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Stochastic characterization of the dynamic response of a pile

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ABSTRACT

The design of pile foundations subjected to large harmonic loads produced by machine vibrations is usually controlled by their service limit state. Verification of this condition under current design philosophy requires the estimation of the probability of failure. However, the deterministic nature of current design procedures does not facilitate such calculations. This paper presents a stochastic model for the dynamic pile response under harmonic loading considering the uncertainty in the mechanical parameters of the soil and the pile itself. The purpose of the model is to quantify the uncertainty in the pile response in terms of its impedance. A sensitivity analysis of the design parameters and stochastic variables is performed to quantify their impact on the uncertainty of the pile response. The effect of pile diameter and uncertainty of its properties, mean soil stiffness, and the selection of distribution models for the random variables are evaluated. Useful insights are gained into the stochastic behavior of the model and the characterization of the pile stiffness and damping components of the pile. This approach may provide a valuable first step towards the development of a comprehensive reliability analysis of this foundation type.

1. Introduction

The design of pile foundations for vibrating machines is usually controlled by the vibration requirements rather than the bearing capacity of the structure or soil. Excessive vibration levels can prevent equipment from functioning properly, produce excessive wear and tear or take it out of service. It can also be a source of discomfort, distraction and health problems for operators and people in proximity [1,2], or lead to structural damage to the foundation itself. This has resulted in the promulgation of standards such as the ISO 2631 [3], ISO 5439 [3] and BS 6841 [4] for human tolerance to vibrations and BS 7385 [5] for the effect of vibrations on buildings.

The dynamic response of pile foundations, even assuming linear mechanical behavior, is still highly complex due to the wave propagation phenomena involved [6]. Analytical and numerical models show responses that depend on the excitation frequency and are sensitive to a variety of input parameters. These models are generally employed in a deterministic manner, evaluating the response of the foundation for a defined set of input parameters that are representative of the problem. Practical design of machine foundation usually incorporates uncertainty of soil stiffness through consideration of upper and lower bound values for the input parameters, without any consideration of the likelihood of each particular scenario. At the moment, there is no clear understanding of whether and how the uncertainty in geotechnical parameters should be considered, which parameters are more relevant, how the soil–pile interaction propagates and is affected by the uncertainty and how to consider this in the design procedure.

Indeed, given the complex nature of the system, intermediate cases between the upper and lower cases may be more disadvantageous. Also, the definition of single upper- and lower-case scenarios is not trivial, since there are several variables involved in the design (e.g., pile mechanical properties, the mechanical properties of the identified soil layers, the distribution of the properties across those layers, the location of the interfaces within the layers, etc.). Even assuming that single upper and lower case can be reasonably defined, whether they satisfy (or not) the serviceability conditions does not provide an estimation of how likely are these scenarios, precluding a proper risk assessment of the design.

A stochastic analysis of the pile response provides a framework to address these issues with ample precedence with applications for other types of structures [7,8]. The capacity to address the probability associated with different vibration threshold (e.g., discomfort, long-term harm, machine wear and tear, foundation damage, etc.) enables the application of performance-based design to these types of foundation. It also enables decision-making processes based on risk analysis. Being able to estimate the reduction in uncertainty in the foundation response, with the implied cost reduction, as a function of reduced uncertainty in the model input parameters also provides the means for a cost–benefit analysis on performing more precise geotechnical testing at the site.

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Reliability analysis conducted for piles foundations under dynamic loads usually consider the ultimate limit state under load conditions that induces a non-linear behavior of the system and results in permanent deformations or structural failure [9–11]. Andersen et al. [12] studied the fatigue limit state. Caswell et al. [13] and Zorzi et al. [14] considered the service limit states for monopile offshore wind turbines but with non-linear soil behavior, which is expected in this type of structure. However, under normal operational conditions, machine foundations are designed so that both the structure and the soil respond elastically to avoid large soil settlements or liquefaction [15] and structural problems. Existing analysis methods usually adopt simplified approaches such as FOSM [9,11] or the response surface method [10]. Damgaard et al. [16] performed a reliability analysis with Monte Carlo simulation for monopile foundations with an elastic soil reaction for the fatigue limit state of an offshore wind turbine. This study is the most directly applicable case to the service limit state of machine foundations. Unfortunately, the mechanical model used is constrained to a single layer stratum with uniform properties and a fixed pile tip.

Deterministic design procedures are based on the capacity of the mechanical models to characterize the complex dynamic behavior of piles and piles foundations [6,17]. In addition to providing a mathematical tool to estimate the foundation properties, these models present the designer with a frame of reference to predict the likely outcome of design decisions in terms of stiffness and damping. Key questions like whether it is convenient to extend the pile length or diameter, to increase the damping, or whether driving piles into a lower stiffer stratum will have the desired effect can be addressed by understanding the expected behavior without the need to carry out further computations. In the same fashion, the development of a stochastic model for the dynamic response of pile foundations should be more than just a mathematical tool for computing probabilities. At its core, it can also provide insight into the likely outcome of design decisions. Where the deterministic approach predominantly deals with outcomes in terms of stiffness and damping, the stochastic approach is interested in predicting the effect of the expected variability on the response.

This paper presents a stochastic model for the dynamic response of piles based on existing and widely used analytical models in the literature. The objective is to understand the stochastic behavior of the model for its further application in the design process by quantifying the effect of the different variables involved and how uncertainty propagates from the input parameters to the pile impedance and the expected outcomes. To that end chapter 2 describes the dynamic mechanical model used for the pile response, chapter 3 deals with the particulars of computing the statistics of the pile impedance which are later used to quantify the variability of the pile response, chapter 4 performs a sensitivity analysis of the pile response to the uncertainty of each of the random variables involved, chapter 5 investigates whether the pile mechanical parameters can be considered as deterministic given their expected lower uncertainty, and lastly, chapter 6 studies the influence of the probability distribution model used for the random variables.

2. Mechanical model

Many models have been proposed for the dynamic response of piles, some of them are purely analytical [18,19], others rely on finite element analysis [14,20] or are a combination of both [12]. An extensive review, evaluation and comparison of these methods is presented by Anoyatis [17]. As noted by Novak [6], these models agree reasonably well with each other, although there are some differences in the predicted behavior. The plane strain solution proposed by Novak [21] following Tajimi [22], models the soil as a Winkler medium consistent a set of horizontal independent springs. This model is adopted herein due to its ease of implementation and low computational requirements. This enables the large number of evaluations required in Monte Carlo simulations.

The model assumes a viscoelastic behavior of the soil. In practice, the soil will experience stiffness degradation and increase in hysteretic damping once the load is applied. This can be accounted for with the consideration of a reduced shear modulus and increased viscous damping with the aid of damage curves [23]. Another source of soil non-linearity is the pile installation process, which can lead to a weakened or compacted region depending on the construction method. El Naggar [24] extended the Winkler model to consider an annular region with a different set of properties in the vicinity of the pile. Also, although not considered in this analysis, El Naggar's model is extended to pipe piles as well as lateral and rocking vibrations [25,26]. An important aspect of the dynamic behavior of soil that is not adequately reflected by this model is the fact that it is a three-phase material, with solid, liquid and gaseous components. The response of single piles and piles groups under vertical [27,28], horizontal [29], and torsional [30] loads. For torsional and horizontal loads the effect of saturation is significant, with a reduction of the stiffness and an increase of the damping across the frequency range with increasing saturation. For vertical loads the effect is less notorious but still present, even when considering fully saturated conditions [31].

2.1. Pile model

The pile is modeled with Bernoulli–Navier beam element [32] embedded in an elastic medium. The stiffness matrix of each element is composed of the horizontal $\mathbf{K}^{\langle u\rangle}$ and vertical $\mathbf{K}^{\langle v\rangle}$ submatrix, with zero coupling stiffness coefficients. For a pile of length L with cross-section of area A and moment of inertia I and Young's modulus E_p, the components of $\mathbf{K}^{\langle u\rangle}$ are given by:

$$K_{11}^{\langle u\rangle} = K_{33}^{\langle u\rangle} = \frac{E_p I \lambda^3}{L^3} \frac{\sin\lambda \cosh\lambda + \cos\lambda \sinh\lambda}{\cos\lambda \cosh\lambda - 1} \tag{1}$$

$$K_{12}^{\langle u\rangle} = -K_{34}^{\langle u\rangle} = \frac{E_p I \lambda^2}{L^2} \frac{\sin\lambda \sinh\lambda}{\cos\lambda \cosh\lambda - 1} \tag{2}$$

$$K_{13}^{\langle u\rangle} = -\frac{E_p I \lambda^3}{L^3} \frac{\sin\lambda + \sinh\lambda}{\cos\lambda \cosh\lambda - 1} \tag{3}$$

$$K_{23}^{\langle u\rangle} = -K_{14}^{\langle u\rangle} = \frac{E_p I \lambda^2}{L^2} \frac{\cos\lambda - \cosh\lambda}{\cos\lambda \cosh\lambda - 1} \tag{4}$$

$$K_{22}^{\langle u\rangle} = K_{44}^{\langle u\rangle} = \frac{E_p I \lambda}{L} \frac{\sin\lambda \cosh\lambda - \cos\lambda \sinh\lambda}{\cos\lambda \cosh\lambda - 1} \tag{5}$$

$$K_{24}^{\langle u\rangle} = -\frac{E_p I \lambda}{L} \frac{\sin\lambda - \sinh\lambda}{\cos\lambda \cosh\lambda - 1} \tag{6}$$

The components of the stiffness matrix for the vertical displacement are:

$$\mathbf{K}^{\langle v\rangle} = \frac{E_p\ A\ \eta}{L} \begin{bmatrix} -\tan^{-1}\ \eta & \sin^{-1}\ \eta \\ \sin^{-1}\ \eta & -\tan^{-1}\ \eta \end{bmatrix}$$

where λ and η are the horizontal and vertical wave numbers given by:

$$\lambda^4 = \frac{\omega^2 \mu_p - i\omega c_p - k_u}{I\ E_p} \qquad \eta^2 = \frac{\omega^2 \mu_p - i\omega c_p - k_v}{A\ E_p} \tag{7}$$

with μ being the weight per unit length of pile, c_p the damping coefficient of the pile material and k_u and k_v the horizontal and vertical stiffness of the soil. The employment of Timoshenko beam elements instead of the Bernoulli-Navier has significant consequences in the dynamic response of the pile [29]. Lastly, throughout the analysis it is assumed that the piles are fully embedded in the ground. Partially embedded piles tend to experience larger deflections under horizontal load due to the coupled effect of a static axial and dynamic horizontal loading [29].

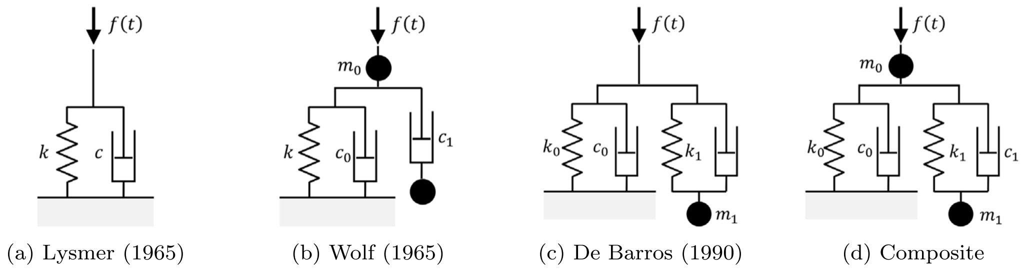


Fig. 1. Hysteretic models developed for vibration analysis of machine foundations, after Deeks and Randolph [38].

2.2. Lateral reactions

The model developed by Beredugo and Novak [33] for the lateral and longitudinal dynamic soil reactions on a cylinder of radius R embedded in a viscoelastic media is used to establish the soil reactions against the pile. For a soil with density ρ_s, shear wave velocity V_s, shear modulus G_s, Poisson's ratio v_s and damping coefficient ζ_s, the vertical k_v and horizontal k_h stiffness are given by:

$$k_v\left(a_0, G_s, v_s, \rho_s\right) = G_s S_v\left(a_0\right) = G_s\left[S_{v1}\left(a_0\right) + S_{v2}\left(a_0\right)\right] \tag{8}$$

$$k_u\left(a_0, G_s, v_s\right) = G_s S_u\left(a_0\right) = G_s\left[S_{u1}\left(a_0\right) + S_{u2}\left(a_0\right)\right] \tag{9}$$

where a_0 is the dimensionless frequency defined as:

$$a_0 = \frac{R\omega}{V_s} \tag{10}$$

and $S_u\left(a_0, v_s\right)$ and $S_\theta\left(a_0\right)$ are complex functions given by [21,33]:

$$S_v\left(a_0\right) = 2\pi a_0 \frac{J_0\left(a_0\right)\ J_1\left(a_0\right) + Y_0\left(a_0\right)\ Y_1\left(a_0\right)}{J_0^2\left(a_0\right) + Y_0^2\left(a_0\right)} + \frac{4i}{J_0^2\left(a_0\right) + Y_0^2\left(a_0\right)} \tag{11}$$

$$S_u\left(a_0,\ v_s\right) = 2\ \pi\ a_0 \frac{q\left(v_s\right)^{-1/2}\ H_2^{(2)}\left(a_0\right)\ H_1^{(2)}\left(x_0\right) + H_1^{(2)}\left(a_0\right)\ H_1^{(2)}\left(x_0\right)}{H_0^{(2)}\left(a_0\right)\ H_2^{(2)}\left(x_0\right) + H_2^{(2)}\left(a_0\right)\ H_0^{(2)}\left(x_0\right)} \tag{12}$$

$$S_\theta\left(a_0\right) = \pi\ \left[1 - a_0 \frac{J_0\left(a_0\right)\ J_1\left(a_0\right) + Y_0\left(a_0\right)\ Y_1\left(a_0\right)}{J_1^2\left(a_0\right) + Y_1^2\left(a_0\right)}\right] + \frac{2\ i}{J_1^2\left(a_0\right) + Y_1^2\left(a_0\right)} \tag{13}$$

with $J_n()$ and $Y_n()$ being the Bessel first and second kind Bessel functions of order n, $H_n^{(2)}$ the n-order Hankel function of the second kind, $q\left(v_s\right) = \left(1 - 2\ v_s\right)/2\left(1 - v_s\right)$ and $x_0 = a_0\sqrt{q\left(v_s\right)}$.

2.3. Pile tip response

The reaction at the pile tip can be obtained from models developed for circular rigid footings resting on the surface of a uniform elastic half-space. Lysmer and Richards [34] proposed a simplified one-dimensional model composed of a single spring and dashpot to capture the frequency dependent impedance of a foundation [34]. This concept was further developed by Velestos and Verbic [35], Wolf [36], De Barros [37] and Deeks and Randolph [38] using different equivalent models as shown in Fig. 1. In these models, the complex impedance of the foundation k_v is computed as the static stiffness multiplied by real k_v' and complex c_v' factors that capture the frequency dependent stiffness and damping response:

$$k_v\left(a_0\, v_s,\ D\right) = \frac{4\ G_s\ R}{1 - v_s}\ \left[k_v'\left(a_0,\ v_s,\ D\right) + i\ a_0\ c_v'\left(a_0,\ v_s,\ D\right)\right] \tag{14}$$

Velestos and Vebric [35] provide semi-empirical expressions for k' and c' considering a uniform viscoelastic half space:

$$k_v' = 1 - \frac{\gamma_1\ \gamma_2^2\ a_0^2\ \left[R' + \sqrt{(R'-1)/2}\ \gamma_2\ a_0\right]}{R' + 2\ \sqrt{(R'-1)/2}\ \gamma_2\ a_0 + \beta_2^2\ a_0^2} - \sqrt{(R'-1)/2}\ \gamma_4\ a_0 - \gamma_3\ {a_0}^2 \tag{15}$$

$$c_v' = \gamma_4\sqrt{(R'+1)/2}\ + \frac{\gamma_1\ \gamma_2^3\ a_0^2\ \sqrt{(R'+1)/2}}{R + 2\ \sqrt{(R'-1)/2}\ \gamma_2\ a_0 + \gamma_2^2\ a_0^2} + \frac{D}{a_0} \tag{16}$$

where $R' = \sqrt{1 + D^2}$; and the coefficients α_1, β_1, β_2, β_3, γ_1, γ_2, γ_3 and γ_4 account for the hysteretic model used for developing the stiffnesses. Fig. 1 presents the conceptual material models considered by Lysmer [34], Wolf [36] or De Barros [37] and Deeks and Randolph [38]. The model parameters are provided in Table 1.

2.4. Pile head reactions

The vertical (8) and horizontal (9) pile shaft reactions, as well as the tip impedance (14) are complex numbers with the real part representing the actual stiffness and the imaginary part the hysteretic and radiative damping of the soil. Upon substituting in the wavenumber equation (7) and the reduction of the pile stiffness matrix to the degrees of freedom at the pile head, the resulting impedances for lateral displacement K_{uu}, vertical displacement K_{vv}, rotation $K_{\theta\theta}$ and coupling $K_{u\theta}$ are also complex numbers, with the real part representing the overall stiffness of the pile and the imaginary part the combination of the hysteretic and radiation damping. For ease of notation, the stiffness and damping components of are noted with lower-case letters:

$$K_i = k_i + i\ c_i \tag{17}$$

The stochastic behavior of the impedance components K_i is a function of the pile model described above, and the probability distribution associated with the input variables of the model.

3. Statistic of pile response

Fig. 2 shows a Monte Carlo simulation for a 10 m long pile with a circular cross-section of 150 cm diameter when considering V_s as the only random variable for a normal distribution with a mean 100 m/s. The simulation was carried out considering a distribution with a coefficient of variation $\delta = \sigma/\mu$ of 0.05, which imply standard deviations of 5 m/s. The deterministic case with the shear wave defaulting to the mean value of the distribution ($\delta = 0$) was also computed. The remaining of the model parameters used in the simulation are $v = 0.3$, $\gamma_s = 22$ kN/m^3, $\zeta_s = 0.03$, $\gamma_p = 24$ kN/m^3, $E_p = 29$ GPa, and $\zeta_p = 0.02$. These parameters and the range of frequencies shown in Fig. 2 were selected since they exacerbate the variability of the impedance response with frequency, helping to illustrate the special considerations that must be made when estimating the statistics of the response or fitting distributions to the simulation results.

Table 1
Parameters of the hysteretic models for a shallow disk on a viscous half-space, after Deeks and Randolph [38].

ν	Model	α_0	α_1	β_0	β_1	γ_0	γ_1
0	Lysmer (1966)	0	0	0.85	0	1	0
	Velestos & Verbic (1973)	0	0.25	0.85	0.25	1	0
	Wolf (1987)	0	0.4	0.8	0.235	1	0
	De Barros & Luco (1990)	0	0.11	0.87	0.23	1	0.31
0.33	Lysmer (1966)	0	0	085	0	1	0
	Velestos & Verbic (1973)	0	0.22	0.75	0.28	1	0
	Wolf (1987)	0	0.35	0.8	0.29	1	0
	De Barros & Luco (1990)	0	0.057	0.84	0.17	0.98	0.5
0.45	Lysmer (1966)	0	0	0.85	0	1	0
	Wolf (1987)	0.105	0.24	0.8	0.16	1	0
	De Barros & Luco (1990)	0	0.063	0.85	0.30	0.99	2.61
0.50	Lysmer (1966)	0	0	0.85	0	1	0
	Velestos & Verbic (1973)	0.17	0	0.85	0	1	0
	Wolf (1987)	0.15	0.15	0.8	0.07	1	0
	De Barros & Luco (1990)	0	0.16	0.85	15.19	0.98	237.85

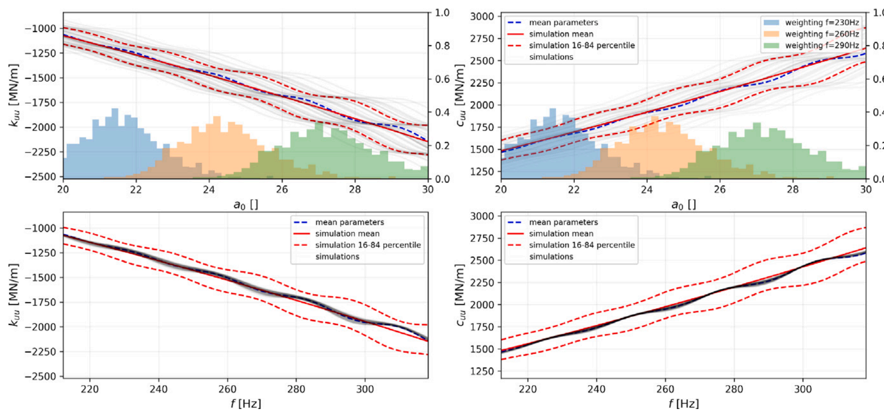


Fig. 2. Simulation of k_{uu} (left) and damping c_{uu} (right) for $L = 10$ m and $\phi = 150$ cm with statistics computed in the dimensionless frequency a_0 domain for a normal V_s distribution with $E[V_s] == 100$ m/s and $\delta V_s = 0.05$ and $\nu = 0.3$, $\gamma_s = 22$ kN/m^3, $\zeta_s = 0.03$, $\gamma_p = 24$ kN/m^3, $E_p = 29$ GPa, and $\zeta_p = 0.02$.

There are aspects for considering the shear wave velocity as a random variable whilst interpreting the simulation results and computing the statistics of the pile impedance. As indicated by Eqs. (11) to (16), the dimensionless frequency a_0 is a convenient way to express the variation of the stiffness and damping of the pile with frequency, since it captures the relative soil to pile stiffness. Its main advantage is that two piles of different radii in different soils (regarding their shear wave velocity) present the same lateral and tip stiffness k_i for a given a_0, when all the other variables remain equal. However, when V_s is a random variable, the values of a_0 of two samples of the simulation corresponding to the same excitation frequency f differ to each other. Therefore, comparison between samples of the simulation is not a straight-forward procedure.

There are two options for performing the Monte Carlo simulation. The first one is to carry out the simulation for the same a_0 values. Each simulation sample, consisting of the value of the different stiffness and damping components across the desired range of dimensionless frequency a_0, can then be converted to the corresponding frequency $f = a_0\, V_s/(2\,\pi\, R)$. This approach enables a straightforward estimation of the stiffness and damping statistics from the simulated sample in the a_0 space. The simulation shown in Fig. 2 was conducted in this way. A subset of the simulated samples is shown with the gray lines, while the estimated mean value and 16–84th percentiles for each a_0 are represented by red solid and dashed lines, respectively. Each simulation sample is also plotted in the f domain, with the frequency range of each sample computed with the sample V_s. The stiffness and damping constants corresponding to the deterministic ($\delta = 0$) case are shown with the blue line.

In the a_0 domain, the 16-84th boundaries are a good representation of the observed dispersion in stiffness k_{uu} and damping c_{uu} for any given frequency a_0. These lines are then translated to the f domain using the mean value of V_s to scale the frequency range. In the f frequency domain, the percentile range is a gross over-estimation of the observed dispersion, and it does not even match the overall behavior of the simulated samples. This demonstrates that statistics computed in a simulation carried out in the a_0 domain cannot be automatically translated into the f domain. The reverse is also true.

The second simulation option is to carry out the simulation in the f domain evaluating all samples in the same f frequency range. Each sample can then be scaled to the a_0 domain using the V_s value. Fig. 3 illustrates this procedure with the statistics estimated in the f frequency domain. In this case, both the mean and percentile range offer a good match to the simulated population in the f domain, but they misrepresent the simulated data in the a_0 domain, in this case under-estimating the overall dispersion.

The error incurred when using the statistics computed from one frequency domain in another domain can be appreciated in Fig. 4 which plots the mean, standard deviation and coefficient of variation computed in both the a_0 and f domains together. The observed discrepancy is attributed to the change of variables from one domain to the other which is probabilistic in nature. In the a_0 space, the change of stiffness for a fixed excitation frequency f due to the change in V_s between different simulation samples is equivalent to a movement in the horizontal axis of the plot for the deterministic case. Keeping a constant f while simulating V_s is identical to picking out values of the damping and stiffness from the deterministic curve in the a_0 domain distributed according to $2\,\pi\, f\, R/V_s$.

The non-linear variation of the deterministic curve combined with the fact that $1/V_s$ does not have a normal distribution means that the resulting distribution for the damping and stiffness will not follow a

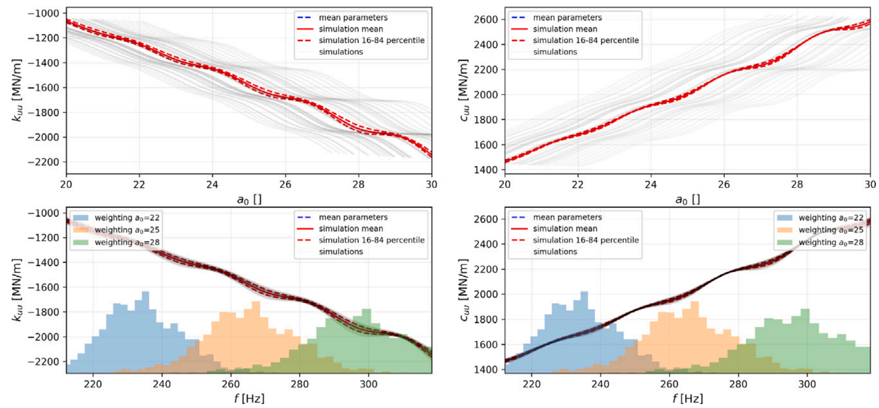


Fig. 3. Simulation of k_{uu} (left) and damping c_{uu} (right) for L = 10 m and ϕ = 150 cm with statistics computed in the f frequency domain for a normal V_s distribution with $E[V_s]$ = 100 m/s and δV_s = 0.05 and ν = 0.3, γ_s = 22 kN/m^3, ζ_s = 0.03, γ_p = 24 kN/m^3, E_p = 29 GPa, and ζ_p = 0.02.

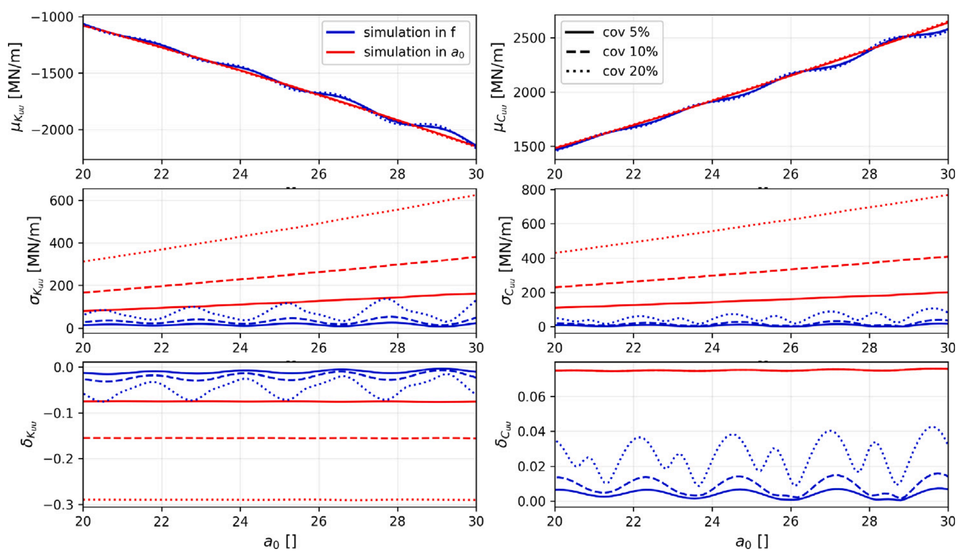


Fig. 4. Comparison of statistics computed in the f and a_0 domains for an L = 10 m and ϕ = 150 cm pile considering a normal V_s distribution with $E[V_s]$ = 100 m/s and δV_s = 0.05 and ν = 0.3, γ_s = 22 kN/m^3, ζ_s = 0.03, γ_p = 24 kN/m^3, E_p = 29 GPa, and ζ_p = 0.02.

normal distribution. Likewise, keeping a_0 constant while carrying out the simulation is equivalent to picking values from the deterministic curve in the f domain distributed according to $a_0\, V_s/(2\pi f)$. Therefore, the mean value of the damping or stiffness in one domain is equivalent to carrying out a weighted average of the deterministic curve across the frequencies in the other domain. Figs. 2 and 3 show examples of the corresponding weighting functions as histograms. This results in a mean curve with a smoother variation, but an increased standard deviation. The higher the standard deviation of V_s the smoother the mean value will be since the averaging window is extended outwards. However, this smoothing of the mean comes at the cost of increased standard deviation since, for the same f, more dissimilar values of a_0 are considered in the weighting. When other variables besides V_s are also included in the simulation this interpretation no longer holds since the reference deterministic curve is no longer unique.

The proper change of variables from one domain to the other is computed using the conditional probability, i.e.,

$$f_K\left(k/a_0\right) = -\int_0^\infty f_k\left(k/\omega\right)\; f_{V_s}\left(\frac{\omega\, R}{a_0}\right)\; d\omega \tag{18}$$

$$f_K\left(k/\omega\right) = \int_0^\infty f_k\left(k/a_a\right)\; f_{V_s}\left(\frac{\omega\, R}{a_0}\right)\; da_0 \tag{19}$$

These equations imply that knowing the probability distribution of the impedance for a given frequency in one domain is not sufficient to determine the probability distribution in the corresponding frequency in the other domain. Consequently, to estimate the statistics for a fixed frequency in one domain, they must be known across the entire frequency range in the other domain. In practice, it is simpler to carry out the simulation in both domains, or to conduct the simulation in one domain for a fixed range of frequencies, then scale for each sample of the simulation the stiffness and damping curve according to the corresponding V_s and interpolate to the desired frequencies. Eq. (19) also confirms that statistics from one domain (mean, standard deviation, coefficient of variation, percentiles, etc.) cannot be directly used in the other.

Simulations carried out in the f domain are useful from a practical point of view, since generally the excitation produced by the machine has a known fixed frequency. Thus, it makes sense to perform the reliability calculations in the f domain. The probabilistic nature of the change of variables between the f and a_0 domain also makes it much easier to measure the distance between the excitation and the resonance frequency in the f domain. On the other hand, the dimensionless frequency domain is of interest when trying to understand the underlying behavior of the foundation system and how it depends on its mechanical properties.

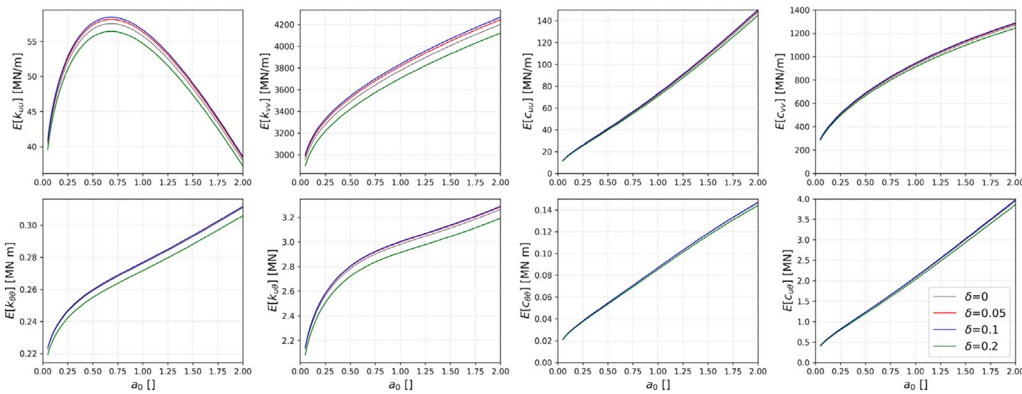


Fig. 5. Mean value of pile stiffness k and damping c considering an $L = 10$ m $\phi = 30$ cm pile with a normal V_s distribution with $E[V_s] = 400$ m/s and $\nu = 0.3$, $\gamma_s = 22$ kN/m^3, $\zeta_s = 0.03$, $\gamma_p = 24$ kN/m^3, $E_p = 29$ GPa, and $\zeta_p = 0.02$.

4. Uncertainty sensitivity analysis

The pile response model is controlled by a set of physical variables, i.e., the pile diameter ϕ and length L, the soil's shear wave velocity V_s and Poisson's coefficient ν_s, or the elastic modulus of the pile material E_p. Each of these variables has a certain degree of uncertainty associated to it, which in a stochastic analysis is modeled by a probability distribution. This uncertainty is a combination of the random variability inherent in the quantity being measured, the error in measurement incurred by the measurement method and the error incurred when a measured parameter is used to infer the value of the desired variable [39]. The stochastic analysis requires that these sources of uncertainty are dutifully addressed. However, not all of them have an equivalent impact on the stochastic behavior of the pile response, measured in terms of the pile top stiffness and damping.

A distinction is generally made where the physical variables are classified into either design parameters or random variables [40]. Although all variables have a certain degree of uncertainty, some variables will be considered constant (deterministic) either because the uncertainty associated with them is relatively small or the pile model is insensitive to the variable dispersion. Variables with low uncertainty are generally design parameters, such as the pile diameter or length. However, not all design variables have relatively small uncertainties, e.g., the mechanical properties of concrete. Also, deterministic variables may not affect the dispersion of the pile response directly, but they can affect the dispersion of the response due to the uncertainties of other variables. For example, the observed range of variability of the pile stiffness or damping for a given probability distribution of the shear wave velocity is affected by the pile diameter, as discussed later.

A sensitivity analysis was carried out to identify which variables should be treated as deterministic or random variables in the stochastic model by evaluating its effect on the mean and spread of the pile stiffness and damping. This analysis considered the soil shear wave V_s, Poisson's coefficient ν_s, unit weight γ_s and damping ζ_s, as well as the pile's unit weight γ_p, Young's modulus E_p and damping ζ_p. The analysis is carried out by considering one variable at the time as a random variable and performing a Monte Carlo simulation of the pile response, while the other variables are assigned their mean value. All random variables are assumed to follow a normal distribution to provide a uniform test across them, otherwise differences in the response may be attributed to the fact that one variable followed one distribution and another variable followed another. The analysis considered 3 levels of dispersion given by $\delta = 0.05$, $\delta = 0.10$ and $\delta = 0.20$.

The pile radius and length are considered as deterministic variables. A 10 m long pile with diameters of 30 and 150 cm are used in the analysis. The pile is assumed to be embedded in a uniform soil layer. Although soil parameters present variability within a given stratum [39,41], and this variability has been proven to be a relevant factor in the reliability analysis of piles [10,13,42], it will not be considered for the analysis. Including these phenomena would add further effects to the mean and deviation of the response that may not be easily distinguished from the sensitivity analysis itself.

Fig. 5 shows the mean value of the stiffness and damping components of the 30 cm pile when considering different levels of uncertainty ($\delta = 0.05,\ 0.10,\ 0.20$) of the soil's shear wave velocity. The mean values of stiffness and damping show a dependency on the dispersion of the physical variables. The damping components are much less sensitive to the dispersion of the input variables than the stiffness, with only very minor changes in the mean observed. Similar behaviors are observed when analyzing other random variables and the 150 cm pile. Although Fig. 5 is useful, it is not easy to disaggregate the effect of the random variable dispersion from the impedance dependence with frequency or pile diameter. It is also hard to contrast different components as they have different magnitudes. A more useful measure of the random variable uncertainty effect is obtained when analyzing the change on the mean relative to the deterministic ($\delta = 0$) case as:

$$\Delta E\left[k_i\right] = \frac{E\left[k_i\right] - \hat{k}_i}{\hat{k}_i} \qquad \Delta E\left[c_i\right] = \frac{E\left[c_i\right] - \hat{c}_i}{\hat{c}_i} \tag{20}$$

where $\hat{K}_i = \hat{k}_i + i\ \hat{c}_i$ is the deterministic impedance computed from the mean values of the random variables. For example, Fig. 6 shows that the relative effect of V_s is uniform (in relative terms) across the frequency range. This is not easily appreciated in Fig. 5. Interestingly, a larger COV of V_s do not translate into larger $\Delta E[k]$ or $\Delta E[c]$ for all components. In some cases, like k_{uu} for $\phi = 30$ cm when considering $\delta_{V_s} = 0.05$ the mean decreases with respect to the deterministic case ($\Delta E[k_{uu}] < 1$), while $\delta_{V_s} = 0.20$ increases the mean $\Delta E\left[k_{uu}\right] \sim 1.02$. This is reversed for $\phi = 150$ cm, where $\Delta E\left[k_{uu}\right] > 1$ for $\delta_{V_s} = 0.05$ and $\Delta E[k_{uu}] < 1$ for $\delta_{V_s} = 0.20$. This shows that the variation of the mean with the variability of the random variables is highly susceptible to the design deterministic variables. It is also worth noticing that for some variables the relative effect of the COV is not constant across the frequency range, as shown by Fig. 7.

Figs. 6 and 7 and the additional figures presented in the online supplemental material, indicate that the effect of the random variable dispersion on the impedance mean is bounded in the ±2% range. Although this may seem a small change, movements of the mean have significant consequences in terms of probability of failure, especially when dealing with values considered for structural reliability analysis. Also, simple analytical methods for reliability analysis such as FOSM assume that the mean value of the failure function can be adequately estimated from the mean values of the random variables [43] disregarding higher movement. Figs. 6 and 7 indicate that this is not the case when considering the pile impedance.

Fig. 8 explores the effect of the input variable dispersion on the impedance components coefficient of variation. As expected, a larger

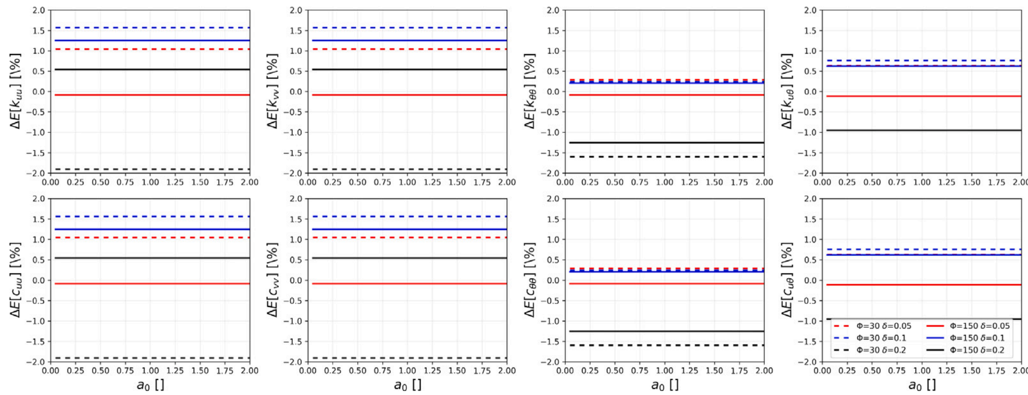


Fig. 6. Variation of the mean value of the pile impedance relative to the deterministic case ($\delta = 0$) considering the uncertainty in the shear wave velocity V_s for an $L = 10$ m pile with $E\left[V_s\right] = 400$ m/s, $\nu = 0.3$, $\gamma_s = 22$ kN/m^3,$\zeta_s = 0.03$, $\gamma_p = 24$ kN/m^3, $E_p = 29$ GPa,and $\zeta_p = 0.02$.

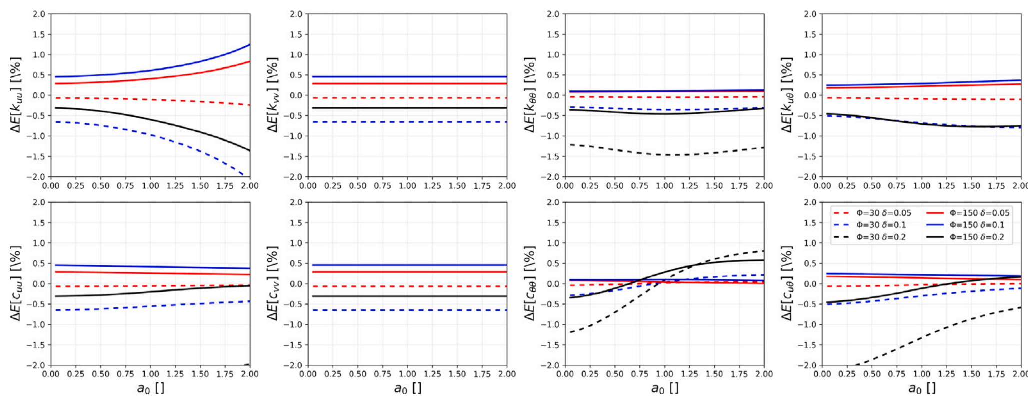


Fig. 7. Variation of the mean value of the pile impedance relative to the deterministic case ($\delta = 0$) considering the uncertainty in the soil unit weight γ_s for an $L = 10$ m pile with $E\left[\gamma_s\right] = 22$ kN/m^3, $V_s = 400$ m/s, $\nu = 0.3$, $\zeta_s = 0.03$, $\gamma_p = 24$ kN/m^3, $E_p = 29$ GPa, and $\zeta_p = 0.02$.

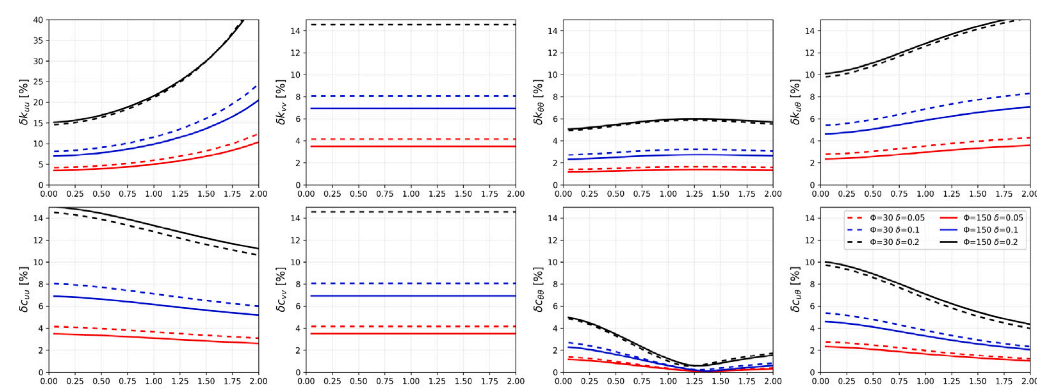


Fig. 8. Coefficient of variation of the impedance components considering the uncertainty in the soil unit weight γ_s for an $L = 10$ m pile with $E\left[\gamma_s\right] == 22$ kN/m^3, $V_s = 400$ m/s, $\nu = 0.3$, $\zeta_s = 0.03$, $\gamma_p = 24$ kN/m^3, $E_p = 29$ GPa, and $\zeta_p = 0.02$.

coefficient of variation of the simulated variable always leads to a larger variability of the pile response. However, this effect is not uniform across all variables. It is clear from the results that the dispersion in the input variables is amplified very differently by the pile model. For example, when the soil Poisson's ratio ν or the damping coefficient ζ_p are considered, the observed dispersion across all stiffness and damping components is significantly lower (<0.1) than for the rest of the variables. The variables whose dispersion causes the largest observed variability in the pile response are V_s, γ_s and E_p. It is also noticeable that in some components the variability of the response (in terms of the coefficient of variation) is larger than that of the input variables, while in others it is smaller. In this regard, k_{uu} and c_{uu} are the most sensitive, especially when V_s is involved.

Fig. 8 also illustrates the effect of the design variables over the stochastic response of the pile. When the mechanical properties of the soil are considered as the random variables one at the time, the δ of stiffness and damping components is larger for the smallest diameter pile. This means that as the soil stiffness and damping become more relevant for the overall pile response, the uncertainty in the soil parameters is amplified. The inverse can be observed with the mechanical properties of the pile material. As the pile diameter increases, the

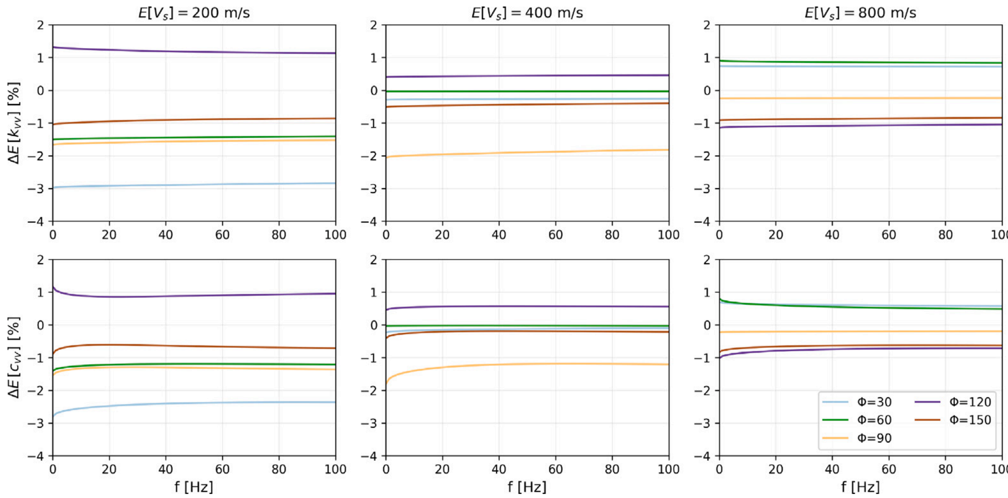


Fig. 9. Effect of considering the pile's material uncertainty in the mean of the pile impedance for vertical vibrations (see Table 2).

Table 2
Random variables considered in the simulation.

Variable	Distribution	Mean	COV
Soil shear wave velocity V_s	lognormal	200, 400, 800 m/s	0.10
Soil unit weight γ_s	lognormal	22 kN/m^3	0.05
Soil Poisson's coefficient ν	normal	0.3	0.10
Pile Young's modulus E_c	lognormal	29 GPa	0.03
Pile unit weight γ_c	lognormal	24 kN/m^3	0.03
Pile damping coefficient ζ_p	normal	0.03	0.10

variation of the pile response will be larger for the same δ value of the pile property.

The above discussion demonstrates that the stochastic behavior of the pile impedance due to the uncertainty of model variables is complex. Both the mean value and coefficient of variation of the stiffness and damping constants depended on dispersion of the input variables. The nature of this relation is also dependent on the design variables.

5. Consideration of uncertainty in the pile mechanical properties

It is reasonable to expect a lower level of uncertainty over the pile mechanical properties than from the soil parameters since the former is an engineered material where quality control and assurance can be performed. The uncertainty sensitivity analysis has also shown that the pile stochastic response is more sensitive to the dispersion of the soil parameters than to the pile itself, especially in terms of its dispersion measured with the coefficient of variation. Therefore, it is worth considering whether the simulation of the concrete properties could be omitted, as it has been done in previous reliability analysis [12,13,16].

To assess the effect of considering or disregarding the variability of the pile mechanical properties, Monte Carlo simulations are carried out for both cases with the distributions listed in Table 2. The mean values are adopted when the pile properties are assumed deterministic. Simulations are carried out for a 10 m long pile with diameters of 30, 60, 90, 120 and 150 cm embedded in a uniform soil stratum. Three different mean values of the shear wave velocity are considered (200, 400 and 800 m/s). The combinations of pile diameter and soil shear wave velocity result in wide range of relative stiffness and damping from the pile and surrounding soil. Frequencies between 0 and 100 Hz are considered in the simulation and 10^3 samples are simulated in each case.

Fig. 9 shows the relative difference in the mean impedance of the pile's vertical degree of freedom between the simulation that considers the variability of the mechanical properties of the pile and the one that does not. A similar behavior can be appreciated for the other degrees of freedom in the figures provided in the supplemental material except for the horizontal stiffness. In the case of k_{uu} with $E\left[V_s\right] = 200$ m/s the stiffness becomes negative after 60 Hz. As k_{uu} approaches 0 the relative difference diverges and is not an adequate measure of the variability effect. This issue also precludes using the coefficient of variation as a measure of dispersion of k_{uu}. Instead, the mean and standard deviation of this component are shown in Fig. 10. For the other components, the relative difference in the mean is below 4% when the uncertainty in the pile's properties is considered. In most cases is significantly smaller.

In general, but not always, the effect of not considering the uncertainty in the pile properties is less significant as the mean shear wave velocity grows. This suggests that when the soil is dominant in the overall stiffness of the pile, the stochastic behavior of the pile response is less susceptible to the uncertainty of pile material. However, if this was strictly true, a similar decrease would be expected in more slender piles with the same V_s. This is not the case, as it can be seen in c_{uu} or k_{vv} with $V_s = 200$ m/s, where slender piles show larger changes in their mean values. In addition, the effect of pile diameter depends on the mean value of the shear wave velocity. While $\phi = 30$ cm shows the largest change in mean value of k_{uu} for a mean $V_s = 200$ m/s, this is not the case for $V_s = 400$ m/s.

The effect of the pile material uncertainty in the dispersion of the pile impedance is illustrated in Fig. 11 in terms of the coefficient of variation of the vertical degree of freedom. Figures for the other DOFs are provided in the online supplement (Appendix A). For all components with all diameters and mean shear wave velocities, considering the pile mechanical properties as deterministic leads to an underestimation of the standard deviation and coefficient of variation. In particular, the $c_{\theta\theta}$ and $c_{u\theta}$ are the most affected, with the standard deviation and coefficient of variation being underestimated by half. Unlike with the mean value where the difference when considering (or not) the concrete in the simulation was diminished with larger mean values of the shear wave velocities, there does not seem to be a consistent effect in either the standard deviation or the coefficient of variation. The same lack of a general consistent effect is observed with the pile diameter. However, there is smaller difference of the coefficient of

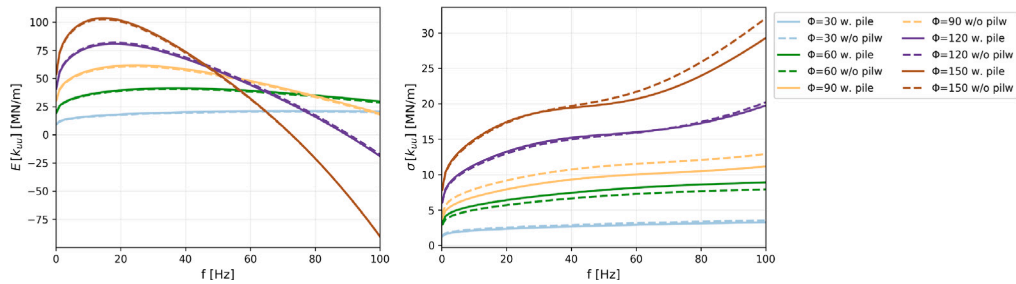


Fig. 10. Estimated mean and standard deviation of k_{uu} with and without considering the pile uncertainty (see Table 2).

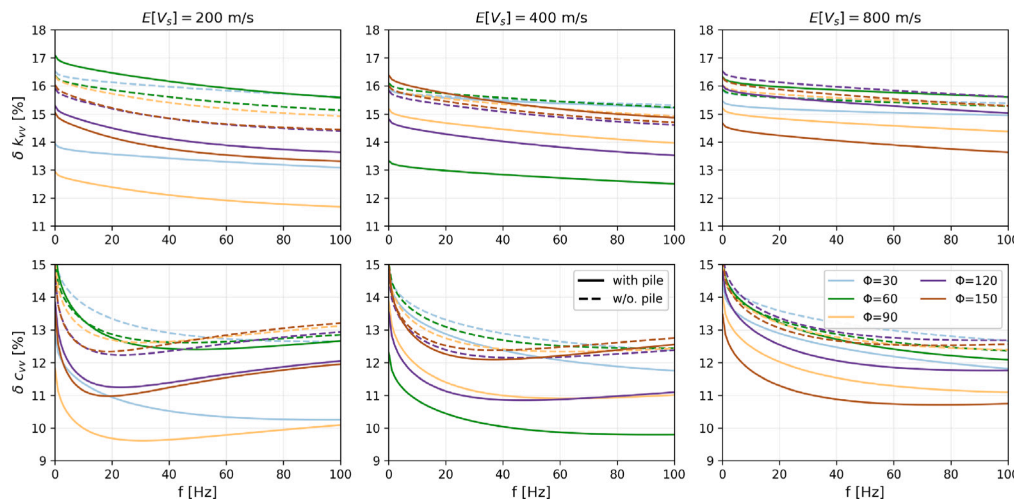


Fig. 11. Effect of considering the pile's material uncertainty in the coefficient of variation of the pile impedance for vertical vibrations (see Table 2).

variation between different pile diameters when the concrete variability is considered than when it is omitted.

Figs. 9 to 11 and similar figures in the supplemental material show that considering the pile's mechanical properties as deterministic variables leads to a significant stochastic misrepresentation of the pile impedance. The pile's uncertainty has a considerable effect over the dispersion of the stiffness and damping components, where its omission would lead to a significant underestimation. It also affects their mean values, albeit in a more reduced way. This indicates that it could be economically advantageous to increase control over the pile material properties ensuring a lower dispersion, thus reducing the uncertainty in the response without changing the expected mean value. Greater certainty in dynamic response would allow more cost-effective designs without affecting the probability of failure.

6. Effect of distribution model

Some parameters used in the model have known probability distributions, e.g., unit weight of soil [7,44] and concrete [8]. For other parameters, distributions need to be fitted based on the available data using methods such as the Pearson test [45], maximum likelihood or Bayesian estimation [46]. When there are no references or little data available, the selection of a probability distribution lies ultimately with engineering judgement and experience. Even when a dataset is available, care should be taken that the fitted distribution does not provide physically impossible values during the simulation. Unbounded distributions such as the normal or log-normal can return values for a variable that are beyond the physical range, such as a negative damping or a Poisson's coefficient above 0.5. When this happens, either these values need to be removed from the simulation or the distribution should be changed to a bounded one, such as a triangular or beta.

To understand the effect of the distribution of the random variables, the pile impedance was simulated using normal, log-normal and triangular distributions with the same mean and standard deviations. The simulations were carried out in the a_0 domain for frequencies between 0 and 100 Hz for a 10 m long pile with a diameter of 30 and 150 cm embedded in a homogeneous stratum. Mean values of shear wave velocity of 200, 400 and 800 m/s were considered for the soil. The mean and coefficient of variation listed in Table 2 where used for the other variables. Each simulation consisted of 10^3 samples.

Fig. 12 analyzes the effect of the distribution shape on the mean values of the simulated stiffness and damping in the horizontal direction. Similar plots are included in the online supplement for the other components (see Appendix A). The effect of the mean is expressed relative to the mean values of the distribution as indicated in Eq. (20). Since the distributions used in the simulations had the same mean, the reference impedance is shared across all cases, which removes the effect of the general increase of stiffness with the diameter or mean shear wave velocity and provides a uniform measure of the change in the mean value.

Fig. 13 examines the effect of the distribution shape on the dispersion of the impedance components, measured in terms of the coefficient of variation δ. Plots for the remaining components are included in the online supplement (see Appendix A). For a given pile diameter and frequency, the effect of the random variable distribution is within $\pm 0.01\%$ in terms of the coefficient of variation of the pile stiffness or damping. The sensitivity of the COV to changes in the distribution is shown to be dependent on the mean shear wave velocity, the pile diameter, and the component under consideration. However, once a mean shear wave velocity and diameter are fixed, the ranking of the coefficient of variation for a given component is the same for all frequencies. Therefore, the mere selection of a distribution does not automatically indicate whether the expected variability in the pile response will be larger or not.

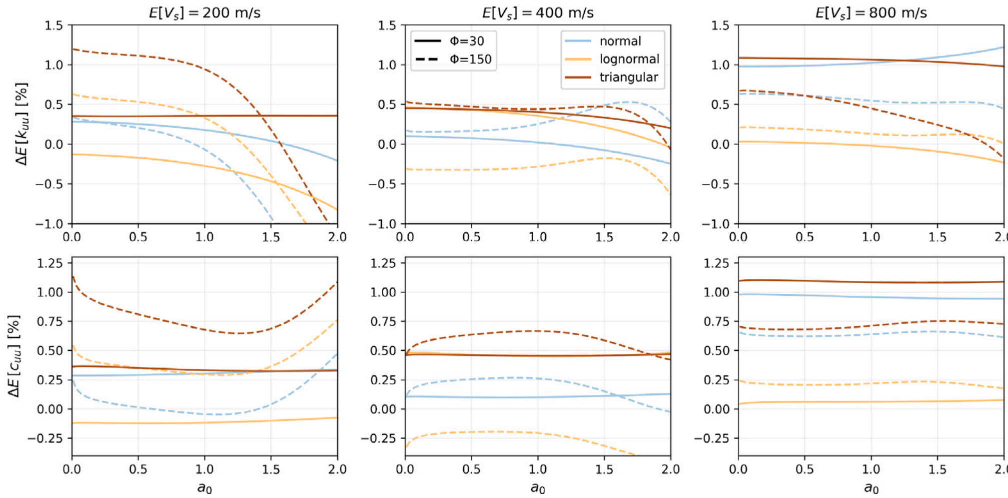


Fig. 12. Effect of random variable distribution on the mean value of the pile impedance (means and COVs from Table 2).

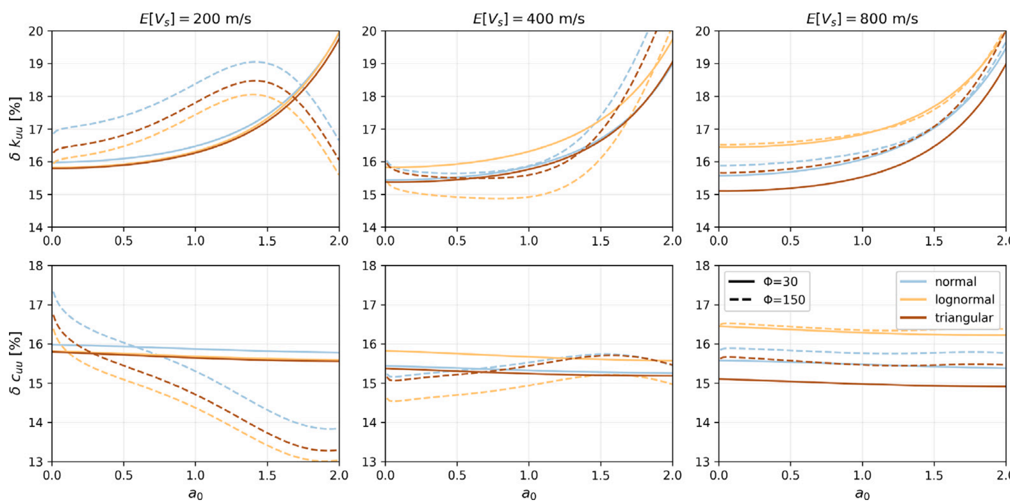


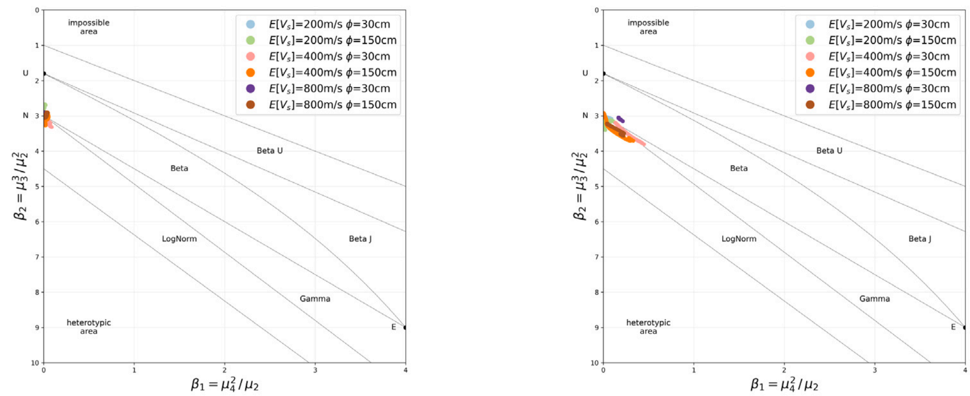
Fig. 13. Effect of random variable distribution on the COV of the pile impedance (means and COVs from Table 2).

Baecher [44] recommended the use of the Pearson distribution system to recognize potential stochastic models for parameters of interest in geotechnical engineering. Pearson [47–49] parameterized a group of 12 commonly used probability distributions by means of a single differential equation. He showed that each of the distributions only covered a single region in the β_1-β_2 domain, where $\beta_1 = \mu_4^2/\mu_2$ and $\beta_2 = \mu_3^3/\mu_2^2$, with μ_k being the kth central moment. This was represented graphically by Ord [45] as the regions shown in Fig. 14. The mathematical expression for the boundaries between the regions is provided by Singh [50].

Fig. 14 show the Pearson distribution plots across the entire frequency range for k_{uu} when considering normal and log-normal distributions for the random variables. Similar plots are produced for the other stiffness and damping components including triangular distributions and are included in the supplemental online material (see Appendix A). When considering the different mean shear wave velocities, pile diameters and distributions for the random variables of the simulation. The plots indicate that for a given pile diameter and mean V_s, the distribution type best suited to represent the simulated stiffness and damping depends on the excitation frequency. Significant differences can be appreciated when either normal, lognormal, or triangular distributions are used for the simulation. Triangular distributions offer the most consistent results, with the beta distribution being the best fit for all the observed dispersion in the stiffness and damping. This is attributed to the bounded nature of the triangular distribution, which leads to bounded stiffness and damping. It is also worth noting that for the normal and log-normal cases, and same shear wave velocity and pile diameter, the best fit may be different for the stiffness and damping.

Pearson plots are limited to the distributions contained in the Pearson system. Thus, even when the skewness and kurtosis values may indicate which of the distributions included in the system is the best suited for representing the simulated results, there may not be a good fit. The goodness of fit therefore needs to be analyzed by a suitable hypothesis test, such as the Kolmogorov–Smirnov test [46]. Fig. 15 shows the corresponding p-value for a normal, log-normal and triangular fit to the simulated stiffness and damping assuming normal, log-normal and triangular distributions for the random variables. None of the distributions fitted to the simulation provide an acceptable fit across all components or frequencies. In general, the log-normal distribution is the best fit, especially when using log-normal distributions for the random variables. Even when the random variables are normal or triangular, the log-normal fit seems to offer a better fit overall. This indicates that the pile response model by itself induces a skewed distribution of stiffness and damping. It is also clear that the normal distribution is not an acceptable fit to the simulated stiffness or damping, except for $k_{\theta\theta}$. This precludes the usage of reliability analysis methods that rely on the normality of the distributions.

The distribution fitting test presented in Fig. 15 shows Kolmogorov–Smirnov test results for c_{uu} when considering lognormal distributions



(a) Simulation with normal distributions. (b) Simulation with lognormal distributions.

Fig. 14. Pearson plots for k_{uu} (means and COVs from Table 2).

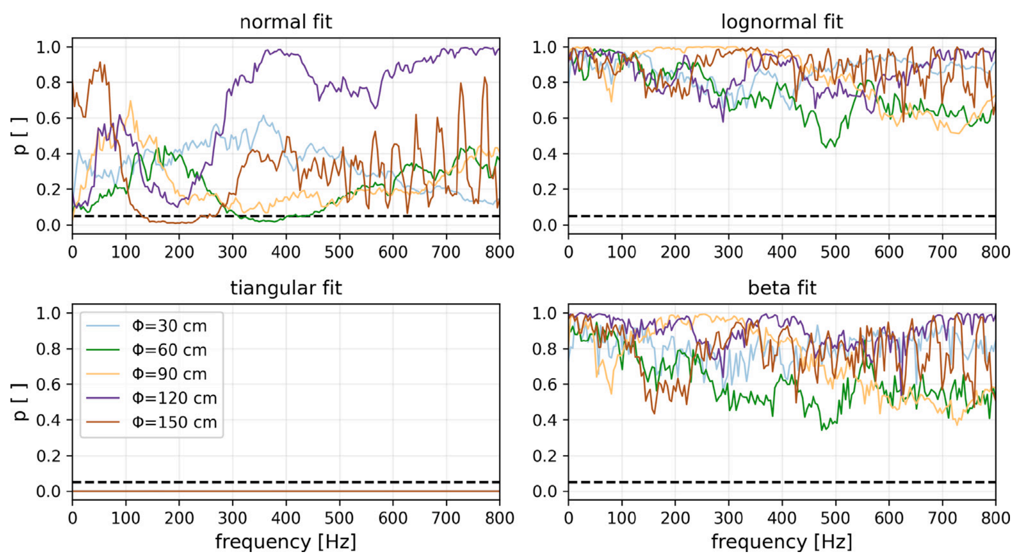


Fig. 15. Kolmogorov–Smirnov p value for fitting a normal, lognormal and triangular distribution to the simulated c_{uu} when using lognormal distributions for the random variables (means and COVs from Table 2).

for the random variables and a mean shear wave velocity of 200 m/s. Each subplot shows the test result for the hypothesis that the simulated values of c_{uu} follows a normal, lognormal, triangular or beta distribution across the frequency range for different pile diameters. Adopting a significance value of $p = 0.05$, the null hypothesis is always rejected for the triangular distribution, and it may or not be rejected in the other cases depending on the frequency and diameter. This same analysis is presented in the supplemental information extended to other components, mean shear wave velocities and distribution used for the simulation of the random variables. In general, triangular distributions do not pass the test even when the random variables are triangular. The only distribution that consistently passes the test is the beta distribution. Normal and log-normal distributions may or not be adequate models depending on the frequency, mean shear wave velocity, random variable distribution, and diameter. No consistent pattern can be seen in the results. Therefore, assuming a priori a distribution shape irrespective of the excitation frequency, V_s or pile diameter is not advisable. This indicates that the application of methods such as PEM [51,52], based on estimating the moment of the stiffness components and using those moments as parameters of an assumed distribution, should be considered carefully.

The spread observed in the Pearson plots (Fig. 14) shows that the selection of a distribution model depends on the higher order moments of the stiffness, which require more simulations for a given confidence level. K–S tests (Fig. 15) only discard a couple of distributions, but do not provide a conclusive answer to which model to select. A more sophisticated model selection technique is required that not only considers the simulated dispersion in the stiffness but also the complexity of the distribution model and whether there is sufficient information to choose one over another. Fig. 16 shows the results of the application of the Akaike information criteria [53,54] to the simulated values of k_{vv}. The criteria indicate that the beta distribution should be adopted, even though it requires more parameters. Not surprisingly the triangular distribution is the worst option. This result is replicated for all components across all frequencies for all pile diameters and mean shear wave velocities considered. Results are presented in the Supplemental information. However, it should be noted that the difference between the AIC score for the normal, lognormal and beta distribution can be very small, and that changing the sample size may change this result.

7. Discussions and conclusion

The stochastic analysis of the dynamic pile response has shown that:

- There is a complex dependency of the pile stiffness and damping on the deterministic and random variables. This dependency precludes a first order estimation of the response moments, which

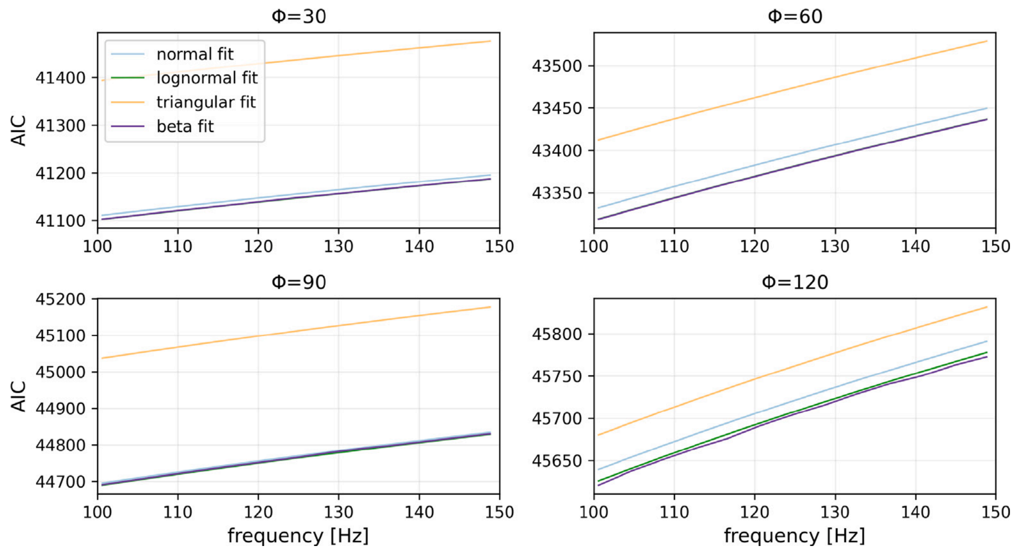


Fig. 16. Akaike information criteria of the simulated c_{vv} when using normal distributions for the random variables and $E[V_s] = 200$ m/s (means and COVs from Table 2).

suggest that simplified reliability methods such as FOSM might lead to significant errors in the estimation of the probability of failure.
- The soil shear wave velocity and unit weight, and the pile Young's modulus are the most influential variables in terms of the observed dispersion of the pile response. The relevance of E_p in the stochastic response depends on the pile-soil relative stiffness.
- The analysis of the effect of the probability distribution selected for the random variables indicates that for a given mean and standard deviation, the distribution shape accounts for relative minor changes in the mean value (<1%) of the pile stiffness and damping, while having a more significant effect on the dispersion as measured by the standard deviation. The effect of the distribution model also depends on the frequency considered.

The expected level of uncertainty in the estimation of soil parameters in a typical site investigation may significantly affect the stiffness and damping of the pile, and thus, the expected dynamic response. Currently, this is not quantitatively considered in the design process. Fortunately, this situation can be easily rectified. Winkler models, such as the one used in this paper, are commonly used in regular geotechnical practice for the design of foundations due to their ease of implementation and modest computational demand, which also made them ideal for reliability analysis.

When considering the uncertainty in the design process, the analysis conducted above provide some useful guidelines:

- Even though the model is relatively simple and well understood, there is a complex interaction between it and the design and random variables. Thus, acceptance of the results requires a detailed inspection to confirm that the random variables distributions are properly quantified the uncertainty in the available data.
- The pile response distribution is heavily dependent on the excitation frequency. Therefore, the estimation of the uncertainty of the responses for a given frequency is not a valid measure of the uncertainty at other frequencies.
- The uncertainty in the pile mechanical properties affects the stochastic behavior of the pile impedance mostly by increasing its standard deviation (up to 200% in the case of flexural damping). The effect on the mean values is much smaller, accounting for a variation limited to 4%. This finding strongly suggests that strict quality controls on the pile material can lead to significant reductions in the dispersion of expected impedance, allowing for more economical designs for a given probability of failure.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.soildyn.2023.108287.

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