Title: Strengthening and Repair of Reinforced Concrete Beams Using Concrete Jackets

Authors: M. Monir Ajan Alhadid ; Maged A. Youssef
ABSTRACT

The necessity to rehabilitate a Reinforced Concrete (RC) structure emerges from several reasons such as new safety requirements, change of structure occupancy, incorrect design calculations and/or degradation of materials with time. One of the most commonly used mitigation practices to strengthen and repair RC beams is the application of RC jackets at either one side or three sides of the beams. The use of these jackets to improve the flexural performance of RC beams is investigated in this study. An iterative incremental algorithm that takes into account the influence of slip along the interface between the old and new concrete layers is developed. In addition, the distribution of interfacial strain gradient, slip and shear stresses along the interface are predicted. A very good agreement is shown between the current proposed analytical model and the published experimental data on RC jacketed beams. Based on an extensive parametric study, effective flexural modification factors of these beams are developed and verified.

INTRODUCTION

Flexural strengthening of RC beams is the process of enhancing their flexural behavior to accommodate certain design requirements when subjected to sagging and/or hogging moments. Composite beams have been used in construction since time immemorial in the form of layered timber planks glued or packed together with ropes to create one entity. Nowadays, jacketing of RC beams presents itself as a prominent method among the various strengthening techniques that are widely adopted by structural engineers worldwide. This process is carried out by attaching new concrete layers to the existing RC member resulting in section enlargement. The added concrete layers are usually reinforced with longitudinal steel bars, stirrups, welded wire mesh or various kinds of fibrous materials [1]. The efficiency of such structural elements chiefly relies on the ability of the sliding surfaces to transfer the generated horizontal shear stresses [2].

Western University, Department of Civil and Environmental Engineering, 1151 Richmond Street, London, ON N6A 3K7, Canada.
The extensive work of early studies culminated in proposing design equations in the 1963 ACI code. Later on, the design philosophy of the current RC building codes, such as the CSA A23.3 [3] and the ACI 318 [4], started to discuss the interfacial behavior between new and existing concrete in view of shear-friction theory. According to this theory, the interface shear strength depends on four main parameters: concrete compressive strength, pressure normal to the interface, transverse reinforcement crossing the interface and roughness of the original concrete surface [5].

The existing-surface treatment is a crucial step in determining the effectiveness of RC jacketing as it governs the slip behavior between the original and new concrete and consequently the overall flexural behavior of the strengthened members. While designing jacketed RC beams, it is common to assume full bond between the existing and new concrete layers. This might be true for some situations. However, relative slip can develop between the concrete layers resulting in inevitable separation of the two surfaces [6]. This separation may have a noticeable effect on the jacketed beam stiffness. Hence, the slip distribution along the interface should be taken into account in the analysis. The magnitude of interface slip along the jacketed beam affects the composite action between the substrate and the enlargement. This results in possible alteration in the overall flexural behavior of the jacketed beam and has to be considered in design.

The aim of this paper is to provide structural engineers with a simplified method to analyze and design jacketed RC beams taking into account the slip distribution at the interface and the actual nonlinear behavior of both concrete and steel. The following sections of the paper summarize the developed calculation algorithm for estimating the behavior of RC beams jacketed with concrete from one side. The material and interfacial mechanical behaviors are estimated from relevant models found in literature. Subsequently, the developed model is validated in view of relevant experimental studies. Slip modification factors are also proposed based on the critical design variables concluded from an extensive parametric study.

MATERIAL MODELS AND INTERFACIAL BEHAVIOR

Material Models

Scott et al. [7] model is used in the analysis as it provides a robust yet simple expression to describe the stress-strain behavior of normal strength concrete in compression. It captures the influence of micro-cracks formation at the vicinity of aggregates as the load increases. After reaching the peak point of compressive stress ($f_c'$) and the corresponding strain ($\varepsilon_{o}$), redistribution of internal stresses takes place preventing sudden failure of concrete as depicted in the descending branch of the model. The tangential Young’s modulus is taken as the first derivative of the concrete stress ($f_c$) with respect to concrete strain ($\varepsilon_c$). The tensile behavior of concrete is predominantly brittle and it is assumed to carry tensile stresses up to the cracking point beyond which the tensile capacity of concrete drops to zero. The steel reinforcement monotonic stress-strain relationship in both tension and compression is modeled as a trilinear relationship that conveniently combines the initial elastic response, yield plateau and strain hardening stages.
Interfacial Slip Behavior

Slip is governed by the ability of the RC beam to transfer the horizontal shear stresses along the interface between the concrete core and the jacket. The shear transfer mechanisms depend on the extent of the relative slip in the partially connected materials. Thus, interfacial shear models are expressed as the summation of concrete contribution (i.e., adhesion, aggregate interlock and friction) and dowel action (effect of transverse reinforcement crossing the interface). The model proposed by Tassios and Vintzeleou [8] is adopted in this study to determine the concrete contribution ($v_c$) in terms of lateral slip ($s$), ultimate slip value at the onset of frictional mechanism failure ($s_{cu}$), ultimate frictional capacity of the interface ($v_{cu}$), steel ratio crossing the interface ($\rho_s$) and the corresponding tensile stress developed in this steel ($f_s$). The roughness of the original concrete is expressed in terms of the coefficient of shear friction at the interface ($\mu$). CSA A23.3 Cl. 11.6 [4] recommends a value of 1.4 for concrete placed monolithically, a value of 1.0 for concrete placed against hardened concrete with the surface intentionally roughened to a full amplitude of at least 5 mm and a value of 0.6 if the original concrete is cleaned but not intentionally roughened. For untreated smooth concrete surface, a value of 0.4 is recommended by Tassios and Vintzeleou [8]. Regarding the dowel resistance, Thermou et al. [9] provides an expression for the resultant dowel force ($V_D$) as a function of the lateral slip between the two concrete surfaces, steel diameter ($D_b$) and the ultimate dowel force ($V_{Du}$).

PROPOSED CALCULATION ALGORITHM

Several assumptions are made in the developed procedure to perform sectional analysis [10] of the jacketed RC beams. Firstly, the Navier-Bernoulli assumption stating that "a cross section remains planar and normal to the deformed middle fiber" is considered implying that shear deformations are small relative to bending deformations. Also, the strain in the embedded steel bars is considered to be identical to the strain in the concrete layer they lie in owing to the assumption that perfect bond exists between the steel reinforcement and the surrounding concrete material. In addition, the failure criterion of the composite beam is defined by crushing of the extreme compression fiber as it reaches the concrete ultimate strain of (0.0035) [3] provided that shear failure and rupture of steel bars are forestalled. Finally, the original RC beam and the added concrete layer are considered to deform by the same curvature through the beam length, as usually carried out in mechanics of materials of composite sections [6,9].

The sectional analysis procedure used by Youssef and Rahman [10] is manipulated in the current analysis with some modifications to capture the behavior of both the original concrete and the added jacket simultaneously. The procedure commences by defining the strengthened section's input variables that correspond to its geometrical properties, cross-sectional dimensions, jacketing type, steel reinforcement ratio, material properties and interfacial shear friction parameters. The composite section is then divided into multiple discrete strips having a maximum height of 0.5 mm, which was found to result in good accuracy. At every loading step, an incremental curvature is applied and the strain at each strip in both the concrete core and the jacket is calculated based on its location from the plastic centroid ($y_i$) of
the jacketed section. Two equilibrium conditions must be satisfied in the analytical procedure; namely, equilibrium between the internal axial forces at any section, and equilibrium between the resultant axial forces at one side of the interface and the resultant shear force acting along the interface. The kinematic and compatibility conditions are considered in light of the material stress-strain relationships and Equation 1, which relates the incremental applied moment (\(\Delta M\)) and axial load (\(\Delta P\)) to the incremental curvature (\(\Delta \phi\)) and axial strain (\(\Delta \varepsilon_a\)) by a defined stiffness matrix. In this equation, \(n\) represents the number of discrete layers, \(E_i\) is the elastic modulus of layer \(i\), \(A_i\) is the area of layer \(i\), subscript (c) represents concrete core and subscript (J) represents concrete jacket.

\[
\begin{pmatrix} \Delta M \\ \Delta P \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} (E_{i,c} A_{i,c} + E_{i,j} A_{i,j}) y_i^2 & -\sum_{i=1}^{n} (E_{i,c} A_{i,c} + E_{i,j} A_{i,j}) y_i \\ -\sum_{i=1}^{n} (E_{i,c} A_{i,c} + E_{i,j} A_{i,j}) y_i & \sum_{i=1}^{n} (E_{i,c} A_{i,c} + E_{i,j} A_{i,j}) \end{pmatrix} \begin{pmatrix} \Delta \phi \\ \Delta \varepsilon_a \end{pmatrix}
\]

(1)

The interfacial slip influence on the flexural behavior of the jacketed RC beam is modeled by modifying Tsioulou and Dritsos [6] procedure through employing the actual moment-curvature relationship obtained from the sectional analysis method to satisfy equilibrium conditions at all loading steps. The developed iterative calculation algorithm is performed for each loading step by defining a general interfacial stiffness coefficient (\(K\)), which is calculated by multiplying the initially assumed values of the interfacial shear stiffness (\(k_s\)); the ratio between the average slip strain along the interface (\(\Delta \varepsilon_s\)) to the maximum slip strain (\(\Delta \varepsilon_{\text{max}}\)); the ratio between the average shear stress along the interface (\(\bar{\tau}\)) to the maximum shear stress (\(\tau_{\text{max}}\)); and the distance from the support to the point at maximum bending moment (\(X_u\)). The slip strain is the strain difference at the interface between the original and the added concrete bodies. It approaches its utmost value at the maximum moment section and vanishes at the support. Figure 1 shows the strain profile and stress distributions in a typical strengthened beam.

![Figure 1](image-url)

(a) Cross-sectional view of the strengthened beam; (b) axial strain profile; and (c) axial and interfacial shear stress distributions.

The procedure continues by dividing the beam into several segments having a maximum length of 50 mm each for enhanced accuracy. The interfacial shear stress distribution is assumed to vary as a cubic function from the support towards the maximum bending moment section. Therefore, the interfacial shear force from the support to any point (x) along the interface can be expressed by Equation 2.
In the developed model, an iterative sectional analysis procedure is carried out only at the mid-span segment of the simply supported beam to obtain two curves, the moment-slip strain relationship and the moment-curvature diagram. With the knowledge of these two curves and the distribution of the bending moment through the beam due to the applied load, the corresponding slip strain at each segment can be determined from Equation 3 in which \( i \) is the load step number, \( j \) is the segment number and \( m \) is the load step number that produces a bending moment in the mid-span segment equals to the moment applied at segment \( j \).

\[
\Delta \varepsilon_{(i,j)} = \Delta \varepsilon_{(m,1)} \left( \frac{x_j}{x_{ul}} \right)
\]

Once the slip strain distribution along the interface is established, both the slip and the shear stress distributions are obtained using equations 4 and 5, respectively. The slip, and consequently shear stress, reach their maximum value at the support and fade away as they approach the maximum bending moment section.

\[
S_{(i,j)} = \sum_{m=1}^{m=j} \left( \Delta \varepsilon_{(i,m)}(x_j) \right)
\]

\[
\tau_{(i,j)} = k_s S_{(i,j)}
\]

In the subsequent step of the proposed model, the ratio between \( \overline{\Delta \varepsilon} \) to \( \Delta \varepsilon_{max} \) and the ratio of \( \overline{\tau} \) with respect to \( \tau_{max} \) are calculated and compared to the initially assumed values. The analysis continues if they are equal, otherwise the whole procedure is repeated with the new calculated values. After that, the interfacial stiffness (\( k_s \)) value is determined from a relevant shear stress-slip model in terms of \( \tau_{max} \). If the obtained value of \( k_s \) matches the assumed one, then the actual moment-curvature diagram at each segment is determined using the sectional analysis method with the knowledge of slip strain at each segment; otherwise, the \( k_s \) value is updated and the entire procedure is repeated until convergence is achieved.

Once the slip effect is incorporated in a unique moment-curvature diagram for each segment, the widely used area-moment method is carried out to determine the deflection at the mid-span point of the simply supported beam using Equation 6 in which \( i \) is load step number, \( j \) is the segment number and \( N \) is the number of segments.

\[
\delta_i = \sum_{j=1}^{N} \left( \phi_{(i,j)}(x_j)(x_u - x_j) \right)
\]

**VERIFICATION OF THE METHOD**

The capability of the present model to predict the flexural behavior of jacketed RC beams is verified in view of the experimental load-deflection curves obtained by
Chalioris et al. [11] and Shehata et al. [12], and the interfacial shear stress distribution determined experimentally by Jones et al. [13] as illustrated in Figure 2. Investigating the influence of the jacket material properties and reinforcement configuration were the common motivation of these studies. Chalioris et al. [11] applied monotonic two point concentrated loads on initially damaged 1.4 m long rectangular RC beams jacketed with 25 mm thick self compaction concrete at three sides. Shehata et al. [12] investigated the influence of steel reinforcement ratio on the flexural behavior of 4.5 m long simply supported beams subjected to one point load. Jones et al. [13] examined the anchorage capacity of steel jackets attached to 2.3 m long RC beams from one side. The stiffness, yield load, ultimate capacity and maximum interfacial shear stress of the beams are well estimated by the model as indicated by the small percent error of about 6%, 4%, 9% and 15% respectively.

(a) load-deflection curves.  
(b) Interfacial shear stress distribution.

Figure 2. Comparison of analytical results with various experimental results.

PARAMETRIC INVESTIGATION

A parametric study is carried out in an attempt to develop a simplified method to predict the actual flexural behavior of jacketed RC beams. The main parameters of investigation are the concrete compressive strength \( f'_c \), steel yield strength \( f_y \), coefficient of friction at the interface \( \mu \), original beam depth \( h_c \), concrete jacket thickness \( h_J \), beam width \( b_c \) and beam span. The concrete compressive strength took values of 25 MPa, 30 MPa and 35 MPa; whereas the steel yield strength were 300 MPa, 400 MPa and 500 MPa. The coefficient of friction took values of 1.4 for monolithically placed concrete, 1.0 for intentionally roughened concrete surface using sandblasting, and 0.4 for smooth concrete surface. The beam depth varied between 300 mm and 600 mm, jacket thickness varied between 100 mm and 200 mm, beam width varied between 200 mm and 400 mm, and beam span varied between 3 m and 5 m. The main steel reinforcement in the concrete core is taken as one half of the balanced steel reinforcement ratio, whereas the steel bars in the jacket were chosen based on practical considerations in accordance with CSA A23.3 [3] as 10M bars with minimum spacing. Two top 10M steel bars were fixed in all beams and stirrups were used to prevent the premature occurrence of shear failure prior to concrete crushing. Figure 3 illustrates the influence of the aforementioned parameters through comparing the load-deflection relationship of selected specimens.
(a) Effect of changing original section depth ($h_c$).

(b) Effect of changing jacket thickness ($h_J$).

(c) Effect of changing beam width ($b_c$).

(d) Effect of changing beam span.

Figure 3. Load-deflection curves of jacketed RC beams.

A total of 2916 jacketed rectangular RC beams having the configuration shown in Figure 1(a) are analyzed to study the slip effect on the produced load-deflection curves. The concrete compressive strength and steel grade have slight influence on the overall flexural behavior of the jacketed beam. The effect of core height and jacket thickness on the yield capacity of both the core and jacket reinforcement are found insignificant. The friction coefficient, beam span and its width are found to have the greatest impact on the capacities of the examined specimens. This conclusion is reasonable because the slip is mobilized by the interfacial properties and the contact area between the two surfaces. A statistical study is carried out based on the results of the 2916 specimens to formulate expressions for slip modification factors that adjust the monolithic load-deflection curves of the jacketed RC beams. The effects of material mechanical properties, friction coefficient ($\mu$), beam span and its width ($b_c$) are accounted for in a multiple regression analysis using the Ordinary Least Square method. Based on the results of the parametric study, Equations 7, 8 and 9 with a minimum coefficient of determination of 90.1% are proposed. Using these factors, the engineers can first plot the load-deflection curve of a jacketed RC beam at jacket yield point, core yield point and ultimate point assuming monolithic behavior. Then, they adjust the produced curve by multiplying the jacket yield deflection, core yield deflection and ultimate deflection by the corresponding slip modification factors $\alpha_{y,J}$, $\alpha_{y,C}$ and $\alpha_{\mu}$, respectively. In these expressions, the mechanical properties are substituted in MPa, the span is substituted in meters and the beam width in mm.
\[
\alpha_{y,l} = \left[ \frac{0.201 \ln(f_y)}{(f'_c)^{0.411}} \right] [1.10 - 0.0247 \mu^3 - 0.0126 \text{Span} - 3.8 \times 10^{-5} b_c] \quad (7)
\]

\[
\alpha_{y,c} = \left[ \frac{0.296 \ln(f_y)}{(f'_c)^{0.161}} \right] [1.04 - 0.0111 \mu^3 - 0.00572 \text{Span} - 1.4 \times 10^{-5} b_c] \quad (8)
\]

\[
\alpha_u = \left[ \frac{0.177 \ln(f_y)}{(f'_c)^{0.003}} \right] [1.03 - 0.0070 \mu^3 - 0.00377 \text{Span}] \quad (9)
\]

**SUMMARY AND CONCLUSIONS**

An analytical procedure for predicting the flexural behavior of jacketed RC beams is presented in the current study. The model introduces the influence of interfacial slip between the original substrate and the added concrete layer on the moment-curvature and load-deflection relationships. The model is verified against experimental results found in literature with very good agreement. Several parameters including surface treatment condition, beam span and its cross-sectional dimensions are considered in a parametric study. The analysis of 2916 specimens culminated in proposing slip modification factors that can be manipulated by engineers to plot the load-deflection curves of jacketed RC beams taking into account slip impact.

**REFERENCES**

4. ACI Committee 318. 2001. "Building Code Requirements for Structural Concrete," ACI 318M-08, American Concrete Institute, Farmington Hills, MI.