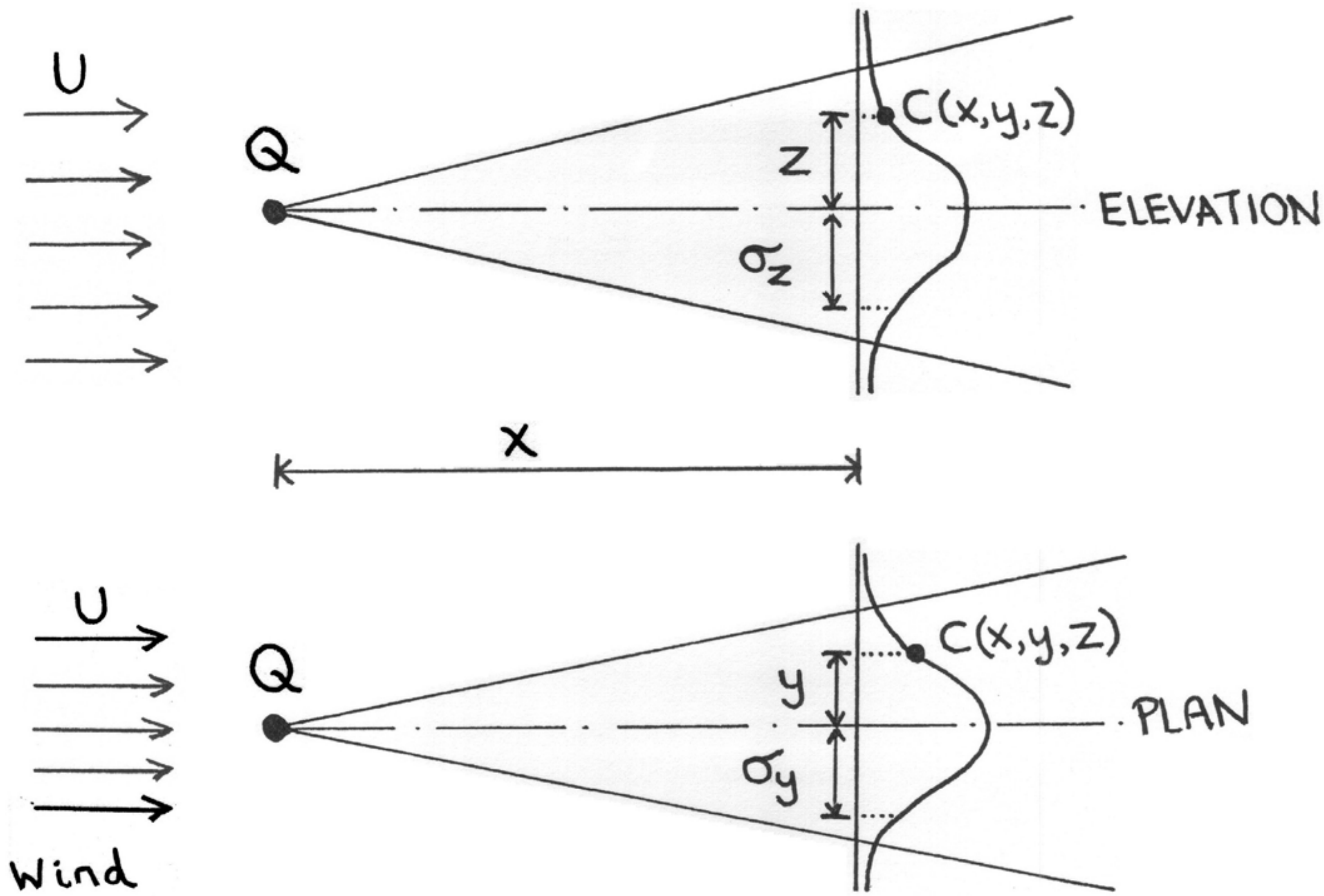
Self-study notes - GAUSSIAN PLUMES

You should read appropriate text books in order to understand the meaning of those words which are given here in italics.

Consider a point source somewhere in the air where a pollutant is released at a constant rate Q (kg/s). The wind is blowing continuously in a direction x (measured in metres from the source) with a speed U (m/s). The plume spreads as it moves in the x direction such that the local concentrations C(x,y,z) (kg/m^3) at any point in space form distributions which have shapes that are *"Gaussian"* or *"normal"* in planes normal to the x direction.



In the lateral, or y, direction the profile shape is given by

$$\frac{1}{\sigma_y \sqrt{2\pi}} e^{\left(\frac{-y^2}{2\sigma_y^2}\right)}$$

whilst in the vertical, or z, direction it is given by

$$\frac{1}{\sigma_z \sqrt{2\pi}} e^{\left(\frac{-z^2}{2\sigma_z^2}\right)}$$

The parameters σ_y and σ_z (m) are the *standard deviations* of these Gaussian distributions, which indicate the spread of the plume in the y and z directions, respectively. They increase with the distance x from the source. The area under the distribution, determined by integration of the functions given above between plus and minus infinity, is equal to unity.

Combining these two-dimensional shape distributions by multiplying the functions together gives us the function for the shape of the distribution in three-dimensions (a kind of "hill" of pollutant).

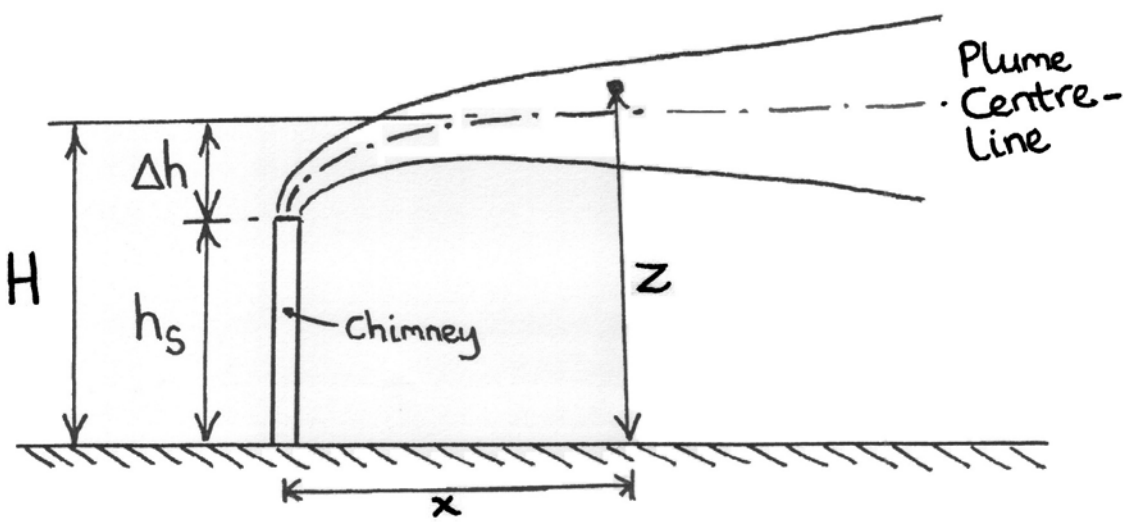
$$\frac{1}{2\pi\sigma_y\sigma_z} e^{\left(\frac{-y^2}{2\sigma_y^2}\right)} e^{\left(\frac{-z^2}{2\sigma_z^2}\right)}$$

The concentration at any point is given by

$$C(x,y,z) = \frac{Q}{U} \frac{1}{2\pi\sigma_y\sigma_z} e^{\left(\frac{-y^2}{2\sigma_y^2}\right)} e^{\left(\frac{-z^2}{2\sigma_z^2}\right)}$$

Hence, the concentration is equal to the rate of emission from the source divided by the wind speed and then multiplied by the shaping function.

This distribution measures y and z normally from the x-axis (the x-axis may also be considered to be the direction of the centre-line of the plume. In practice, the source will usually be raised above the ground (for example the exit of a chimney). Hence we need to modify the z coordinate so that it is measured from the ground.

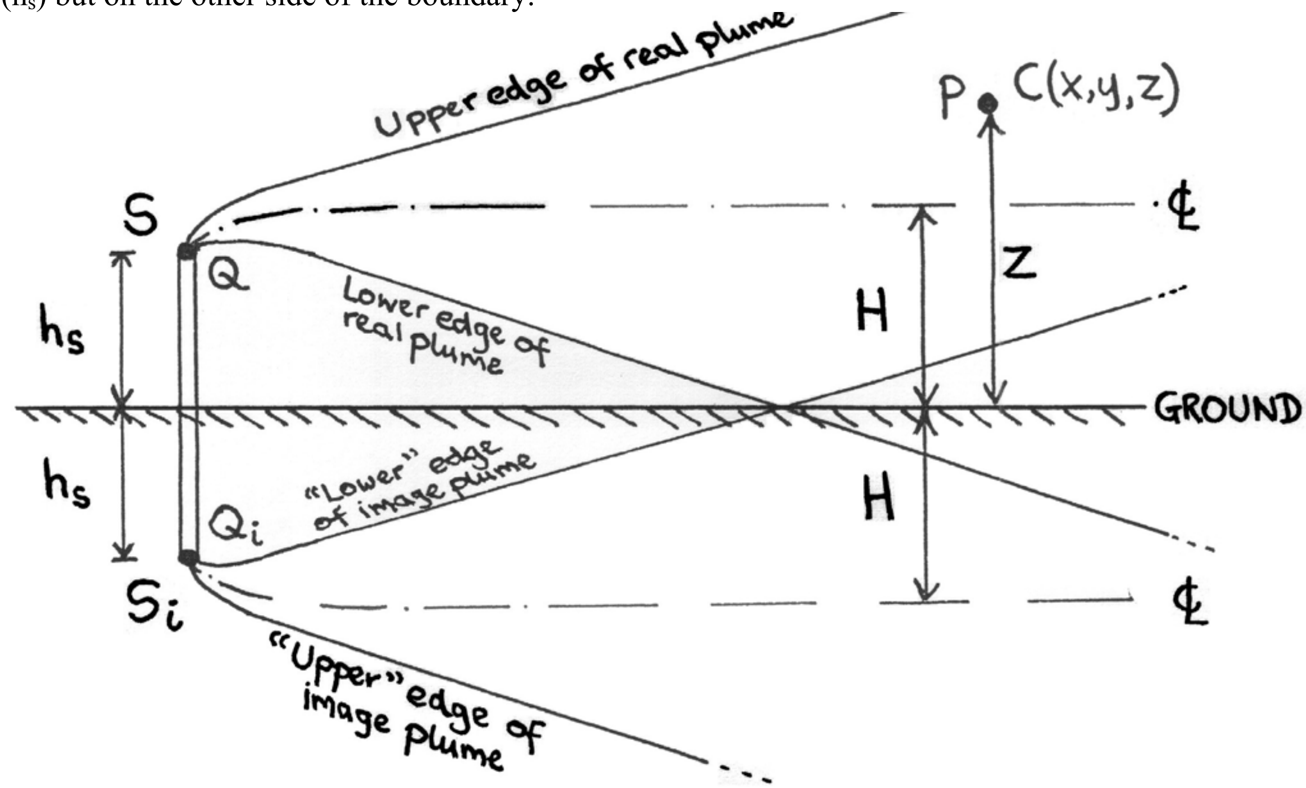


H = effective height of plume centre-line (m)
h_s = height of source above ground (m)
Δh = initial plume rise (m)
z = coordinate measured vertically from the ground to a point in the plume (m)

Hence, relative to the plume axis the new vertical coordinate is (z-H) giving

$$C(x,y,z) = \frac{Q}{U}\frac{1}{2\pi\sigma_y\sigma_z} e^{\left(\frac{-y^2}{2\sigma_y^2}\right)} e^{\left(\frac{-(z-H)^2}{2\sigma_z^2}\right)}$$

There is one further modification to be made to this equation. Unlike a plume spreading in free air, most plumes will be emitted close to the ground, like the case of the chimney shown above. Hence, as the plume spreads downwards (as well as upwards) as it moves downwind from the source, it will eventually "hit" the ground. Clearly, the plume cannot continue to spread into the ground! Instead, it is "reflected" back into the air above the ground. The effect of the ground boundary is included in the concentration equation mathematically by using a fictitious "mirror-image" source ($S_i = S$) of the same strength ($Q_i = Q$) placed at the same distance from the ground (h_s) but on the other side of the boundary.



Hence, at any point P there is a contribution to the concentration C(x,y,z) from both the real source (S) and the imaginary source (S_i). The vertical distance to P from the centre-line of the real plume is (z-H). The vertical distance to P from the centre-line of the imaginary source is (z+H). The lateral distance (y) into the page is the same for both sources.

Hence, the total concentration at P is

$$C(x,y,z) = \frac{Q}{U}\frac{1}{2\pi\sigma_y\sigma_z} e^{\left(\frac{-y^2}{2\sigma_y^2}\right)} \left[e^{\left(\frac{-(z-H)^2}{2\sigma_z^2}\right)} + e^{\left(\frac{-(z+H)^2}{2\sigma_z^2}\right)} \right]$$

You should remember this basic equation!

If only concentrations at ground level are required (for example in assessing the exposure of crops or humans to the pollutant) then we can simplify the equation by setting z=0. This gives

$$C(x, y, 0) = \frac{Q}{U} \frac{1}{\pi \sigma_y \sigma_z} e^{\left(\frac{-y^2}{2 \sigma_y{}^2}\right)} e^{\left(\frac{-H^2}{2 \sigma_z{}^2}\right)}$$

It should be noted that the maximum concentration occurs when

$$\sigma_z = \frac{H}{\sqrt{2}}$$

It is useful to remember this little rule.

At large distances from the source, where σ_z is much larger than H, the concentration varies in proportion to $1/(\sigma_y \cdot \sigma_z)$.

If only concentrations at ground level on the centre-line of the plume (along the x-axis direction) are required then the equation is simplified further since both z=0 and y=0. This gives

$$C(x, 0, 0) = \frac{Q}{U} \frac{1}{\pi \sigma_y \sigma_z} e^{\left(\frac{-H^2}{2 \sigma_z{}^2}\right)}$$

Using the basic equation, if we know the rate of emission from the source (Q), the prevailing wind speed (U) and direction (x) and the height of the centre-line of the plume above ground (H), we can determine the concentration (C) at any point (x,y,z). However, to do this we need information about the plume spread by obtaining values for σ_y and σ_z.

There are many formulae and semi-empirical expressions available for determining σ_y and σ_z under different conditions of *atmospheric stability*. A reasonable approximation in regions near to the source when the source is elevated above the ground (such as at the top of a chimney) is

$$\sigma_y = I_y \cdot x \qquad \text{and} \qquad \sigma_z = I_z \cdot x$$

where I_y and I_z are the turbulent wind speed fluctuations (*turbulence intensities*) in the y and z directions, respectively.

Under *neutral* atmospheric conditions it has been found that, over a range of heights corresponding to the vertical plume spread, centred at approximately h_s, I_y and I_z may be estimated as

$$\mathbf{I_y} = \frac{\mathbf{0.88}}{\mathbf{\ln\left(\frac{h_s}{z_o}\right) - 1}}$$

and

$$\mathbf{I_z} = \frac{\mathbf{0.50}}{\mathbf{\ln\left(\frac{h_s}{z_o}\right) - 1}}$$

where "ln" is the natural log, h_s is the release height and z_o is the aerodynamic roughness representing different topographic ground conditions (see notes on atmospheric boundary layers). This simple model for the turbulence intensities bears some similarity with the approximations we used earlier in the course for I_x, I_y and I_z and either may be used in practice. For general cases of different atmospheric conditions the following typical values apply

Thermal stratification	Lateral intensity (I_y)	Vertical intensity (I_z)
Extremely unstable	0.40 - 0.55	0.15 - 0.55
Moderately unstable	0.25 - 0.40	0.10 - 0.15
Near neutral	0.10 - 0.25	0.05 - 0.08
Moderately stable	0.08 - 0.25	0.03 - 0.07
Extremely stable	0.03 - 0.25	0 - 0.03

It may be seen that the turbulence intensities, especially the vertical wind speed fluctuations, increase as atmospheric conditions become more unstable.

One other factor that needs to be considered in practice is the *plume rise* (Δh). This is the path or trajectory of the plume centre-line after it leaves the source. Its course depends upon atmospheric conditions and the amount of *buoyancy* and vertical momentum in the initial plume at the source.

Buoyancy forces causes the plume rise to vary with $x^{2/3}$
Momentum forces cause the plume rise to vary with $x^{1/3}$

Hence, the shape of the trajectory will depend on which forces dominate the plume. If the chimney plume is buoyant the plume rises to a maximum level of $\Delta h = 0.2h_s$ to $\Delta h = 0.6h_s$ above the source, depending on atmospheric conditions. If the initial momentum of the plume dominates then an approximate expression for the final plume rise is

$$\mathbf{\Delta h = \frac{3\,D\,W}{U}}$$

where D = diameter of the source, e.g. chimney exit diameter (m)
W = initial vertical velocity of the plume (m/s)

In practice, the initial vertical velocity is often similar to the prevailing wind speed. So, for a typical chimney diameter of, say, 3m the final plume rise is of the order of 9m. In many cases the value of the plume rise is so small, relative to the magnitude of the release height and the size of the plume, that the plume centre-line may be considered to be horizontal for simplicity.

Gross Screening Analysis – a simple initial assessment

It is often useful to perform a simple screening analysis *before* applying a more refined analysis. A *gross screening analysis* will quickly identify the order of magnitude of the expected concentrations and may even show that no problem exists, in which case more advanced modelling is unnecessary. A useful formula for estimating worst case mean concentrations downwind of a point source is:

$$C_{wc} = \frac{10^9\ Q}{U\ H_{wc}\ W_{wc}} \qquad (1)$$

where:

Q = source strength or emission rate of gas or particulate [kg/s]

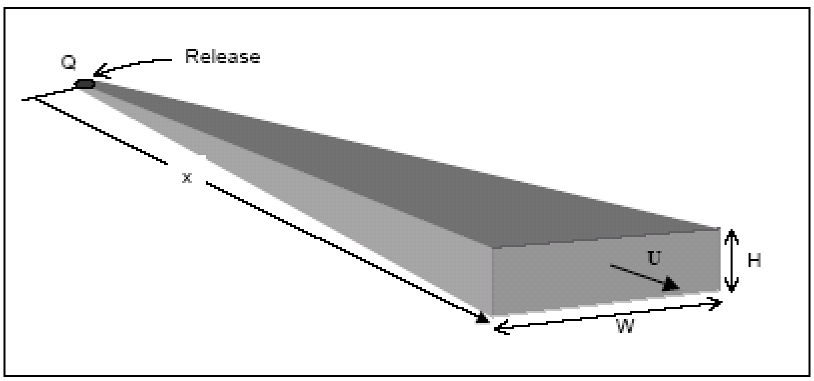
C_{wc} = worst case concentration [$\mu g/m^3$]

U = worst case wind speed at height z = 10 m, usually 1 m/s

W_{wc} = worst case cloud width [m] (usually we assume W = 0.1x, where x is distance from the source)

H_{wc} = worst case cloud depth (usually we assume H = 50 m in worst case)

This equation is essentially a statement of the conservation of pollutant mass, but it illustrates many of the basic parameter dependencies in dispersion modeling. Referring to Figure 2, we assume a uniform concentration in the plume passing through the downwind plane HW.



Equation (1) follows from the fact that the flux of pollutant through any plane must equal the source rate Q. Equation (1) illustrates several important dependencies that should be satisfied by all plume models:

1. The mean concentration is inversely proportional to mean wind speed.
2. The mean concentration is directly proportional to the release rate.
3. The mean concentration is inversely proportional to the plume cross-sectional area.

As an example of the above, suppose a small amount (1-kg) of ammonia is released over a period of 30 minutes in an accidental release. Assuming a light wind of 1 m/s, does this release pose any risk to the occupants of a hospital located 5 km downwind? For this example the estimate of the plume width is W_{wc}= 0.1 × 5000 = 500 m, thus,

$$C_{wc} = \frac{10^9\ Q}{U\ H_{wc}\ W_{wc}} = \frac{\left(10^9\ \mu g/kg\right) x \left(1\ kg/1800\ s\right)}{1\ m/s\ x\ 50m\ x\ 500m} = 22.2\ \mu g/m^3$$

This concentration turns out to be equivalent to 0.032 ppm (and is, in fact, 1500 times below the personal exposure limit (PEL) associated with negative health effects due to prolonged exposure to ammonia). Therefore, we can safely say that there is no risk. In such a case there is also no need to perform any advanced modelling to assess the risk.

Questions

(1) Sulphur dioxide is emitted at a rate of 2kg/s from the top of a chimney that is 120m high. The plume initially rises vertically a further 10m above the chimney exit, before being convected horizontally by a wind speed of 15m/s under conditions of neutral stability. The surrounding terrain is flat with a roughness length z_o of 0.01m. Calculate;

(a) The concentration (kg/m^3) on the plume centre-line at a distance of 800m downwind of the chimney.[Answer: 5311 $\mu g/m^3$]

(b) The ground level concentration at a distance of 800m downwind of the chimney (that is, along the x-axis). [Answer: 257 $\mu g/m^3$]

(c) The location (x) where the maximum ground level concentration occurs downwind of the chimney on the x-axis. [Answer: 1542.3 m]

(d) The concentration at this location. [Answer: 1051 $\mu g/m^3$]

(2) A 100m tall chimney stack emits hydrogen chloride (density = $1.64 kg/m^3$) at a rate of $1 m^3/s$. The plume initially rises a further 5m directly above the exit before being convected horizontally by a wind blowing at a speed of 10m/s under neutral atmospheric conditions. The terrain has a roughness length of $z_o = 0.03$m. A small housing development commences at a location which is a distance of x = 1500m downwind of the stack and y = 500m from the centre-line of the plume.

(a) What is the pollution concentration (kg/m^3) at ground level at the start of the housing development? [Answer: 43 $\mu g/m^3$]

(b) If the wind direction changed so that the plume axis pointed directly towards the housing development, what would be the new ground level concentration at the same location as before ? (Assume that the wind speed and the rate of emission remain unchanged). [Answer: 1537 $\mu g/m^3$]

(3) A 100m tall chimney emits sulphur dioxide at a rate of 2.5kg/s in a highly buoyant plume. The plume rise (Δh) after exiting the chimney follows a trajectory which is given by the equation: $\Delta h = 0.13\ x^{2/3}$ (all dimensions in metres). The wind speed is 10m/s and the local ground roughness length is $z_o = 0.01$m. What is the ground level concentration along the x-axis at a distance of 2.5km down wind of the chimney?

[Answer: 1400 $\mu g/m^3$]

(4) A ground level accidental release of 0.5 g/s occurs at a 4 m height in terrain with a roughness length of z_o = 0.01m. Assuming a wind speed of 1 m/s what is the maximum ground level concentration 4 km downwind estimated from the gross screening method and from the gaussian plume equation?

[Answers: 25 $\mu g/m^3$, 0.56 $\mu g/m^3$]