Vortex shedding from slender surface mounted pyramids

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Abstract

The current investigation examines periodic vortex shedding from surface mounted pyramids with taper ratios (2*height/base diameter) of 7.6 to 14.3 in a thin boundary layer using surface pressure measurements along the obstacle faces. Over the lower portion of the pyramids (y/h<~0.5) regular periodic vortex shedding was observed. Near the tip, a large time-scale structure was observed. This structure exhibits a strongly modulated period. In general, the period decreases with increasing taper ratio. The results are found to be similar to tapered plates of similar taper ratios and pyramids with smaller taper ratios. The discrepancy between taper plates and pyramids with taper ratios less than 7.6 are attributed to the inclined leading edge and the afterbody effect of the pyramids.

List of Symbols

ζ	Apex angle
d_2	Base diameter of tapered geometries
δ	Boundary Layer Thickness
ρ	Density
U_∞	Free Stream Velocity
υ	Kinematic Viscosity
h	Length of the geometry
P	Pressure
P_∞	Static Pressure
f	Vortex shedding frequency (Hz)

$\zeta = 2\arctan\left(\frac{1}{TR}\right)$ Apex angle

$AR_T = \frac{2h}{d_2}$ Aspect Ratio for tapered geometries

$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho U_\infty^2}$ Pressure Coefficient

$\mathrm{Re} = \frac{U_\infty d_2}{\nu}$ Reynolds number

$St = \frac{fd_2}{U_\infty}$ Strouhal number for tapered geometries

$TR = \frac{2h}{d_2}$ Taper Ratio

Introduction

Vortex shedding in the wake of two-dimensional geometries, with various cross sections, have been studied extensively over the past 100 years. It is only recently, however, that vortex shedding formed by three-dimensional geometries has been studied in detail. Typically, three-dimensional bodies are given a linear taper along their length, by decreasing the taper ratio the body become more three-dimensional. The following study will examine tapered bodies which have a free end condition at their tip.

Castro and Watson[i] investigated tapered plates with taper ratios of 0.58 to 40 (ζ= 120° - 5.72°) with a free tip condition. For taper ratios less than 1, it was found that there was no vortex shedding; in this case the three dimensionality of the body was sufficient to suppress the shed vortices. For taper ratios ranging from 1-7.5 (ζ= 90°-15.2°), a single Von Kármán type shedding cell was found to span the entire length of the plate. In addition, it was found that plates of this taper ratio also contained an in-phase vortex cell near the tip of the body, which is shed at a lower frequency than the base cell. The next regime spans from a taper ratio of 7.5 to 20 (ζ= 15.2°-5.72°) and exhibits similar behaviour to the previous region. However, the Von Kármán type vortex cell spans only the lower portion of the body and the in phase cell found near the tip has a lower frequency than that of the previous regime.

Experiments conducted by AbuOmar and Martinuzzi[ii,iii] on surface mounted pyramids with taper ratios of 0.58 to 7.6 (ζ= 120° - 15°) have shown that for taper ratios less than 1 vortex shedding is completely suppressed. For taper ratios between 1 and 7.6, the periodic vortex shedding is found only over the lower half of the pyramid height. Along the side face of the pyramid, there is a vortex pair arrangement: the primary shed vortex is deflected downstream of the body by one of these secondary vortices generated at the leading edge of the front face. The other secondary vortex joins with a similar vortex from the opposing side directly behind the tip of the pyramid similar to an arch type vortex found in low aspect ratio cylinders[iv]. However, in this case the arch vortex is a mean flow structure that is not shed from the body. Figure 1 shows the vortex arrangement formed by surface mounted pyramids. Since the vortices are symmetric about the centre line of the pyramid, Figure 1 only shows the vortices from a single side.

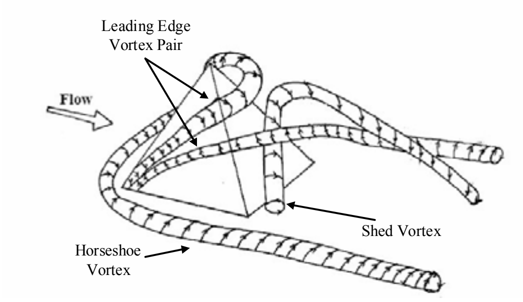

Figure 1 Vortex Model for surface mounted pyramids, Modified from Martinuzzi and AbuOmar[iii]

These results indicate that while the flows generated by tapered flat plates and pyramids are similar, there are important differences in the shedding behaviour between taper ratios of 1 to 7.6 (ζ= 90°-15°). The discrepancies in the flow behaviour between the tapered plates and pyramids are believed to be due to the inclined leading edge and afterbody of the pyramid. Consequently, it is expected that as the pyramid taper ratio is increased the influence of the inclined leading edge should diminish and the behaviour between tapered plates and pyramids should begin to match. Interestingly, the tapered plates show a transition at a taper ratio of 7.5 (ζ=15.2°) where the behaviour seems to exhibit similar behaviour to pyramids with smaller taper ratios. The next question that arises is whether the pyramids also show a change in behaviour at this taper ratio. The present study attempts to further understand the effects of taper ratio on the vortex shedding process from surface mounted pyramids when the tip is contained within the flow. Of specific interest are the differences in observed behaviour between the surface mounted pyramids and the tapered plates and also if these differences continue to higher taper ratios.

Experimental Setup

Three pyramid models were tested with taper ratios of 7.6, 9.5, 14.3 (ζ= 15°, 12°, 8°). All models were 0.914m in height, and had base width ranging from 0.13m to 0.24m. The pyramids were instrumented with pressure taps 1.6mm in diameter along the centre line of the faces perpendicular to the flow (side faces). The pyramids with taper ratios of 7.6 and 9.5 (ζ= 15°, 12°) had twenty-four taps spread over the centreline of each side face, and two reference taps located at the mid height of the upstream and downstream faces giving a total of fifty taps on each model. The 14.3 taper ratio (ζ= 8°) pyramid had a similar pressure tap arrangement, except the tap closest to the tip was not present due to manufacturing (space) limitations.

The measurements were conducted in the suction-type Boundary Layer Wind Tunnel I of the University of Western Ontario, which has a cross-section at the entrance of 1.5m x 2.4m and a total length of 33m. The tunnel has an inlet contraction ratio of 2:1 and the cross section expands to 2.15m x 2.4m at the exit to maintain a zero pressure gradient. The models were mounted on the sidewall of the working section with the centre line of the pyramid located 0.46m above the tunnel floor and 1.4m downstream from the inlet in order to minimize the boundary layer thickness. Figure 2 contains a schematic of the experimental set-up and nomenclature.

A full two-dimensional profile was taken at this downstream location using Hot-wire Anemometry (HWA) to verify the inlet flow conditions. At the measurement location the boundary layer thickness was approximately 0.08m or 8.3% of the model height. Outside the boundary layer, the turbulence intensity was found to be approximately 1%. The wind tunnel speed, while conducting these measurements, was approximately 10m/s, corresponding to a Reynolds number ranging from 80,000 to 150,000. The data was sampled at 400Hz for 170 seconds using four Pressure Systems Inc. model ESP-16 solid-state pressure transducers. The frequency response of the transducers and tubing was found to be flat to 200Hz. This sampling time produces a minimum of 850 shedding cycles for the current geometry. Each transducer scans sixteen pressure taps sequentially creating a maximum time lag in measurements of approximately 0.0025 seconds. The free stream velocity and the static pressure were measured using a Pitot-static tube, which are located in the centre of the working section of the wind tunnel and used to calculate the pressure coefficient.

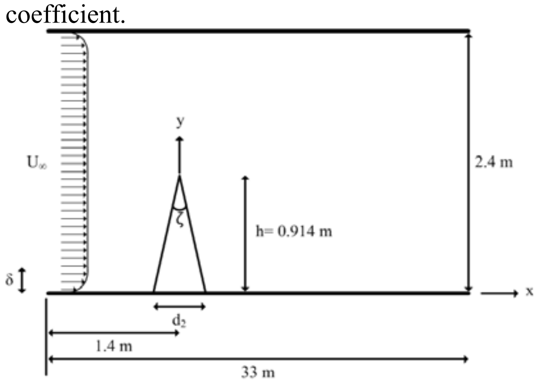

Figure 2 Schematic of the test configuration

Results

Power Spectrum of Pyramids

The power spectra obtained using a fast Fourier transform (FFT) algorithm are presented in the this section. Due to symmetry, the power spectra from either side are nearly identical and, to avoid duplication, only the spectra from one side of the models are shown. The y-axis of the power spectra is offset to allow comparison between measurements at different locations.

The power spectra, obtained from the 7.6 (ζ= 15°) taper ratio pyramid, are shown in Figures 3-5 and are typical of the pyramids tested in this study. The spectra are dominated by a strong peak, located at 5.7 Hz, indicating a strong periodic component. Further along the span of the pyramid, closer to the tip, the strength of the peaks diminishes as y/h increases and a peak cannot be identified at y/h>0.686. The shedding frequency for the 9.5 and 14.3 taper ratio (ζ= 12°, 8°) pyramids is 6.8 Hz and 9.7 Hz respectively and the disappearance of the peak occurs at y/h=0.625 for the 9.5 and y/h=0.564 for the 14.3 taper ratio pyramid. Generally vortex shedding is characterized by the diameter and, the base diameter has been shown by previous authors[i,ii] to be a good scaling for the frequency of the base vortex cell for tapered geometries. By normalizing the length of the base vortex by the base diameter, the base cell increases in length relative to its base diameter as shown in Table 1 for increasing taper ratio. The lengthening of the base cell as the taper ratio increases may be due to the pyramids approaching the two-dimensional (uniform cross-section) case where the base vortex spans the entire length of the cylinder.

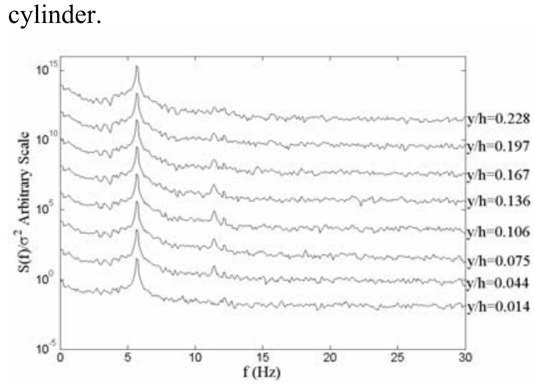

Figure 3 Power spectra for TR=7.6

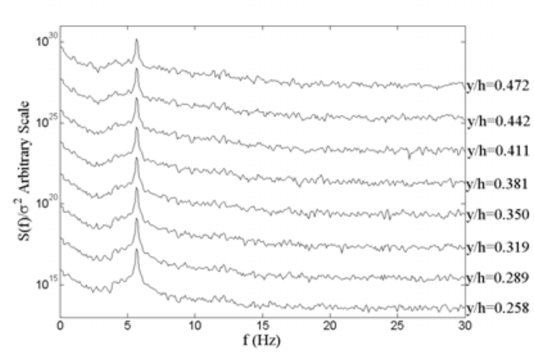

Figure 4 Power spectra for TR=7.6

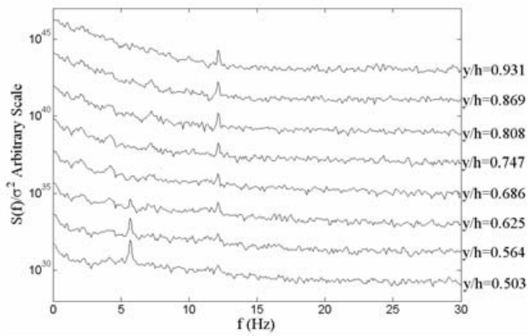

Figure 5 Power spectra for TR=7.6

Taper Ratio	Base Vortex Height (y/h)	Base Vortex Height (# diameters)
7.6	0.686	2.61
9.5	0.625	2.97
14.3	0.564	4.03

Table 1 Scaling of the base vortex length

Base Vortex Cell

Cross correlations were calculated using equation 1 with the reference fixed at the base of each pyramid for the entire set of correlations:

$$R_{p1p2}(\tau) = \frac{\int_0^T (C_{p1}(t) - \overline{C_{p1}})(C_{p2}(t+\tau) - \overline{C_{p2}})}{C_{p1}{}' C_{p2}{}' T} \quad (1)$$

Where C_{p1} is considered the reference signal and C_{p2} is considered the primary signal. The primary signal was located on the opposite side from the reference and each correlation is for different y/h locations. Figure 6 shows R_{p1p2b} at different heights for the 7.6 taper ratio (ζ= 15°) pyramid and is typical for all pyramids tested in this study. The cross correlations show a local minimum at τ=0 indicating that the periodic signals found near the base of the pyramids are 180˚ out of phase with the opposing side, which is consistent with the formation of alternately shed vortices, similar to a Von Kármán vortex street.

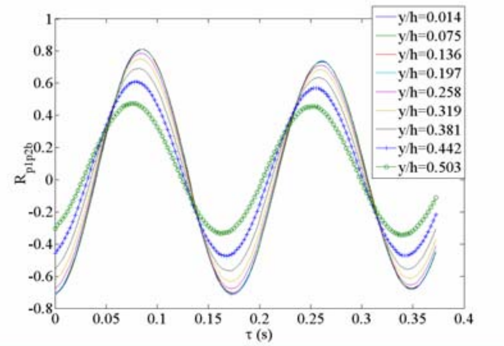

Figure 6 Cross correlations of the 7.6 taper ratio pyramid reference at the base

Figure 7 shows a comparison between Strouhal numbers obtained from the base vortex cell from the current investigation and those found from previous studies[i, ii]. The Strouhal number for a two-dimensional square cylinder has been found to be approximately 0.129 in several independent studies[v] and has been included in Figure 7. The base vortex cell found in each model corresponds well with those found from previous authors. However, there is large discrepancy in Strouhal number between tapered plates and pyramids at low taper ratios (TR<2), which is likely due to the influence of the afterbody. For pyramids with taper ratios less than two, the shear layer is found to reattach to the body and periodicity is observed in the wake but not on the surface of the pyramid. As the taper ratio is increased, the Strouhal number approaches that for the two-dimensional square cylinder case: indicating that the effects of taper on the base shedding frequency are reduced. This result is perhaps not surprising, since the two-dimensional case is the limiting case for an infinite taper ratio.

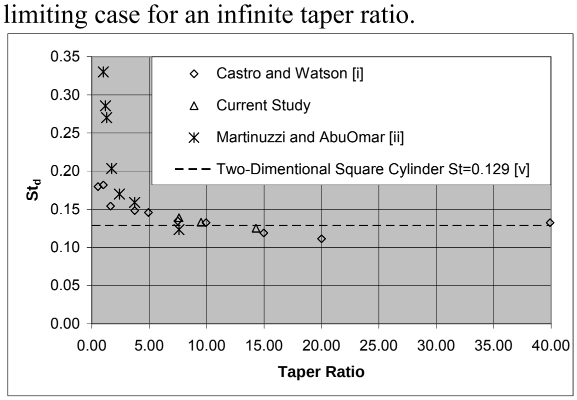

Figure 7 Comparison of base Strouhal number for triangular plates and pyramids.

Flow near the tip of the pyramids

Cross Correlations, R_{p1p2y},calculated using equation (3.1) and where both the primary signal and the reference signal are located at the same y/h location but on opposite faces across the pyramid centre line, are shown in Figure 8 for the 7.6 taper ratio (ζ= 15°) pyramid. These results are typical to other pyramids. The cross correlations show little correlation in time. However, the spatial correlation, *i.e* .the first term of the cross correlation R_{p1p2y} (τ=0), is found to be increase with height and approaches unity indicating that the two signals are similar at any given instant in time. For comparison, the maximum magnitude of $R_{p1p2y}(0)$ approaches 0.8 in the base region where periodic shedding is strongest. The increase in $R_{p1p2y}(0)$ near the tip cannot be explained simply by the physical proximity of the pressure taps and supposes the existence of a common tip flow structure which influences taps simultaneous on opposite sides of the pyramid. In previous studies, by both AbuOmar and Martinuzzi and Castro and Watson, a tip flow structure was identified. In the case of surface mounted pyramid, a "rotor" vortex is found in the mean flow field, where as for tapered plates a periodic "in-phase" vortex cell that is shed from the body has been reported. The nature of the structure observed in these experiments, however, appears to be different.

Examination of the present power spectra has found little evidence of periodicity in the pressure signals found near the tip of the pyramid, which suggest that if a flow structure does exist near the tip it is not periodic.

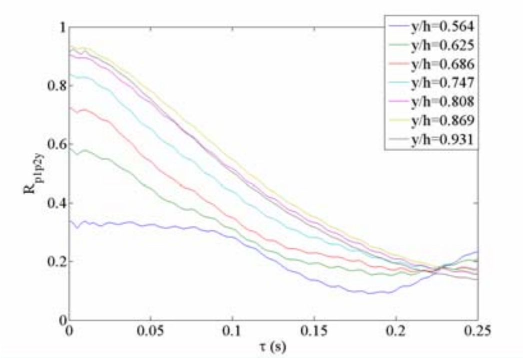

Figure 8 R_{p1p2y} Cross correlations for the 7.6 taper ratio pyramid

Figure 9 present contours of Cp(t) variations in both space (along the pyramids height) and in time for the 7.6 taper ratio (ζ= 15°) pyramid. At the base of the pyramid (y/h<0.5), the fluctuations due to periodic vortex shedding are clearly visible. Above the base vortex shedding region (0.5<y/h<0.8), these fluctuations gradually decay. Near the tip of the pyramid, where high spatial correlations are found, a large time scale structure is observed as indicated on the figure. Several instances of this structure can be found in the time series; however, they do not exhibit a regular period. The period of the structure near the tip appears to decrease with increasing taper ratio, indicating that this tip vortex is being shed from the body more frequently with increasing taper ratio.

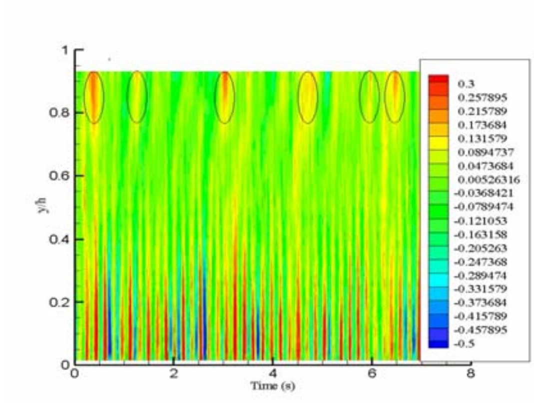

Figure 9 Contours of Cp for TR=7.6

Discussion

The introduction summarized the differences in behaviour between surface mounted pyramids and tapered plates with taper ratios ranging from 1 to 7.5 (ζ= 90° to 15.2°). In the case of tapered plates the leading edge is kept perpendicular to the oncoming flow regardless of the taper ratio, and vortex shedding is observed over the entire length of the plate. In the case of the pyramids the leading edge is inclined in the streamwise direction, the inclined front face and afterbody of the pyramid results in a complex vortex pair arrangement. It is expected that this effect is more significant on smaller taper ratio pyramids since the angle of incline of the front face is the greatest and it is expected that at larger taper ratios (smaller apex angle) this effect will be reduced and eventually eliminated as the body approaches the two-dimensional case. At the point where this effect is eliminated it is expected that the behaviour of the surface mounted pyramids will match the behaviour from the tapered plates. However, the current results have shown that the behaviour of pyramids with taper ratios of 7.6 to 14.3 (ζ= 15° to 8°) is consistent with that of pyramids with smaller taper ratios from previous studies. In addition, tapered plates with taper ratios of 7.6 to 14.3 (ζ= 15° to 8°) now exhibit similar behaviour to the surface mounted pyramids with the base vortex shedding spanning only the lower portion of the plate. The fact that the tapered plates undergo a transition and now exhibit behaviour similar to surface mounted pyramids indicates that the effects of the inclined leading edge are more complex than first anticipated.

The studies of the flows over two dimensional bodies with a free end condition have shown that a tip vortex is formed adjacent to the free end of the body[iv,vi]. This tip vortex has been shown to locally suppress the vortex shedding from the body up to three diameters below the free end. A similar tip vortex pattern has been observed in studies involving both surface mounted pyramids and tapered plates. The formation of shedding vortices adjacent to the tip is influenced by the local shear layer the distance away from the stronger base shear layer and the tip vortex. The tip vortex has been found to suppress vortex shedding in two-dimensional bodies. It is expected that the effect of the base shear layer will diminish with increasing taper ratio i.e. more distance between the base and the tip.

The suppression of the vortices near the tip is then dependent on the strength of the tip vortex, the strength of the shear layers adjacent to the tip and the distance away from the base. For tapered geometries it is expected that the strength of the side shear layers near the tip and the tip vortex structure when compared to the strength of the base shear layer will be much less than that of two-dimensional bodies with a free end condition due to the reduction in body size near the tip. For the case of the tapered plates with a taper ratio less than 7.6, it is expected that the relative strength of the base shear layer compared with the tip vortex is sufficient to prevent the local suppression of the shed vortices near the free end due to the tip vortex. However, as the taper ratio is increased the distance between the tip and the base increases, this results in the base vortex having less influence over the flow near the tip. At this point the strength of the tip vortex is sufficient to locally suppress vortex shedding near the tip, resulting in the observed transition at a taper ratio of 7.6 (ζ= 15.2°). Unlike the tapered plates, the surface mounted pyramids with taper ratios of 1 to 14.3 (ζ= 90° to 8°) show consistent behaviour with vortex shedding suppressed near the tip of the pyramid.

In the case of the pyramids, there is an added complication due to the after body and inclined leading edge, the effects of the after body with taper ratios of 1 to 7.6 (ζ= 90° to 15°) have been documented by AbuOmar and Martinuzzi using oil film visualizations and PIV vector fields. Figure 10 shows a two-dimensional vector field from a pyramid with taper ratio of 1.73 (ζ=60°) which has been presented as a typical case over the tested taper ratio range. The plane is a distance of z/h=0.2 away from the tip cutting the pyramid part way along the side face. This figure shows that the flow runs along the side face of the pyramid and is attributed to the formation of the arch vortex discussed in the introduction. The flow along the side face of the pyramid seems to enhance the circulation strength of the tip vortex. This enhancement of the tip vortex allows for the local suppression of vortex shedding near the tip.

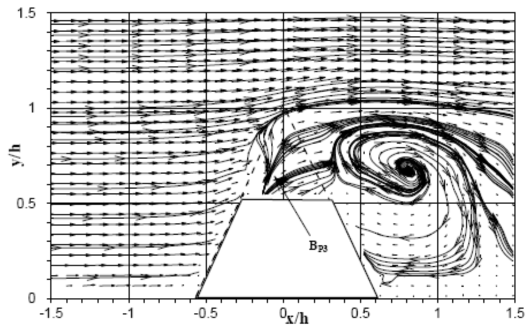

Figure 10 PIV vector field for TR=1.73 taken from AbuOmar and Martinuzzi[iii]

Conclusions

The periodic flow behaviour in the wake of surface mounted pyramids with taper ratios of 7.6 to 14.3 (ζ= 15° to 8°) were investigated. Surface pressure tap measurements were collected from tappings along the centre line of the side faces of each pyramid. The periodic vortex shedding region was found to span the lower half of the body. Near the tip of the pyramid a flow structure was found to be shed from the body intermittently. However, the time scale of this structure was found to be larger than that of the vortex shedding found at the base. The flow behaviour of these pyramids is consistent with that of tapered plates of similar taper ratio and pyramids of smaller taper ratio. The discrepancies between pyramids and tapered plates with taper ratios of 1 to 7.5 (ζ= 90° to 15°) have been discussed. The discrepancy in flow behaviour is due to the complex vortex pair arrangement generated by the inclined leading edge and after body of the pyramid allowing for the suppression of vortex shedding near the tip at lower taper ratios than for the tapered plates. There are several questions that remain unanswered such as the precise vortex interactions of the base vortices near the mid height of the pyramid. Furthermore, the behaviour of pyramids with taper ratios greater that 14.3 still need to be investigated.

Acknowledgements

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