simulations of multimodel ensembles or multi ensemble experiments to account for the uncertainties in extreme hydrological events due to the impacts of climate change. Fowler et al. (2005) used HadRM3H model for the simulation of future changes in extreme rainfall across UK using both regional and grid box frequency analysis. Frei et al. (2006) performed inter comparisons of scenarios from six regional climate models (RCMs) to examine for future change of precipitation extremes in Europe and noted differences between model outputs. Kharin et al. (2007) evaluated changes in temperature and precipitation extremes in the
IPPC Ensemble of Global Coupled Model Simulations for climate extremes of 20 year return values of annual extremes of near-surface temperature and 24-h precipitation amounts. They found that there is a very large intermodel disagreements in the Tropics which suggests that some physical processes associated with extreme precipitation are not well represented in models hence reduces confidence in the projected changes in extreme precipitation. Kendon et al. (2008) assessed the reliability of future changes in heavy rainfall over the UK and found that in winter, increases in atmospheric moisture associated with warming are likely to be dominant in driving changes in extreme precipitation across the UK while in summer, warming is no longer the dominant mechanism as the over all changes in extreme precipitation in summer are sensitive to large-scale circulation changes which are uncertain, varying considerably between different models. Finally, they concluded with their belief that ensemble probabilistic approaches combined with a mechanistic understanding are the best way to provide climate change predictions. However, Hall et al. (2007) argued that probabilistic climate scenarios may misrepresent uncertainty and lead to bad adaptation decisions emphasizing that considerable caution is required in the interpretation of probability distributions and in their use for adaptation decision making, e.g. in the field of flood defense or water resource management.

Another approach followed to study the impacts of climate change is based on the trend analysis of historical observations. An example of how regional analysis could be extended to incorporate a trend in terms of time dependent parameters of a distribution is proposed by Katz et al. (2002). Kharin et al. (2005) examined changes in transient climate change simulations performed with the second-generation coupled global climate model of the Canadian center for Climate Modelling and Analysis from a fitted GEV distribution by including trends in terms of time-dependent location, scale and shape parameter. They noted that changes in precipitation extremes are due to changes in both the location and scale of the extreme value distribution and exceed substantially the corresponding changes in the annual mean precipitation. Boo et al. (2006) investigated the change in extreme events of temperature and precipitation over Korea using regional
projection of future climate change and found that the increasing trend of temperature is associated with an increasing trend of precipitation with an increase in the number of the days of heavy precipitation as well as the corresponding amount. Arnbjerg (2006) analyzed significant climate change of extreme rainfall in Denmark through statistical trend analysis and found that the majority of trends are towards a positive trend, indicating more frequent and more heavy rain and concluded that studies should be done on how to implement the time-varying nature of IDF curves due to climate change in the design practices of urban drainage.

The impacts of climate change on the frequency analysis of extreme precipitation events and hence on the design and management of urban water infrastructure have become a concern in many parts of the world including North America and hence research has been done on the impact assessment and adaptation strategies based on future projections or historical trends of climate for the area of the study. Grum et al. (2006) evaluated the effect of climate change on urban drainage in Denmark based on regional climate model simulations for point rainfall extremes and remarked that the results and conclusions rely heavily on the regional model’s suitability in describing extremes at time scales relevant to urban drainage. He et al. (2006) examined the impacts of changes in design storm magnitudes on the design of urban infrastructure with a focus on changes arising from climate change. They noted that extreme precipitation events that exceed the design storm are likely to become more frequent in the future. Overeem et al. (2007) examined the effects of dependence between the maximum rainfalls for different durations on the estimation of depth-duration-frequency (DDF) curves and the modelling of uncertainty of these curves for stations in the Netherlands. Mailhot et al. (2007) assessed the future changes in IDF curves for annual maximum rainfall depth series of 2-, 6-, 12- and 24-h durations over southern Quebec (Canada) using the Canadian Regional Climate Model (CRCM) and found that the return periods of extreme rainfall depths for all durations decreases but remarked that the results obtained remain model dependent and multi-model ensemble systems need to be analyzed in order to quantify the associated uncertainties. Mailhot et al. (2009 in press) proposed design criteria of urban drainage
infrastructure under climate change by considering the expected life time of the infrastructure and future evolution of intensity or frequency of extreme events. They remarked that many IDF users have expressed concerns that there is no systematic upgrade of IDF curves covering Canada in recent years. Nguyen et al. (2007; 2009 in press) presented a spatio-temporal downscaling approach to the assessment of the impact of climate change on the estimation of design storms at a local site based on a combination of spatial downscaling method to link large-scale climate variables given by GCM simulations with daily extreme precipitation at a site, and a temporal downscaling procedure to describe the relationships between daily and sub-daily extreme precipitation based on the scaling GEV distribution and to construct IDF curves for present and future periods for Dorval (Canada). Coulibaly et al. (2005) identified the effects of climate change on future highway drainage infrastructure in Ontario (Canada) and suggested the importance of revisions of the existing design standards to include the anticipated climate change effects in order to avoid an increase in the risk of “failure” of highway structures in Ontario. Guo et al. (2006) derived an up to date IDF relationship for Chicago and emphasized the requirements of updated IDF relationship for synthesizing design storms that reflect changing climate conditions. Markus et al. (2007) conducted sensitivity analysis to examine the effects of various factors on 100-year, 24-h precipitation at Aurora College station in Northeastern Illinois, in particular the effects of selecting different periods of the precipitation record, different regions, and different underlying distributions. Young et al. (2006) developed revised precipitation frequency estimates for the Kansas City Metropolitan areas in United States of America and obtained significant differences (i.e. for the purposes of design and analysis) in rainfall depth predictions from different methods currently in use though there is no evidence that these differences are significant in a statistical sense. They noted that characterization of the uncertainty in these predictions is important.
2.4. Uncertainty assessment methods

Several uncertainty assessment methods which are based on stochastic simulation, Bayesian theory, reliability and fuzzy-logic approaches are available pertaining to the impacts of climate change.

2.4.1. Monte Carlo based simulation methods

Monte Carlo method is a stochastic method of repeated sampling of random variables from a specified probability distribution to measure the response of the stochastic system of interest. New et al. (2000) developed a hierarchical impact model in Bayesian Monte Carlo simulations to address the uncertainty about future greenhouse gas emissions, the climate sensitivity and limitations and unpredictability in GCMs. Prudhomme et al. (2003) described a methodology to quantify the uncertainty of climate change impact on the flood regime of small UK catchments by a Monte Carlo simulation using several Global Climate Models, SRES-98 emission scenarios and climate sensitivities. Burn (2003) used Bootstrap resampling to derive confidence intervals associated with estimation of extreme flood quantiles for both at-site and pooled frequency analysis. Other applications of bootstrap resampling include Prudhomme et al. (2003) and Fowler et al. (2005).

2.4.2. Reliability Ensemble averaging (REA)

The reliability ensemble averaging (Giorgi and Mearns, 2002) takes into account two “reliability criteria”, the performance of the model in reproducing present-day climate called “model performance criterion” and the convergence of the simulated changes across models (“model convergence” criterion) to represent the best estimated responses. Giorgi and Mearns (2003) extended the REA method to calculate the probability of regional climate change exceeding given thresholds based on the ensembles of different
model simulations for application to transient experiments of different emission scenarios and GCMs of climate change projections in terms of temperature and precipitation.

2.4.3. **Bayesian model averaging (BMA)**

Downscaled outputs of a single GCM with a single climate change scenario represent a single trajectory among a number of realizations derived using various scenarios with GCMs and such a single trajectory alone, therefore, cannot represent a future hydrologic scenario and will not be useful in assessing hydrologic impacts due to climate change (Ghosh and Mujumdar, 2007). Climate change scenario projections that are obtained from several combinations of emission scenarios and GCMs can provide a range of possible outcomes which may be considerably different. Ensembles averaging, which is based on the arithmetic ensemble mean (AEM) has been used as a tool for predictive model uncertainty. However, AEM gives equal weights for all the models under consideration and also provides only a deterministic output so that predictive uncertainties can’t be expressed in a probabilistic manner. Assigning weights to the models used based on the predictive performances of the projected outcomes can reduce the uncertainty and also representation of these simulated outcomes in probabilistic terms is suitable for application in risk and adaptation studies. Hoeting et al. (1999) and Raftery et al. (2005) proposed Bayesian model averaging (BMA) as a useful tool to assess the predictive performance of model ensemble and individual ensemble members as it considers model uncertainties by not conditioning on a single “best” model, but on the entire ensemble of statistical models considered and it produces probability distributions which are the weights for the predictive performance of different models for the training period considered. Raftery et al. (2003) demonstrated BMA as a statistical post processing of ensemble output to produce calibrated and sharp predictive PDFs of model forecasts of temperature and sea level pressure by approximating the conditional PDFs as Normal distributions to estimate the model parameters (weighting factors and variances) by maximum likelihood and expectation-maximization (EM) algorithms. Lee et al. (2005) described a Bayesian analysis of the evidence for human-induced climate change
in global surface temperature observation as climate change detection and attribution assessment. Min et al. (2007) noted that if one has a high enough number of ensemble members forming a sample of the climate model population, BMA will give a realistic estimate of the modeling uncertainty of the climate system but this assumption probably does not hold due to an undersampling of the ‘climate model space’: even available models cannot be regarded as independent because they share components or are from the same institution. Sloughter et al. (2006) extended BMA to probabilistic quantitative precipitation forecasting using a gamma distribution as a predictive PDF of precipitation. Other works related to Bayesian analysis and BMA include Reis et al. (2005), Vrugt et al. (2006; 2007; 2008) and Wohling et al. (2009).

2.4.4. Fuzzy (alpha-cut) uncertainty analysis

The Fuzzy Extension principle provides a mechanism for the mapping of the uncertain parameters (inputs) defined by their membership functions to the resulting uncertain output (dependent variable) in the form of a membership function. The direct application of the Extension principle very often involves a computationally intensive procedure; therefore it is generally carried out in practice using the so-called α-cut (Maskey, 2004). Some application of extension principle for uncertainty propagation is presented by Hall et al. (2007) which combined fuzzy emission scenario uncertainty and imprecise probabilistic representation of climate uncertainties (i.e. climate sensitivity) for global mean temperature anomaly.

2.5. Risk based design and Risk management

In order to account for uncertainties of different sources and hence reduce the risks of failure related to urban water infrastructure, risk based approaches for design and management are appropriate. Due to the changing climate, currently some researchers such as Arnbjerg (2006) argued that the traditional return period based design of urban
water infrastructure turns out to be obsolete. The classical notions of “probability of exceedance” and “return period” are no longer valid under non-stationarity (Khaliq et al., 2006). The Risk-based design procedure integrates the procedures of uncertainty and reliability analyses in the design practice by considering the tradeoff among various factors such as failure probability, economics and other performance measures (Tung et al., 2006). Risk management builds on risk assessment by seeking to determine a management policy (Simonovic, 1997).

In stochastic approaches of risk management, several tools and methods pertinent to risk assessment, stochastic simulation and stochastic optimization are utilized. Equations which are pertinent to risk analysis and stochastic optimization entail representation of the uncertainty of the random variables in terms of probability density function. The main uncertain task relevant to the design and management of urban water infrastructure is the estimation of design storm and hence design flood. A perfect knowledge about the distribution of storm or flood is assumed which is not generally the case in reality (Tung et al., 2006). Under the assumption of stationarity for extreme precipitation events, the distributions and the return period are assumed to remain the same throughout the design life of urban water infrastructure but this assumption may not be valid under the impacts of anticipated climate change. Therefore, quantification of the uncertainties pertaining to these assumption will be useful for the risk based design and management of urban water infrastructure.
3. RESEARCH PROPOSAL AND PROGRESS

In this section, the proposed research methodology and research progress to data are provided. The research work to date includes identification of different sources of uncertainties, bootstrap resampling method for uncertainty assessment and discrete Bayesian method of averaging quantiles. The identified procedures are applied for a case study for the City of London (London climate station, Ontario). Delineation of homogeneous pooling groups for extreme precipitation events (i.e. annual maximum series) of different durations based on focused pooling approach (the region of influence) similarity measure and L-moment pooled frequency analysis, identification of the “best-fit” at-site and regional distributions have been executed. Quantiles are averaged to obtain deterministic quantiles from estimates obtained from at-site and different pooled observations. The results of uncertainty assessment pertinent to the delineation of homogeneous pooling groups are summarized in the form of quantile estimates and IDF curves with 95% confidence intervals estimated from non-parametric balanced bootstrap resampling of the sample observations. Box plots of the quantile estimates are also provided.

3.1. Research methodology

The main task in this research is to assess the uncertainties in the frequency analysis of extreme precipitation events in the form of intensity-duration-frequency (IDF) curves. The sources of uncertainties considered are those associated with the impacts of climate change, sampling uncertainty in terms of delineation of homogeneous pooling groups and the inherent uncertainties in the frequency analysis of extreme precipitation events. Projected scenarios of future extreme precipitation events are commonly used to assess the impacts of climate change on the frequency and magnitudes of extreme precipitation events. But these modelled estimates of future extreme precipitation values are uncertain themselves due to the uncertainty in the GCMs (process, or model uncertainty) and uncertainty in the future emission scenarios that may occur (input, or data uncertainty).
Uncertainties also arise from the downscaling approach used; the latter source of uncertainty will not be examined in this research.

IDF curves can be obtained from extreme precipitation data at a single site (at-site analysis) or by combining information from several similar sites (pooled or regional analysis). For the latter, this research will use the focused pooling approach (Burn, 1990) in which a potentially unique collection of sites is identified for estimating extremes at each target site. Both at-site and pooled IDF curves will be developed in the proposed research and the results compared. It is expected that the larger amount of information available in pooled analysis will lead to reductions in the uncertainty of the estimates of precipitation extremes (Burn, 2003).

Annual extreme precipitation series that will be obtained from the projected future climate scenarios and downscaled to scales relevant to design and management of urban water infrastructure will be used to estimate the quantiles and intensity-duration-frequency (IDF) curves for at-site and different homogeneous pooling groups. Then the uncertainties pertinent to the impacts of climate change and climate scenario projections, sampling in terms of delineation of pooling groups and the choice of statistical distribution will be assessed. Since each combination of GCM, emission scenario, homogeneous pooling group and statistical distribution will result in a potentially different estimate for a point on an IDF curve; the results from the ensemble of runs of different combinations will lead to a distribution of outcomes. For example, rather than having a single value for the design rainfall intensity corresponding to a 12-hour storm with a 25-year return period, the research results will give one result for each ensemble realization allowing the fitting of a probability density function (pdf) to these values. An alternative approach that can be followed is obtaining the weighted average of projected extreme precipitation events from the ensemble of outcomes of different GCMs and emission scenarios. Then this deterministic outcome will be used with respective homogeneous pooling groups and statistical distribution considered.
3.2. Research Progress

3.2.1. Identification of potential sources of uncertainties in frequency analysis of extreme precipitation events

The main sources of uncertainties in the frequency analysis of extreme precipitation events which are relevant to the estimation of design storm quantiles and IDF curves are identified and shown in the flow chart (Figure 3.1).

3.2.2. Uncertainty Assessment in quantile estimation of extreme precipitation events (from historical observations)

3.2.2.1. Data quality

- Tests for trend, stationarity and independence

In traditional statistical frequency analysis of extreme precipitation events, it is assumed that observational data are independent, identically distributed (iid) and stationary. Under stationarity assumption, the statistical distribution, parameters of the distribution, the frequency or return period and the corresponding magnitudes of extreme precipitation events will not change under future conditions. But under anticipated climate change, evaluation of the significance of trends in terms of change in the frequency and magnitude of extreme precipitation events, change in statistical distributions and their combined impact on the estimation of design storm quantiles and IDF curves is important. If the earth’s climate is changing, then its extremes are most likely those of a non-stationary process, which will have characteristics that change through time (Wang et al., 2004).
Figure 3.1: Flow chart for main sources of uncertainty in frequency analysis of extreme precipitation events relevant to the estimation of quantiles and IDF curves
The presence of significant non-stationarity in hydrologic time series can not be ignored when estimating design values for future time horizons (Cunderlik and Burn, 2003). Non-parametric approaches of trend tests have been extensively implemented. Bradley (1998) in his work on regional frequency analysis for evaluating changes in hydrological extremes determined that there is strong evidence for climate-related non-randomness in extreme precipitation in the Southern plains of the United States and also mentioned that many factors may influence the observed departures from the random model of hydrologic extremes including regional heterogeneity, distributional assumptions and intersite correlation. Adamowski et al. (2003) investigated the existence of trend in annual maxima rainfall data for various durations ranging from 5 minute to 12 hr in the province of Ontario (Canada) using the regional average Mann-Kendall S trend test and detected significant trends in precipitation records. Crisci et al. (2002) carried out a comprehensive analysis of the extreme rainfalls in Tuscany (Italy) with the objective of understanding and quantifying the uncertainties due to trends connected with the estimation of the design storms by using Pearson linear correlation coefficient and the Mann-Kendall test. They demonstrated that the hydrological consequences of this kind of climate variability have a major impact on the design of hydraulic works in the basin. Also, parametric approaches of trend analysis based on non-stationary extreme value analysis (Coles, 2001; Katz et al., 2002; Kharin and Zwiers, 2005; El Adlouni et al., 2007; Wang et al., 2004; Khaliq et al., 2006; Pujol et al., 2007) considered time as a “covariate” to model the impacts of climate change on the parameters of the distribution of extreme precipitation events.

Correlation can occur as spatial correlation or serial correlation. Spatial correlation (Hosking and Wallis, 1988; 1997; Mikkelsen et al., 1996; Martins and Stedinger, 2002; Madsen et al., 2002; Bayazit et al., 2004; Castellarin et al., 2008) can have an effect on the homogeneity test statistics in regional or pooled frequency analysis. The effect of intersite dependence on the regional L-moment algorithm is to increase the variability of the regional averages and this increases the variability of estimated growth curve (Hosking and Wallis, 1997). Madsen et al. (2002) in their extreme rainfall analysis based
on partial duration series (PDS) for Denmark noted that in general, the correlation is a
decreasing function of distance and the correlation being larger for larger durations.
Extreme precipitation events of longer durations and hence low intensity are usually the
result of frontal activities and cover larger areas while short durations of high intensity
storms are usually of localized convective storms. Therefore, higher correlation is
expected for longer duration storms than shorter duration storms. If there are long
concurrent records among the stations used in pooled frequency analysis, the effect of
spatial correlation can be studied. The problem of serial correlation is one of the
limitations for the partial duration series (peaks over thresholds) method of sampling of
extreme precipitation events. Hosking and Wallis (1997) noted that a small amount of
serial dependence in annual data series has little effect on the quality of quantile
estimates.

3.2.2.2. Homogeneity of pooling groups, choice of frequency distribution and
parameter estimation

Frequency analysis from historical extreme observations is widely used for prediction of
extreme precipitation quantiles. For sites with sufficient record length as compared to the
return period of the extreme quantile of interest, at site frequency analysis could give a
reliable estimation. But sufficient records are not usually available for reliable prediction
of quantiles corresponding to larger return periods hence data augmentation from the
regional observations through the method of pooled frequency analysis has been widely
practiced for both flood data (Hosking and Wallis, 1993; 1997; Burn, 1988; 1990; 1997;
2003; Burn et al., 2000) and precipitation data (Gellens et al., 2002; Nguyen et al., 2002;
Adamowski et al., 2003; Fowler et al., 2003; Kysely et al., 2007; Gaal et al., 2008). The
approach is most advantageous for variables such as precipitation that exhibit high and
largely random spatial variability (Kysely et al., 2007). Therefore, in frequency analysis
at a site with insufficient data, it should be preferred to base the analyses on all the
extreme precipitation data for homogeneous region which is called regional or pooled
frequency analysis. For pooled frequency analysis, the regional L-moment algorithm developed by Hosking and Wallis (1997) is commonly used. However, there is always uncertainty in quantile estimation which is associated with the delineation of homogeneous pooling groups, choice of statistical distributions and parameter estimation methods. Resampling or bootstrap approach can be employed to quantify these uncertainties in terms of confidence intervals of quantile estimates.

3.2.2.2.1. Non-parametric balanced bootstrap resampling (BBRS)

In non-parametric bootstrap (Efron 1982), the samples are drawn with replacement from the original sample. Sampling with replacement means that every sample is returned to the data set after sampling. So a particular data point from the original data set could appear multiple times in a given bootstrap sample. Davison et al. (1986) discussed balanced bootstrap resampling simulation as a method for improving the efficiency of simulation in Efron’s (1982) non-parametric bootstrap method. That is, one that reuses each of the sample observations exactly the same number of times. In balanced bootstrap, when resampling one makes sure that number of occurrences of each sample point is the same i.e. if we make B bootstrap, the number of \( x_i \) is equal to B in all bootstrap samples. Of course, in each sample some of the observation will be present several times and other will be missing. But for all of them all sample points are present and their number of occurrences is the same. Burn (2003) explained the main advantage of resampling or bootstrap approach that the bootstrap resampling approach of constructing confidence intervals does not require assumptions, in contrast to conventional approaches that are typically based on an asymptotic formula and require a distributional assumption and does not rely on the available sample size being large enough to ensure that the asymptotic behavior of the approach applies. Also the initial spatial correlation of the data from different sites is not affected (Pujol et al., 2007).
Balanced bootstrap resampling method can be used with any parameter estimation method. The procedure for balanced bootstrap resampling based on L-moment parameter estimation method to construct $100(1-2\alpha)$ % confidence intervals of quantile estimates, following Faulkner and Jones (1999) and Burn (2003) is given as below:

i. Create an original sample from the site (s) observations for each year from at-site or pooling groups of interest.

ii. By repeating each year of data $B$ times ($B = 999$ in this case) we would get a $NY*B$ rows by $NS$ columns matrix, where $NY$ is the number of years for which we have data at one or more sites and $NS$ is the number of sites.

iii. The resampling then involves random permutation selection of $NY$ rows of data for which we can then calculate at-site or regional L-Moments and quantiles for the statistical distributions and return periods of interest. This process is then repeated $B$ times.

iv. Calculate bootstrapped residuals ($E_i$), which are the deviations of each estimated quantile from the quantile estimate from the original sample. $E_i = QT_{i\text{est}} - QT_{\text{sam est}}$, where $QT_{i\text{est}}$ is quantiles estimated from bootstrapped samples and $QT_{\text{sam est}}$ is quantile estimated from the original sample corresponding to a $T$-year return period.

v. Rank these deviations in ascending order to find $E(u)$ and $E(l)$ for 95% confidence interval (i.e. significance level = 0.05) where $u = \alpha*(B+1)$ and $l = (1-\alpha)*(B+1)$ which corresponds to the upper and the lower confidence levels respectively, where $\alpha = \frac{1}{2}$ of the significance level. For $B = 999$, $u$ corresponds to 25th and $l$ corresponds to 975th bootstrap residuals.

vi. Finally, the confidence intervals for the quantile estimate can be constructed as $(QT_{\text{sam est}} - E(u), QT_{\text{sam est}} - E(l))$. This confidence interval follows a test-inversion approach (Faulkner and Jones, 1999; Carpenter, 1999).
3.2.2.2. Averaging of quantile estimates

As discussed earlier, there can be wide ranges of uncertainties in the estimation of quantiles and IDF curves. Therefore, deriving a single quantile estimate which may be the best representative or reliable estimate can be considered as an approach to minimize these uncertainties. The procedure for Bayesian approach of averaging to minimize the uncertainty due to the delineation of homogeneous pooling groups through obtaining a weighted deterministic quantile estimate from the set of estimates obtained from at-site and different pooling groups is described below. The same procedure can be followed for the choice of frequency distribution and parameter estimation methods. For a random variable \( Y \) which contains a set of observations \( y^n = (y_1, y_2, \ldots, y_n) \), the Bayesian rule for continuous distribution can be expressed by Bayes’s Theorem (Wasserman, 2000):

\[
f(\theta | y^n) = \frac{f(\theta) f(y^n | \theta)}{\int_{\Theta} f(\theta) f(y^n | \theta) d\theta} \tag{3.1}
\]

If the \( y_i \)'s are independent, the likelihood function can be written as (Wasserman, 2000):

\[
f(y^n | \theta) = \prod_{i=1}^{n} f(y_i; \theta) \tag{3.2}
\]

For a discrete case (Vrusias, 2005), suppose rules and inferences are represented in the following form: IF \( E \) is true, THEN \( H \) is true with probability \( p \). This rule implies that if event \( E \) occurs, then the probability that event \( H \) will occur is \( p \). \( H \) represents a hypothesis and \( E \) denotes evidence to support this hypothesis. The Bayesian rule for discrete case expressed in terms of hypotheses and evidence for multiple evidences and multiple hypotheses is (Vrusias, 2005):

\[
p(H_i | E_1, E_2, \ldots, E_n) = \frac{p(E_1 E_2 \ldots E_n | H_i) \times p(H_i)}{\sum_{k=1}^{N} p(E_1 E_2 \ldots E_n | H_k) \times p(H_k)} \tag{3.3}
\]

Two evidences namely the convergence of the estimate to the overall mean of estimates and the widths of 95% confidence intervals, which are estimated from BBRS, are
considered. Therefore, the conditional probabilities \( p(E|H) \) for the evidences have been determined based on the following two metrics:

i. The absolute difference between the T-year quantiles obtained from an individual at-site or pooled data and the arithmetic average of all the quantiles obtained from at-site and pooled data; and

ii. The width of the 95% confidence intervals determined from BBRS for the quantile estimates for each at-site and pooled data.

The premise for selecting these two metrics is based on the convergence criteria of Giorgi and Mearns (2002) and narrow confidence intervals correspond to sharp prediction or low predictive uncertainties of Raftery et al. (2003).

Computation of the posterior probabilities requires obtaining the conditional probabilities of all possible combinations of evidences for all hypotheses. Therefore, the conditional independence between the evidences is assumed (Vrusias, 2005) and for two evidences we obtain:

\[
p(H_i|E_1 E_2) = \frac{p(E_1|H_i) \times p(E_2|H_i) \times p(H_i)}{\sum_{k=1}^{N} p(E_1|H_k) \times p(E_2|H_k) \times p(H_k)} \quad \text{.........................(3.4)}
\]

\( p(H) \) are prior probabilities for possible hypotheses.

\( p(E|H) \) are conditional probabilities for observing evidence E if hypothesis H is true.

\( p(H|E) \) are posterior probabilities of hypothesis H upon observing evidence E.

With no prior evidence, an equal prior probability of being the best quantile estimate is assumed for each at-site and different pooled data. Hence for N total number of quantile estimates obtained from at-site and pooled observations, \( p(H) = 1/N \).

Arithmetic mean quantile, \( Q_{mean} = \frac{1}{N} \sum_{i=1}^{N} Q_i \) 

\( \quad \text{.................................(3.5)} \)
The absolute distance from the arithmetic mean, $AbsDmeanQ_i = |Q_i - Q_{mean}|$ ...(3.6)

The confidence interval width, $CIWQ_i = UCLQ_i - LCLQ_i$ ..................................(3.7)

The absolute distance from the mean ($AbsDmeanQ_i$) and the width of 95% confidence interval ($CIWQ_i$) are normalized to values between [0, 1] based on “score-range normalization for minimum” so that quantile estimates with small absolute difference from the arithmetic mean and with narrow width of 95% confidence interval is assumed to be likely the best estimate and hence larger normalized weights are assigned to them than those with large absolute differences from the mean and wide confidence intervals.

$$AbsDNmeanQ_i = \frac{\text{Max} (AbsDmeanQ_i) - (AbsDmeanQ_i)}{\text{Max} (AbsDmeanQ_i) - \text{Min} (AbsDmeanQ_i)}$$ ...........................................(3.8)

$$CIWNQ_i = \frac{\text{Max} (CIWQ_i) - CIWQ_i}{\text{Max} (CIWQ_i) - \text{Min} (CIWQ_i)}$$ ......................................................(3.9)

Normalization of the likelihood approach by Giorgi and Mearns (2003) is adopted to estimate the respective conditional probabilities of the evidences from the normalized weights:

$$p(E_1 | H_i) = \frac{AbsDNmeanQ_i}{\sum_{k=1}^{N} AbsDNmeanQ_k}$$ ..........................................................(3.10)

$$p(E_2 | H_i) = \frac{CIWNQ_i}{\sum_{k=1}^{N} CIWNQ_k}$$ ..........................................................(3.11)

Then the posterior probabilities are calculated from equation (3.4) that would be used as weights to calculate the weighted quantiles. The weighted quantiles which are obtained from the quantile estimates of the at-site and pooled data are computed as follow:
\[ WeightedQ = \sum_{i=1}^{N} \left( p \left( H_i \mid E_1, E_2 \right) \times Q_i \right) \]

\[ \sum_{i=1}^{N} p \left( H_i \mid E_1, E_2 \right) = 1 \]

3.2.3. Case Study

The proposed research seeks to develop procedures for quantifying uncertainty in extreme precipitation and it is anticipated that the procedures developed will be applicable to sites for which IDF curves need to be developed. The procedures developed will be applied to an area within the Upper Thames River Basin (UTRB) with a particular focus on the City of London, Ontario. The Upper Thames River Basin, with a drainage area of 3,450 km\(^2\), outlets to the Lower Thames Valley segment of the Thames River, which is a tributary to Lake St. Clair. The significant urban centre in the watershed is the City of London.

The Thames River Basin has experienced severe flooding and drought events in the past. Severe flooding in 2000 and more recent flooding in January of 2008 have raised concern over design storm standards for floodplain management and urban storm water infrastructure design. These concerns are similar to those of many Conservation Authorities across the Province and water management agencies in other parts of the country as well. Some municipalities in the UTRB are currently dealing with litigation as a result of flooding in recent years that considerably exceeded design expectations. As a result, substantial infrastructure reinvestment is being considered. However, concern is that system capacities may be increased without an adequate basis for design or a proper accounting for the uncertainty in the estimates of design storm magnitudes.
3.2.3.1. Frequency analysis of extreme precipitation events for London CS

Geographical location of the sites (X and Y co-ordinates) and their elevations above mean sea level can be selected as attributes for the region of influence (ROI) similarity measure. In analysis of rainfall extremes, Schaefer (1990) for Washington State, Mikkelsen and Harremoës (1993) for Denmark and Adamowski et al. (1996) for Canada found that certain statistical characteristics of rainfall extreme varies systematically with mean annual precipitation (MAP). In this case study, the precipitation data used are from the months of April to October, which covers precipitation in the form of rainfall, hence mean annual rainfall (MAR) is chosen as the fourth attribute for similarity measure. Hence, the similarity among the sites is measured by a weighted and scaled Euclidian distance metric in P-dimensional space (Burn et al., 2000) defined by sets of:

i. Geographical location (X and Y co-ordinates) of the stations;

ii. Elevations of the stations above mean sea level; and

iii. Mean annual rainfall at the stations.

The dissimilarity measure defined in equation (2.10) is used. Equal weights are assigned to all of the four attributes mentioned above. A total of 16 gauging stations in Southern Ontario are selected and tested for similarity with the target site for this case study, which is London climate station (the City of London, Ontario).

Mean annual rainfall for all the sites is calculated from the rainfall data obtained from Environment Canada data archive of 1961 to 2000. Annual maximum series precipitation data are also obtained from Environment Canada data archive (IDF files) for the sites considered. The whole record period range from 1926 to 2003 while most of the records are within the periods of 1963 to 2003. The at-site record lengths range from 23 to 74 years. Adamowski et al. (2003) and Bougadis et al. (2006) noted that storm durations of 5-, 10-, 15-, and 30 min are typical times of concentration for small urban watersheds and 1-, 2-, 6-, and 12 hr durations are typical times of concentration for larger rural watersheds for the province of Ontario. For this case study, extreme precipitation events corresponding to durations of 15-, and 30 min, 1-, 2-, 6-, 12-, and 24h are analyzed.
These extreme precipitation events from the stations considered are pooled and tested for homogeneity based on Hosking and Wallis homogeneity test and discordancy measures. Two sites, out of the total sixteen sites tested, appeared to be discordant and one other site is heterogeneous and hence the total number of sites for further pooled frequency analysis is 13. Location map of the case study area and precipitation stations is given in Figure 3.2.

Although pooled approach augments the at-site short records of extreme precipitation events, the homogeneity of pooling groups is indispensable in order to obtain reliable quantile estimates. By utilizing the ROI similarity measure and L-moment heterogeneity measures or H-statistics of Hosking and Wallis (1997), pooling groups from different combinations of sites appeared to be homogeneous but the estimated quantiles for each pooling group are different. Therefore, it is crucial to check the significance of the uncertainty pertinent to the selection of pooling groups which can be attributable to the uncertainty due to sampling of data. Hence to assess this uncertainty, quantiles for the annual maximum series precipitation intensity have been estimated for at-site (London climate station) observations and 12 sets of pooled observations from homogeneous pooling groups containing a total of 2, 3, ......, 13 sites. Then these homogeneous sites are ranked according to their proximity to the target site and are given in Appendix A. The target site has a record length of 55 years. The total homogeneous pooling groups containing 13 sites has a total pooled observation length of about 515 years. Therefore, the at-site record and the pooled records may give reliable quantile estimates corresponding to a recurrence interval (return period) of up to 10 years and 100 years respectively according to a 5T guideline (Jacob et al., 1999), which suggests that a pooling group should contain at least 5T station-years of data so as to obtain reasonably accurate estimates of the T-year quantile.
In statistical frequency analysis of extreme values, usually the “best-fit” statistical distribution is selected based on indices which indicate the goodness-of-fit. Goodness-of-fit tests are hypothesis testing of whether the data represent the assumed distribution or not. But it is not possible to certainly know that the selected “best fit” distribution represents the true parent distribution of the population since the inferences are drawn based on limited numbers of observed sample. In addition, different types of statistical distributions may reasonably fit to the data based on different goodness-of-fit measures yet may yield different quantile estimates. In order to minimize the uncertainty due to the choice of statistical distributions, some commonly used statistical distributions for frequency analysis of extreme precipitation events such as generalized extreme value

Figure 3.2: Location map of the case study area and precipitation stations
(GEV), extreme value type 1 (EV1), Pearson type III (PE3), generalized logistic (GLOG), generalized Pareto (GPAR) and Normal distributions (GNOR), have been tested by different goodness-of-fit measures such as Z-values and L-skewness versus L-kurtosis L-moment ratio diagram (Hosking and Wallis, 1993; Peel et al., 2001) for pooled frequency analysis based on L-moment parameter estimation. In most cases, the distribution being tested will have location and scale parameters which can be chosen to match the regional average mean and “coefficient of L-variation” (L-CV). The goodness-of-fit will therefore be judged by how well the L-skewness (L-CS) and L-kurtosis (L-CK) of the fitted distribution match the regional average L-skewness and L-kurtosis of the observed data (Hosking and Wallis, 1993). The sample regional average L-moment ratios are compared with the population values of the frequency distributions and the distribution that gives the best fit to the data is chosen among the candidate distributions. In addition, choice of the at-site “best fit” distribution for target site of London CS has been done based on Chi square ($\chi^2$) and Kolmogorov-Smirnov (K-S) goodness-of-fit tests (Rao and Hamed, 2003) for ML and PWM parameter estimation methods. Results for heterogeneity statistics and goodness-of-fit measures are given in appendix B. Plots for regional L-moment diagrams are given under appendix C. Comparison of statistical distributions for at-site records of London CS are given under appendix D. Based on the different goodness-of-fit measures described earlier, the distributions given in Table 3.1 appeared to be the “best-fit” for respective durations of extreme precipitation events.

However, it is expected that distributions of extreme value types and Pearson type III distribution are more robust than the Normal and Generalized Pareto distributions for environmental data like extreme precipitation events sampled from annual maximum series observations. Furthermore, it is observed that there are larger differences among the quantile estimates from GPAR distribution and other distributions considered. For extreme precipitation intensity of 1 hr duration, generalized extreme value distribution is the second “best fit” based on the regional L-moment diagram and the “best fit” for at-site data based on $\chi^2$ and K-S tests. Therefore GEV distribution can be selected as a robust distribution for 1 hr duration. Similarly, Pearson type III distribution which is the
“best-fit” based on regional L-moment diagram and KS test and also the second “best-fit” distribution based on the Z-value can be selected as the robust distribution for 24 hr duration extreme precipitation intensity.

### Table 3.1: “Best-fit” distributions based on different goodness-of-fit measures for annual maximum precipitation intensities of different durations of London CS

<table>
<thead>
<tr>
<th>Durations</th>
<th>LM: regional Z – values</th>
<th>LM ratio diagram</th>
<th>ML: at site $\chi^2$</th>
<th>K-S $\chi^2$</th>
<th>PWM: at-site K-S</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 min</td>
<td>GEV/GLOG</td>
<td>GEV</td>
<td>GEV</td>
<td>EV1</td>
<td>PE3</td>
</tr>
<tr>
<td>30 min</td>
<td>GEV/GNOR</td>
<td>GEV</td>
<td>PE3/EV1</td>
<td>GEV</td>
<td>GLOG</td>
</tr>
<tr>
<td>1 hr</td>
<td>GNOR</td>
<td>LOGNOR/GEV</td>
<td>GEV</td>
<td>GEV</td>
<td>GEV</td>
</tr>
<tr>
<td>2 hr</td>
<td>GEV</td>
<td>GEV</td>
<td>GLOG</td>
<td>GEV</td>
<td>GLOG</td>
</tr>
<tr>
<td>6 hr</td>
<td>PE3</td>
<td>PE3</td>
<td>EV1/PE3</td>
<td>PE3</td>
<td>PE3</td>
</tr>
<tr>
<td>12 hr</td>
<td>PE3</td>
<td>PE3</td>
<td>EV1</td>
<td>EV1/GLOG</td>
<td>GEV/PE3</td>
</tr>
<tr>
<td>24 hr</td>
<td>GPAR</td>
<td>PE3</td>
<td>EV1</td>
<td>PE3</td>
<td>EV1</td>
</tr>
</tbody>
</table>

In order to be able to visualize and compare the magnitudes of the uncertainties attributable to the methods of parameter estimation, choice of frequency distribution and delineation of pooling groups, quantile estimations from three parameter estimation methods (ML, MOM and LM); three statistical distributions (EV1, GEV and PE3) and two datasets (at-site and pooled data from 13 sites) for 1 hr and 12 hr durations of annual maximum precipitation events are displayed in Figures 3.3 and 3.4 respectively.
It is indicated in Figures 3.3 and 3.4 that the uncertainty in quantile estimation due to the delineation of homogeneous pooling group dominates those due to parameter estimation methods and the choice of statistical distributions. Moreover, the uncertainty due to the choice of frequency distribution is expected to be minimized since selection of the “best-fit” and robust distribution has been done based on combinations of different goodness-of-fit measures. Further analyses will be based on the two distributions namely GEV and PE3 as given in Table 3.2, which are selected as the “best fit” based on the goodness-of-fit measures and reasonable judgments as described earlier. It is observed that different types of statistical distributions appeared to be the “best-fit” for different durations of extreme precipitation events. Therefore, it can be concluded that rigorous distribution selection methods should be followed for extreme precipitation events of different durations rather than fitting a single statistical distribution for all durations.

Table 3.2: Selected “best fit” distributions for annual maximum precipitation intensities of different durations for London CS

<table>
<thead>
<tr>
<th>Durations</th>
<th>“Best-fit” distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 min</td>
<td>GEV: Generalized extreme value</td>
</tr>
<tr>
<td>30 min</td>
<td>GEV: Generalized extreme value</td>
</tr>
<tr>
<td>1 hr</td>
<td>GEV: Generalized extreme value</td>
</tr>
<tr>
<td>2 hr</td>
<td>GEV: Generalized extreme value</td>
</tr>
<tr>
<td>6 hr</td>
<td>PE3: Pearson type III</td>
</tr>
<tr>
<td>12 hr</td>
<td>PE3: Pearson type III</td>
</tr>
<tr>
<td>24 hr</td>
<td>PE3: Pearson type III</td>
</tr>
</tbody>
</table>

Though the H-statistics homogeneity criteria of Hosking and Wallis (1997) is satisfied for the regions to be considered homogeneous as displayed in appendix B, there are significant variations among quantile estimates from at-site and different pooled data.
which shows that the homogeneity based on H-statistics alone can’t guarantee reliable quantile estimation and hence uncertainty due to the delineation of homogeneous pooling groups needs to be addressed.

3.2.3.2. Uncertainty assessment of delineation of homogeneous pooling groups

In order to express the uncertainties due to the delineation of homogeneous pooling groups in terms of confidence intervals of quantile estimates, non-parametric balanced bootstrap resampling from at-site and pooled observations for different pooling groups have been performed to estimate 95% confidence intervals for quantile estimates corresponding to different durations and return periods of interest. Annual maximum series of precipitation data, the selected statistical distributions given in Table 3.2 and L-moment method of parameter estimation are used.

The quantile estimates for London CS from the AMS precipitation events of different durations from at-site and pooled original samples are displayed in the Figures 3.5, 3.6 and under appendix E. In addition, dispersions of quantile estimates are displayed in the form of box plots and presented under appendix F. The bottom and top of box shows the lower and upper quartiles respectively and the line through the box shows the median. Whiskers extend from each end of the box to the most extreme values within 1.5 times the interquartile range from the ends of the box. Outliers are data with values beyond the ends of the whiskers which are displayed with a + sign.

It is observed that for frequent events (for a return period of up to 10 years), there is no real difference in the quantile estimates from at-site and pooled observations. Therefore, for such events at-site approach can give reliable quantile estimate. But for quantiles corresponding to higher return periods, there are larger differences in quantile estimates that increase with the return period. As can be observed from Figure 3.7, quantile estimates from some pooled data exceed the 95 % upper confidence limit of the at-site estimate which shows that even the 95% interval estimate for the at-site data doesn’t
capture the full range of uncertainty in the delineation of homogeneous pooling groups considered. Though it is expected that adding more data in the pooling group increases the information content of the data and may provide reliable estimation, in the contrary there is an associated uncertainty due to the possible increase in the heterogeneity of pooled data as additional sites are added. In addition, it is indicated that pooling has different effects on the results of frequency analysis of extreme precipitation events of different durations. For instance, for 15 min duration the higher values of quantile estimates correspond to the pooled data from small number of sites while for 12 hr duration it is the lower values of quantile estimates that correspond to pooled observations from small number of sites. The reason may be since different durations of extreme precipitation events may have different degrees of spatial variability and heterogeneity.

The uncertainty in delineation of homogeneous sites for extreme precipitation events may be partially attributable to the choice of site characteristics (attributes) for similarity measures. Therefore, further research work is required if reduction of the uncertainty due to delineation of pooling groups can be achieved through identification of the climatic (metrological) attributes which can better explain the similarity in extreme precipitation events of different durations.

### 3.2.3.3. Averaging of quantiles for London CS

Based on the Bayesian averaging of quantile estimates, which is discussed under 3.2.2.2.2, the weighted mean estimates of Intensity-Duration-Frequency relationships with their 95% lower and upper confidence levels have been found and summarized in Table 3.3 and Figure 3.8 contains IDF curves with 95% confidence intervals. The LCL and UCL correspond to the arithmetic averages of the 95% confidence widths estimated from the at-site and pooled data.
Return period (years)
Table 3.3: Weighted mean annual maximum precipitation intensity quantiles with 95% confidence levels for London CS

<table>
<thead>
<tr>
<th>Durations</th>
<th>Weighted mean quantile estimates and 95% CI from AMS precipitation intensity (mm/hr)</th>
<th>Return periods (T) in years</th>
<th>“Best-fit” distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimates</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>15 min</td>
<td>Estimates</td>
<td>63.144</td>
<td>84.005</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>54.900</td>
<td>73.820</td>
</tr>
<tr>
<td></td>
<td>UCL</td>
<td>68.408</td>
<td>92.426</td>
</tr>
<tr>
<td>30 min</td>
<td>Estimates</td>
<td>40.656</td>
<td>55.487</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>36.161</td>
<td>49.262</td>
</tr>
<tr>
<td></td>
<td>UCL</td>
<td>45.172</td>
<td>61.993</td>
</tr>
<tr>
<td>1 hr</td>
<td>Estimates</td>
<td>24.044</td>
<td>33.283</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>20.700</td>
<td>28.964</td>
</tr>
<tr>
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4. CONCLUSIONS AND PLANS FOR FUTURE WORK

Different sources of uncertainties in the frequency analysis of extreme precipitation events and estimation of associated IDF curves and methods for the assessment of these uncertainties are identified. Procedures for non-parametric bootstrap resampling approach for uncertainty assessment of delineation of homogeneous pooling groups have been given. Discrete Bayesian approach of averaging quantile estimate to obtain deterministic quantile values and reduce the uncertainty in quantile estimation due to the delineation of homogeneous pooling groups has been discussed.

The proposed methods have been applied as a case study for London climate station (the City of London, Ontario) in the upper Thames river basin. Uncertainties in the frequency analysis of extreme precipitation events and associated IDF curves using historical (past) observations of annual maximum series of precipitation events have been assessed. Different goodness-of-fit measures are employed for the choice of statistical distribution with the main intent to minimize the uncertainty due to the choice of frequency distributions. Similarly, different parameter estimation methods have been used to evaluate the magnitude of uncertainties attributable to parameter estimation methods. It is observed that the uncertainty in quantile estimates due to sampling in terms of the delineation of homogeneous pooling group exceeds the uncertainties attributable to the choice of statistical distributions and parameter estimation methods. Then the uncertainties which are attributed to sampling in terms of delineation of homogeneous pooling group have been quantified and displayed in the form of quantile estimates and their 95% confidence intervals, which are estimated by non-parametric balanced bootstrap resampling method and also displayed as box plots.

It is indicated that there are wide ranges of uncertainties in the frequency analysis of extreme precipitation events of particular durations and return periods. The uncertainties associated with the impacts of anticipated climate change (i.e. future climate scenarios) have not been included in the work to date. It can be inferred that incorporation of the
uncertainties pertinent to the impacts of climate change and climate change scenario projections will further widen the range of uncertainty in the frequency analysis of extreme precipitation events (i.e. estimation of quantiles and associated IDF curves). Therefore, quantification and presentation of the overall uncertainties by including the uncertainties associated with the impacts of climate change is indispensable to obtain results that can be useful as inputs to derive procedures and guidelines for risk based design and management of urban infrastructure.

Most of the research publications related to the impacts of climate change on future precipitation events have focused on addressing the impacts of climate change on mean precipitation. Also, in the research work related to extreme precipitation events and the design and management of urban water infrastructure, little or no effort has been done to incorporate the uncertainties in the actual design and life time management practices of urban water infrastructure as adaptation strategies for the anticipated climate change. Therefore, the research gaps which will be addressed in the remainder of this research are:

i. Identification of appropriate climatic (metrological) attributes for improved measurement of regional similarity in extreme precipitation events of different durations to reduce the uncertainties in the delineation of homogeneous regions for pooled frequency analysis;

ii. Quantification and representation of the overall uncertainties pertinent to the impacts of climate change, delineation of homogeneous pooling groups and the inherent uncertainties in statistical frequency analysis of extreme precipitation events in the form of IDF curves. The use of ensembles of projected climate change scenarios to estimate probabilistic IDF curves and methods for deriving reliable deterministic IDF curves will be investigated;

iii. Evaluation of the performances of both at-site and pooled frequency analysis approaches for both historical observations and future climate conditions;
iv. Temporal disaggregation of quantile estimates from projected extreme precipitation events of coarser temporal scales to obtain quantile estimates for fine temporal resolutions which are relevant for the design and management of urban water infrastructure; and

v. Outlining approaches how to incorporate the overall uncertainties pertinent to the frequency analysis of extreme precipitation events in the risk based design and management of urban water infrastructure with an objective of minimizing the risks of flooding throughout the design life.
## 5. Schedule

Table 5.1: Schedule of work

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<td></td>
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<tr>
<td>Model development for frequency analysis</td>
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<td>Writing Thesis proposal</td>
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<td>Data acquisition (projected scenarios)</td>
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<tr>
<td>Model development for assessment of uncertainties associated with the impacts of climate change, sampling and inherent uncertainties in frequency analysis of extreme precipitation events</td>
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<tr>
<td>Quantification and presentation of uncertainties and evaluation of the performances of at-site and pooled frequency analysis</td>
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<tr>
<td>Outlining methods how to incorporate the overall uncertainties in risk based design and management of urban water infrastructure</td>
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6. References


59


Guo, Y. (2006). Updating rainfall IDF relationships to maintain urban drainage design


64


Planton, Serge; Déqué, Michel; Chauvin, Fabrice; Terray, Laurent. (2008). Expected impacts of climate change on extreme climate events. C.R. Geoscience 340, 564-574.


7. Appendices

Appendix A: Stations homogeneous to London CS for pooled frequency analysis from annual maximum precipitation observations

<table>
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<th>At-site</th>
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<td>Preston WP CP</td>
</tr>
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<td>Delhi CS</td>
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<td>Waterloowellington</td>
<td>Waterloowellington</td>
</tr>
<tr>
<td>Hamilton A</td>
<td>Hamilton A</td>
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<tr>
<td>Brantford MOE</td>
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<td>Windsor A</td>
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<table>
<thead>
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<th>Number of sites and names of sites in pooling groups</th>
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Appendix B: Summary results for homogeneity tests and choice of distributions for frequency analysis of annual maximum precipitation intensities (mm/hr) of different durations for London CS

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<p>| 30 min                   | 2                      | 1.16 | 0.43 | 0.15 | -0.4 | -1.32 | GNOR | -0.3 | -0.1 | -0.5 |
|                          | 3                      | 0.87 | 0.09 | -0.19 | -0.7 | -1.8 | GEV | -0.6 | -0.8 | -1.1 |
|                          | 4                      | 0.69 | -0.3 | -0.56 | -1.2 | -2.5 | GEV | -1 | -1.3 | -1.6 |
|                          | 5                      | 0.75 | -0.3 | -0.6 | -1.2 | -2.7 | GEV | -0.6 | -1.5 | -2.1 |
|                          | 6                      | 1.42 | 0.22 | -0.14 | -0.8 | -2.6 | GEV | -1 | -1.7 | -1.7 |
|                          | 7                      | 1.02 | -0.3 | -0.61 | -1.3 | -3.3 | GEV | -1.1 | -1.8 | -1.7 |
|                          | 8                      | 1.84 | 0.32 | -0.06 | -0.8 | -3.2 | GNOR | -1.2 | -1.8 | -1.1 |
|                          | 9                      | 2.23 | 0.58 | 0.27 | -0.4 | -3.1 | GNOR | -1.3 | -1.5 | -0.4 |
|                          | 10                     | 2.53 | 0.69 | 0.38 | -0.4 | -3.4 | GNOR | -1.6 | -1.7 | -0.7 |</p>
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<td>-0.41</td>
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<td>0.01</td>
<td>-0.36</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

At site GEV
2 | 0.75 | 0.12 | -0.18 | -0.7 | -1.44 | GEV  | -0.1 | -0.6 | -0.4 |
3 | 1.9  | 1.11 | 0.74  | 0.08  | -0.85 | PE3  | -0.5 | -1.1 | 0.25 |
4 | 2.82 | 1.76 | 1.36  | 0.62  | -0.79 | PE3  | 0.47 | -0.7 | 0.18 |
5 | 2.31 | 1.36 | 0.86  | -0  | -1.05 | PE3  | 1.38*| -0.2 | 0.69 |
6 | 1.74 | 0.81 | 0.3   | -0.6 | -1.54 | GNOR | 0.74 | -0.7 | 0.35 |
7 | 1.47 | 0.41 | -0.09 | -1 | -2.2  | GNOR | 0.95 | -0.3 | 0.33 |
8 | 1.46 | 0.36 | -0.14 | -1 | -2.3  | GNOR | 0.67 | -0.8 | -0.2 |
9 | 1.7  | 0.37 | -0.14 | -1.1 | -2.8  | GNOR | 0.56 | -0.5 | -0   |
10 | 2.05 | 0.52 | 0.02  | -1 | -3.1  | GNOR | 0.35 | -0.3 | 0.15 |
11 | 2.56 | 0.73 | 0.25  | -0.7 | -3.5  | GNOR | 0.31 | 0.03 | 0.39 |
12 | 2.75 | 0.79 | 0.27  | -0.8 | -3.7  | GNOR | 0.11 | -0.4 | -0.1 |
13 | 2.9  | 0.74 | 0.22  | -0.9 | -4.2  | GNOR | 1.39*| -0.3 | -0.3 |

At site GEV
2 | 0.2  | -0.5 | -0.68 | -1.1 | -2.1  | GLOG | -1.2 | -1.3 | -1.5 |
3 | 1.05 | 0.2  | -0.06 | -0.6 | -1.8  | GNOR | -1  | -1.7 | -0.9 |
4 | 1.72 | 0.55 | 0.3   | -0.2 | -2.1  | PE3  | 0.31 | -0.8 | -0.3 |
5 | 1.63 | 0.6  | 0.2   | -0.5 | -1.9  | GNOR | 1.16*| 0.1  | 0.29 |
6 | 0.57 | -0.3 | -0.75 | -1.5 | -2.6  | GEV  | 0.37 | 0.03 | 0.43 |
7 | 0.81 | -0.4 | -0.75 | -1.4 | -3.2  | GEV  | 1.19*| 1.03 | 1.31 |
8 | 0.93 | -0.3 | -0.66 | -1.4 | -3.1  | GEV  | 1.11*| 0.87 | 1.07 |
9 | 1.07 | -0.3 | -0.73 | -1.5 | -3.6  | GEV  | 0.89 | 0.57 | 0.7  |
10 | 1.36 | -0.2 | -0.64 | -1.5 | -3.9  | GEV  | 0.71 | 0.72 | 0.82 |
11 | 1.71 | -0  | -0.49 | -1.4 | -4    | GEV  | 0.57 | 0.23 | 0.26 |
12 | 1.84 | -0.1 | -0.57 | -1.5 | -4.6  | GEV  | 0.59 | 0.81 | 0.62 |
13 | 1.85 | -0.3 | -0.71 | -1.7 | -5    | GEV  | 0.8  | 0.54 | 0.2  |

At site EV1/PE3
2 | 1.32 | 0.59 | 0.3   | -0.3 | -1.17 | PE3  | 1.39*| -0.1 | -0   |
3 | 2.26 | 1.4  | 1.04  | 0.39 | -0.7  | PE3  | 0.92 | -0.6 | 0.08 |
4 | 2.75 | 1.76 | 1.32  | 0.53 | -0.67 | PE3  | 0.52 | -1.1 | -0.7 |

6 hr |     |     |     |     |     |

At site EV1/PE3
2 | 1.32 | 0.59 | 0.3   | -0.3 | -1.17 | PE3  | 1.39*| -0.1 | -0   |
3 | 2.26 | 1.4  | 1.04  | 0.39 | -0.7  | PE3  | 0.92 | -0.6 | 0.08 |
4 | 2.75 | 1.76 | 1.32  | 0.53 | -0.67 | PE3  | 0.52 | -1.1 | -0.7 |
|  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 |  |  |  | 2.49 | 1.54 | 1.03 | 0.12 | -0.85 | PE3 | 0.56 | -0.8 | -0.5 |
| 6 |  |  |  | 2.23 | 1.34 | 0.77 | -0.2 | -1.0 | PE3 | 0.01 | -0.6 | -0.4 |
| 7 |  |  |  | 2.13 | 1.09 | 0.52 | -0.5 | -1.56 | PE3 | -0.1 | -0.5 | -0.4 |
| 8 |  |  |  | 2.58 | 1.4 | 0.85 | -0.1 | -1.51 | PE3 | -0.1 | -0.2 | 0.02 |
| 9 |  |  |  | 2.83 | 1.45 | 0.88 | -0.2 | -1.9 | PE3 | -0.3 | -0.1 | -0.1 |
| 10 |  |  |  | 3.58 | 1.95 | 1.38 | 0.3 | -1.9 | PE3 | -0.2 | 0.24 | 0.45 |
| 11 |  |  |  | 3.79 | 2.1 | 1.43 | 0.2 | -2.0 | PE3 | -0.5 | 0.15 | 0.15 |
| 12 |  |  |  | 3.88 | 2.02 | 1.35 | 0.08 | -2.4 | PE3 | -0.5 | 0.18 | 0.05 |
| 13 |  |  |  | 3.73 | 1.87 | 1.12 | -0.3 | -2.6 | PE3 | -0.5 | 0.24 | 0.13 |

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<th>GEV/ PE3</th>
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<td>2.71</td>
<td>1.53</td>
<td>1.36</td>
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<tr>
<td>4</td>
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<td>1.66</td>
<td>1.03</td>
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<td>10</td>
<td>4.85</td>
<td>2.59</td>
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</table>
Bold Z-values: $|Z^{DIST}| \leq 1.64$ (fit distributions), *: $H_1 > 1.0$ – “possibly heterogeneous”
by Hosking and Wallis (1997) if any other distribution doesn’t fit.

Appendix C: L-moment ratio (L-CS vs. L-CK) diagram for pooled frequency analysis of
annual maximum precipitation intensity of different durations from 13 stations (Target
site-London CS).
Appendix D: Distributions comparison for frequency analysis of annual maximum precipitation intensity of different durations for London CS (at-site)
Appendix E: Annual maximum precipitation intensity quantile estimates for London CS from at-site and pooling groups with 2 to 13 sites and EC-at site (EV1 distribution) for different durations.
Appendix F: Box plots for annual maximum precipitation intensity quantile estimates for different durations for London CS from at-site and pooling groups with 2 to 13 sites.
Precipitation intensity quantiles (mm/hr)

Return period (years)