Theory Behind Geotechnical Centrifuge Modelling

Geotechnical centrifuge modelling is a physical modelling technique used to reproduce the stress conditions that occur in a full-scale soil–structure system within a small-scale laboratory model. The fundamental principle is to subject a reduced-scale model to an enhanced gravitational acceleration so that the stresses within the model are equivalent to those in the corresponding prototype. This is particularly important in geotechnical engineering because soil behaviour is governed strongly by stress, rather than simply by geometric dimensions.

Consider a prototype with a characteristic length L_p and a centrifuge model with a corresponding length L_m. The geometric scale factor is commonly defined as

$$N = \frac{L_p}{L_m}.$$

If the model is tested under an acceleration of Ng, where $g$is Earth's gravitational acceleration, the vertical stress at a depth $z_m$in the model is approximately

$$\sigma_m = \rho(Ng)z_m$$

where $\rho$is the soil density. Since $z_p = Nz_m$, the prototype stress is

$$\sigma_p = \rho g z_p = \rho g(Nz_m)$$

Thus,

$$\sigma_m = \sigma_p$$

This equality is the central theoretical basis of centrifuge modelling. A model that is, for example, 1/50 of the prototype dimensions is therefore tested at approximately 50g to reproduce the prototype's self-weight stresses.

The importance of this approach arises from the nonlinear nature of soil behaviour. Conventional small-scale models tested at 1 g experience stresses that are much lower than those in the prototype. Consequently, soil stiffness, strength, consolidation, permeability-related behaviour, and soil–structure interaction may not be properly represented. Centrifuge testing increases the body force acting on the model soil and consequently reproduces realistic prototype stress levels.

A major consideration is **scaling laws**. Under an acceleration of Ng, lengths scale as $1/N$, while areas and volumes scale as $1/N^2$and $1/N^3$, respectively. Forces associated with soil self-weight scale approximately as $1/N^2$. Stress, however, is maintained at the prototype level. Time scaling depends on the physical process being investigated. For example, because drainage distances are reduced in the model, consolidation and seepage processes can occur much faster. For diffusion-controlled processes such as consolidation, the appropriate time scale is approximately:

$$t_m = \frac{t_p}{N^2}$$

if the soil permeability and other relevant properties are appropriately scaled. Other processes, such as dynamic loading, require different scaling relationships.

Similarity of soil behaviour is another fundamental requirement. The model soil should reproduce the important mechanical and hydraulic characteristics of the prototype. In many cases, the same soil can be used because centrifuge modelling reproduces prototype stress levels rather than relying solely on geometric similarity. However, particle size introduces an important limitation. If the model dimensions are reduced substantially while the soil grain size remains unchanged, the ratio of particle size to model geometry increases. This can produce scale effects, particularly for foundations, piles, retaining walls, and other systems where the characteristic dimension may be only a few tens of particle diameters.

Centrifuge modelling is particularly powerful for investigating **soil–structure interaction**. Foundations, retaining structures, tunnels, slopes, pipelines, offshore wind foundations, and other geotechnical systems can be constructed at reduced scale and tested under realistic stress conditions. Instrumentation can measure quantities such as displacement, acceleration, pore-water pressure, bending moment, load, and soil stress. These measurements provide information that can be difficult or impossible to obtain at full scale.

Nevertheless, centrifuge models are not exact replicas of prototypes. Scaling distortions can arise from particle size, model boundaries, imperfections in material scaling, differences in stress history, and limitations in reproducing construction processes. The finite dimensions of the model container can also influence the results through boundary effects. Careful selection of the acceleration level, model geometry, soil preparation method, boundary conditions, and instrumentation is therefore essential.

Overall, the theory of geotechnical centrifuge modelling is based on **stress similitude**: reducing the physical dimensions of a geotechnical system while increasing the gravitational acceleration so that the model experiences prototype-equivalent stresses. By satisfying appropriate scaling relationships and controlling potential scale effects, centrifuge modelling provides a powerful means of studying complex geotechnical behaviour under controlled and repeatable laboratory conditions.