Influence of Specimen Size in Engineering Practice

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ABSTRACT: A combined laboratory experimental and numerical analysis is presented to investigate the influence of specimen size and scale effect on engineering analysis and design. Laboratory triaxial compression and direct shear tests show that the size of the specimen has a significant influence on the stress-strain behavior of sands with larger specimens mobilizing smaller shear strengths. Shear strengths measured in laboratory direct shear tests are incorporated in a FEM and slope stability analyses to evaluate and compare the shear stress distribution and deformational behavior of a slope case study. The numerical analyses are conducted using ABAQUS and mohr-coulomb failure criteria. The performance of the slope under load application due to staged highway embankment construction is also evaluated. The analyses results show that the shear stresses and performance of slope and highway embankment are influenced considerably by the size of the triaxial specimens. This would have significant implications on engineering design and the choice of a representative sample size. In order to apply the shear strengths in design, it is suggested to employ larger specimen sizes to achieve the critical state strengths of the soil and better representation of field deformations.

INTRODUCTION

Granular soils are widely employed as backfill material for earth-retaining structures, trenches and highway embankments, as they provide high shear strength, suitable compaction and drainage properties. In these soils, friction angle plays a major role in their strength behavior and stability. Accurate assessment of shear strength parameters of cohesionless soils plays a vital role in analysis and design of geotechnical structures, earth retaining walls and foundations. However, shear strength testing of coarse sands can be problematic since most testing equipments are of small size, relative to the size of particles in the soil. In particular, widely different specimen sizes are nowadays employed by different soil shear testing laboratories, while the impact of differences in specimen size on shear strength parameters and geotechnical analyses is largely overlooked. Estimation of shear strength parameters can be highly affected by the mechanical boundary restraints and gradation of soil in both triaxial and direct shear tests. This study presents a combined laboratory and numerical investigation on the scale and specimen size dependency of sand friction angle and implications in stability analysis of a highway embankment.

BACKGROUND CONCEPT

The mechanical behavior and shear strength mobilization in cohesionless soils essentially depends on the interaction among soil particles and the amount of particle movement, rearrangement, reorientation and possible particle crushing. Accordingly, soil stress-strain response which is a fundamental soil behavior should be essentially unrelated to the size of the specimen. This makes stress-strain behaviors of specimens of different sizes appear different as the strain is calculated based on normalization with respect to specimen dimension. For illustration, FIG. 1 compares the stress-strain behavior of a quartz sand (D_{50} = 0.22 mm) in triaxial compression tests on different specimen sizes. The cylindrical specimens were prepared at diameters of 38, 50, and 70 mm with a length to diameter ratio of one to reduce shear strain non-uniformity at large strains. While irrespective of specimen size the minimum deviator stress is mobilized at about 7% axial strain, a larger minimum strength (65 kPa) is mobilized in the smaller specimen (D = 38 mm) as it undergoes a smaller amount of shear displacement and particle movement compared to the other two specimen sizes. This appears as sand behavior mainly depends on amount of shear displacement and therefore at a particular level of strain, the strength of the specimen would be different as a result of different displacement levels in specimens of different sizes.

SPECIMEN SIZE AND SCALE EFFECT ON FRICTION ANGLE

According to the conceptual illustration of FIG. 1, specimen size affects shear strength of cohesionless soils and therefore friction angle. Several studies have investigated the effect of specimen size and scale on the friction angle of cohesionless soils. Almost all of these studies have found that the shear strength and friction angle increase with decreasing specimen size (Hight and Leroueil 2003; Cerato and Lutenegger 2006; Nakao and Fityus, 2008; Wang and Gutierrez 2010; Moayed and Alizadeh 2011). Table 1 presents the results of direct shear tests on three specimen sizes (59.9 mm, 101.6 mm, and 304.8 mm) of Sand A (D_{50} = 0.7 mm), Sand B (D_{50} = 0.6 mm), and a gravel (D_{50} = 2.8 mm) (Cerato ad Lutenegger 2006). For all of these sands, the friction angle (ϕ') decreases (up to 10^{o}) with increasing dimensions and size of the specimen (from 60 mm to 305 mm box size). In fact, a larger specimen size provides a larger space for the sand particles to move and rearrange and therefore improving the propagation of a shear band. This is a significant reduction in friction angle which can largely affect analysis and design procedures in which friction angle plays an important role (e.g. slope stability analysis, retaining structures design, bearing capacity calculations).

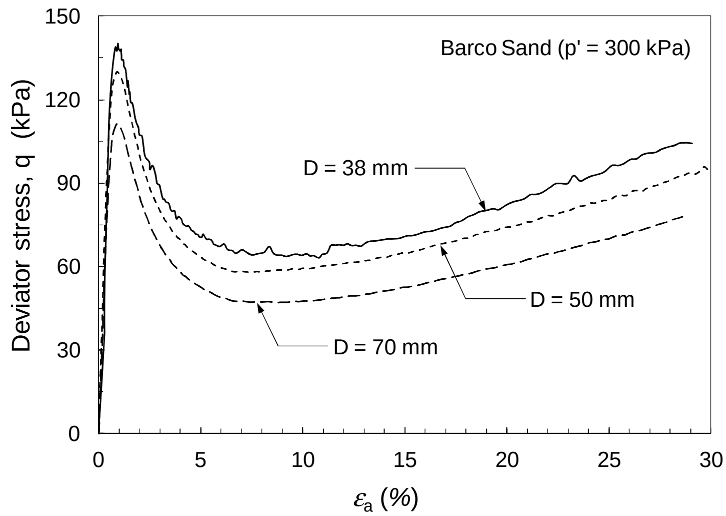

FIG. 1. Specimen Size Effect on Stress-Strain Response of a Quartz Sand in Triaxial Compression Tests at p' = 300 kPa

In order to evaluate the implication of obtaining different shear strength parameters from different tested samples, performance of a highway embankment construction was analyzed using ABAQUS Finite Element program. A series of slope stability analyses using SLIDE program were also conducted to evaluate the stability of the embankment slopes. Shear strength parameters for Sand A, Sand B and a gravel obtained from direct shear tests on three shear box sizes (see Table 1) are used in these analyses.

FINITE ELEMENT MODELING

Here the influence of specimen size is evaluated on the results of slope stability analyses based on the laboratory data of Table 1. A two dimensional finite element simulation of an embankment construction on a 5% slopping ground in three sequential layers with each layer being 5 m high is conducted. The utilized side slopes of the embankments are 2H:1V. The geometric boundary conditions of this model are determined far enough to avoid any boundary effect on performance analyses of the embankment. The bottom and side boundaries are in 2D (D is the width of the embankment at the ground surface) and 1.5 D distance from the embankment, respectively. The snapshot of model geometry is shown in FIG. 2. The materials are simulated using Mohr-Coulomb constitutive model.

The material parameters assumed in the analyses are shown in Table 1. The foundation soil stiffness is assumed to change with depth. Foundation soil is assumed in five layers with the average thicknesses of 5, 10, 20, 50 and 125 m from top to bottom of the model. The Young's moduli (E) for each layer is calculated using maximum shear modulus relations ($E = 2G_{max}(1+ \nu)$) and other soil parameters from

Table 1. The stiffness of the foundation soils is assumed to increase with depth due to increase in confining pressures using following equations (Sadrekarimi 2013).

$$G_{max} = 6.6\frac{(2.17-e)^2}{1+e}(\sigma'_c)^{0.52}, \text{ for } \sigma'_c < 9.8 \text{ kPa} \quad (1)$$

$$G_{max} = 8.2\frac{(2.17-e)^2}{1+e}(\sigma'_c)^{0.43}, \text{ for } \sigma'_c > 9.8 \text{ kPa} \quad (2)$$

Where, G_{max} is shear modulus, e is void ratio, σ'_c is effective consolidation stress, and ν is soil Poisson's ratio.

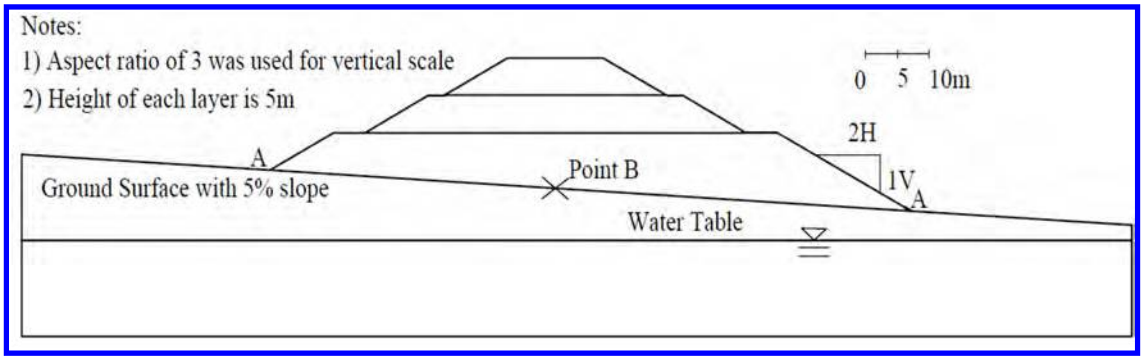

FIG. 2. CAD Drawing of Embankment Model Geometry

Table 1. Experimental Test Results

Material	ϕ (degrees)			Void ratio (e)	Unit weight (kN/m³)	Permeability k (m/s)	ν
	Analysis I (59.9 mm sample)	Analysis II (101.6 mm sample)	Analysis III (304.8 mm sample)				
Sand A	47.4	42	40.8	0.568	20.3	0.0004	0.35
Sand B	36.5	28.5	26.1	0.867	18.7	0.0009	0.35
Gravel	43.5	36.5	34	0.790	18.6	0.0225	0.35

The FEM analysis is conducted in four steps involving sequential construction of embankment in three layers (see FIG. 3). The initial stress state of the foundation soil is analyzed using geostatic conditions in the first step. Single type of material was considered for foundation and embankment soil for simplicity. For each material type three analyses I, II and III (see Table 1) were conducted to evaluate different shear strength parameters. As an example, the global deformation response of the FEM model for Sand A is shown in FIG. 4. The deformation profiles are unsymmetrical and propagate to deeper depths with the maximum deformations at the center zone of embankment.

The vertical and horizontal deformations at the end of embankment construction along the line A-A (See FIG. 2 for location) are shown in FIG. 5 and FIG. 6, respectively. The estimated deformations are varied along the line from left toe of embankment to the right. The deformations profile is not symmetric as the ground surface is sloped. The maximum vertical deformations are greater for Analysis I, which represents shear strength parameters of smallest size samples.

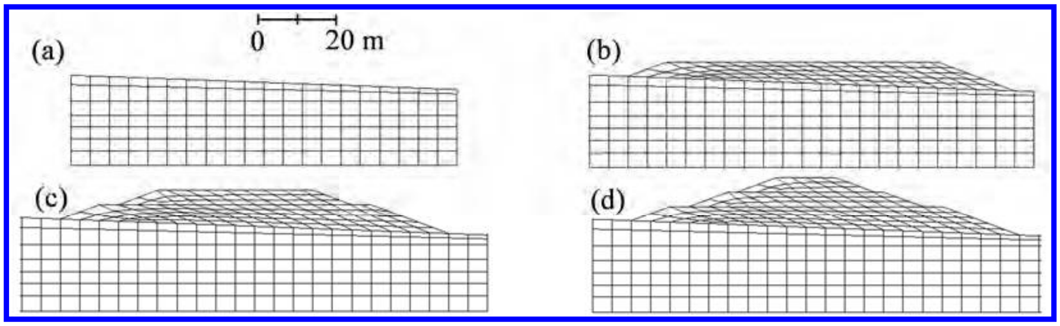

FIG. 3. Finite Element Mesh Showing Construction Sequence (a) Before Construction, (b) After Placement of First Layer, (c) After Placement of Second Layer, (d) After Placement of Third Layer

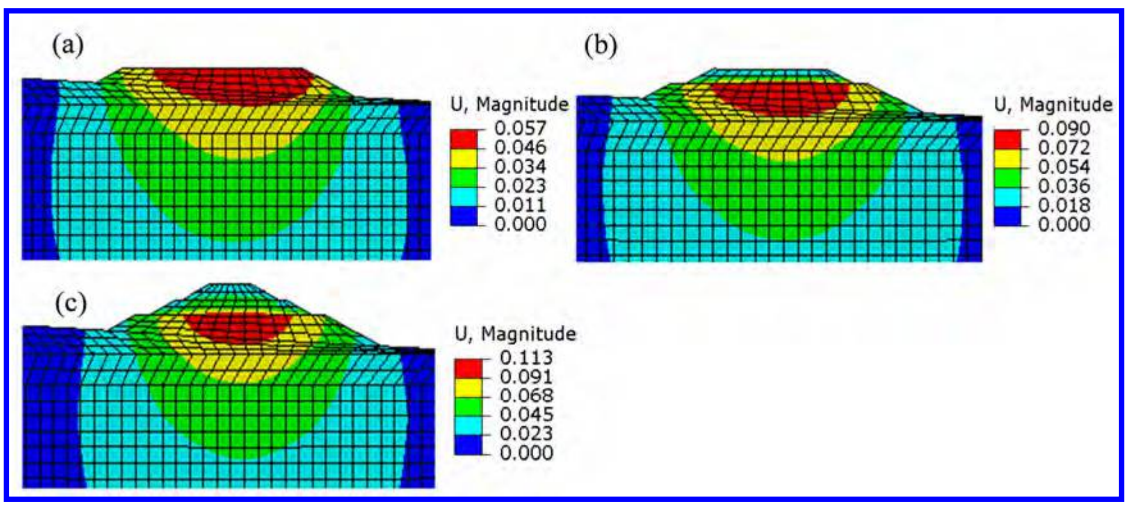

FIG. 4. Global Deformation Response of Sand A in Meters for (a) After Placement of First Layer, (b) After Placement of Second Layer, (c) After Placement of Third Layer

The maximum vertical deformation in Analysis I is 9% to 11% greater than Analysis II and 11% to 14% greater than Analysis III along line A-A. Therefore, it is expected to need 452 m^3/km and 585 m^3/km more compacted material based on Analysis I results comparing to Analysis II and III, respectively.

The horizontal displacements are zero in center zone of embankment and increase to the sides of the embankment. The maximum horizontal deformation is obtained at about 0.25D (D is width of embankment at ground surface) from the center line of embankment on both sides.

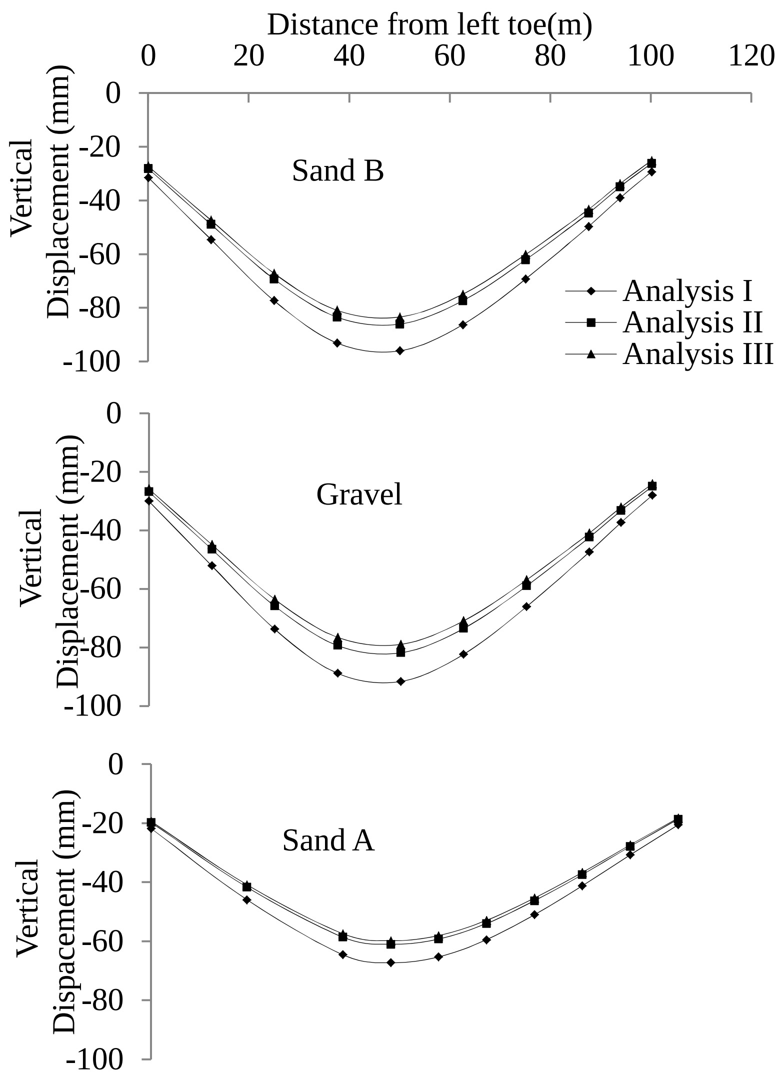

FIG. 5. Vertical Displacement Graphs of Three Sand Sample Along Line A-A

Table 2 shows FEM analysis results at Point B (shown in FIG. 2) on the embankment for the three samples. It presents the comparison of changes in stress, strain and displacements values between different box size samples. The increase or decrease in stress, strain and deformation in analysis II and III are referenced to the ones obtained from Analysis I. As it is expected the vertical stresses are not significantly affected using parameters obtained from different shear box samples. The horizontal stresses are somewhat affected for Sand B. There is a minimal change in horizontal stresses observed for Sand A and gravel using difference sample sizes.

The horizontal and vertical strains show that they are more subjected to change. The magnitudes of horizontal and vertical strains increase with using parameters obtained from smaller shear box sizes. The maximum vertical and horizontal strains in Analysis I are both about 11% and 14% higher than the ones obtained from Analysis II and III, respectively. The maximum horizontal deformation in Analysis I is about 10% and 13% higher than the ones obtained from Analysis II and III, respectively.

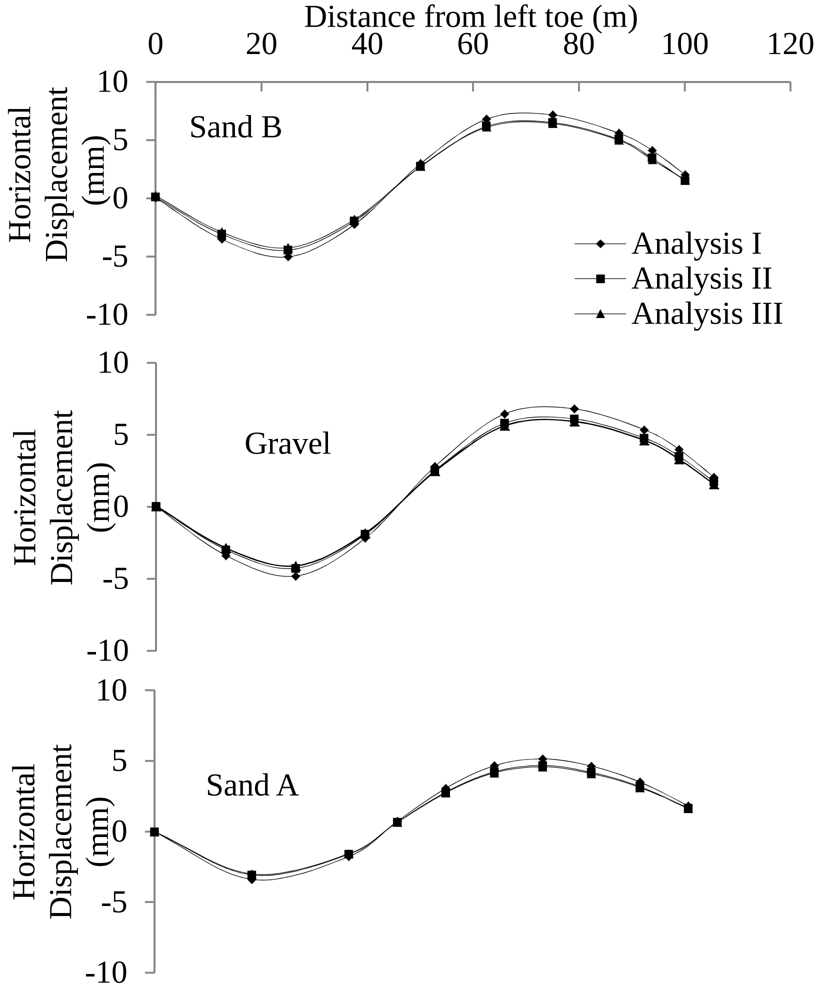

FIG. 6. Horizontal Displacement Graphs of Three Sand Sample Along Line A-A

Table 2. Stress, Strain and Displacement Comparative Table for Three Sand Samples

Studied Parameter	**Percentage increase in values from Analysis II to Analysis I**			**Percentage increase in values from Analysis III to Analysis I**		
	Sand A	**Sand B**	**Gravel**	**Sand A**	**Sand B**	**Gravel**
Horizontal Stress	-0.5	-1.85	-0.75	-0.64	-7.87	-1.02
Vertical Stress	0.0071	0.03	0.033	0.012	0.031	0.0417
Horizontal Strain	9.1	10	10.61	10.84	12.5	14.01
Vertical Strain	9.24	10.4	10.7	11.19	13.32	14.265
Horizontal Displacement	9.24	9.1	10.24	11.24	10.3	13.2
Vertical Displacement	9.26	10.37	10.7	11.05	13.13	13.86

SLOPE STABILITY ANALYSIS

The slope stability of the embankment slopes is determined for all three sets of shear parameters for all three sample types. The geometry of the FEM model was used in the SLIDE program for slope stability analyses. The minimum safety factors were determined using Spencer's method (Spencer, 1967) and running different shear failure surfaces. The critical shear failure surfaces were typically passing through the toe and the crest of the embankment.

Table 3 shows the minimum calculated safety factor for the different samples. In general, Sand A shows greater safety factors against slope stability due to its higher friction angle regardless of sample sizes. The safety factors are up to 50% greater when shear strength parameters obtained from smaller shear box samples are used. With increase in sample sizes from 59.9 mm to 304.8 mm, the safety factors decrease from 21% to 34% for the analyzed materials. The greatest safety factor of 3.67 was calculated for analyses that used shear parameters from Analysis I for Sand A. The smallest minimum factor of safety of 1.5 is calculated for Sand B with a friction angle of 26.1 degrees in Analysis III. The ratio of safety factors from consecutive analyses show that, the safety factor ratio between Analyses I and II is 28.9% higher than the ratio between Analysis II and III.

Table 3 Minimum Factor of Safety for Different Materials and Different Sample Sizes

	Minimum Factor of Safety		
	Analysis I	**Analysis II**	**Analysis III**
Sand A	3.67	2.75	2.65
Sand B	2.27	1.66	1.5
Gravel	2.9	2.26	2.28

DISCUSSION

As demonstrated above, specimen size and scale can influence soil shear strength (see FIG. 1) and friction angle (Table 1). This could significantly affect slope stability analysis and whether or not to stabilize a slope. An undersized specimen could lead to large over estimation of shear strength parameters and unconservative analysis and design of a slope. A similar mechanism is associated with the increasing bearing capacity factor, $N\gamma$ with footing size (Kimura et al. 1982). Specimen size effect could be minimized by ensuring that specimen size with respect to the mean particle diameter is adequately large that would allow complete particle rearrangement and reorientation and the full development of shear band. The length and thickness of the shear band are the main factors causing the specimen size effects of FIG. 1 and Table 1. As presented by many experimental studies (e.g. Roscoe 1970; Sadrekarimi and

Olson 2010) shear band thickness is about 10 to 14 times of D_{50} of the sand. Accordingly, the size of the specimen should be several times larger than (10-14) D_{50} of the test sand in order to allow complete particle rearrangement, reorientation and the full development of the shear band to minimize specimen size effect.

The current ASTM testing standard for direct shear tests (ASTM D3080) requires a minimum specimen width of 10 times the maximum particle diameter. Several laboratory and DEM numerical studies have recommended using specimen (L) to particle size (D_{50}) ratios of up to 100 (e.g. Scarpelli and Wood 1982; Cerato and Lutenegger 2006; Wang and Gutierrez 2010). While the sands of Table 1 (for the minimum shear box size of 59.9 mm, Sand A: L/D_{50} = 86; Sand B: L/D_{50} = 86 ; gravel L/D_{50} = 21) and the quartz sand presented in FIG. 1 (for the minimum shear box size of 59.9 mm: L/D_{50} = 272) meet these criteria, there is still significant specimen size effect. Although the results presented here do not specifically show a particular specimen to particle size ratio at which specimen size effect disappears, it is possible that the effect of specimen size could be minimized beyond a specimen size to D_{50} of greater than 300 as suggested by Jewell and Wroth (1987). This could be considered as the minimum thresholds where the specimen size effects could be avoided.

CONCLUSION

This study shows the potential effects of sample size scale on the performance analyses of a highway embankment. The experimental results on sands and gravel show that soil shear strength and friction angle reduce with increasing specimen size. A large specimen allows complete particle rearrangement and development of the shear band which is consequently a more accurate representation of field strength conditions. The finite element analyses results for different sample sizes show that the strain and displacements values were greater when shear strength parameters obtained from 59.9 mm sample sizes were used. The stresses do not show a major change. The slope stability analyses show a greater minimum factor of safety values using shear strength parameters from 59.9 mm box sizes. While using small size samples for determining shear strength parameters might result in unconservative safety factors, they can result in greater ground deformations. It is suggested to use specimen sizes of at least 300 times larger than mean particle diameter in determining strength parameters of cohesionless soils and application in engineering analysis or design.

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