Weighted OFDM With Block Codes for Wireless Communication

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Abstract— **One of the major disadvantages with OFDM system is its high peak-to-average power ratio (PAPR). In this paper we investigate the potential of some of the well known linear block codes in reducing the PAPR. We present a method of biasing the codewords to minimize PAPR. The influence of combined weighting and block coding on the PAPR is investigated. A class of block codes capable of both error correction and PAPR reduction together with several weighting functions is considered. We investigate the interplay of various weighting functions with block codes in order to minimize PAPR. Proposed schemes reduce PAPR significantly as is evident from the simulation results.**

I. Introduction

In recent years, Orthogonal Frequency Division Multiplexing (OFDM) [1], [2] has become a good candidate for wireless multimedia communication by virtue of its excellent properties in frequency-selective fading environment. In OFDM, data is transmitted over several parallel low data rate channels. Thus it provides data integrity due to fading, relative to modulation methods that employ single channel for high data rate transmission. Among other benefits of OFDM is that it fully exploits the advantages of digital signal processing concepts [3].

OFDM transmitter and receiver structures are shown in Figs. 1 and 2 respectively. The serial input data is encoded using channel encoder before being sent to the mapper. The mapper could be PSK, QAM, or DPSK etc. In this paper we have chosen BPSK as the mapping technique. The mapper output is applied to a serial-to-parallel converter and then to IFFT, which is the most important block of an OFDM system as it outputs orthogonal signals on its N sub channels. After up converting the resulting signal to desired carrier frequency the signal is transmitted.

At the receiver, the process described above is reversed. As shown in Fig. 2, the process starts with down converting the received signal and passing through several blocks to eventually obtain the transmitted data sequence. In the absence of noise and fading, transmitted data is recovered without errors.

In OFDM, as the number of sub carriers increase, the effective waveform approaches that of a sample function from a Gaussian process. This results in occasional peaks in the transmitted signal. Peak-to-average power ratio (PAPR) is a good measure of these peaks. A baseband OFDM signal with N sub-channels has a $PAPR = N$ [4]. When passed

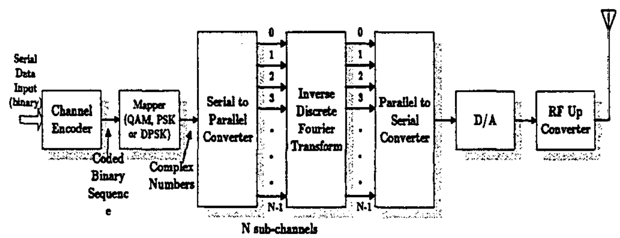

Fig. 1. OFDM Transmitter

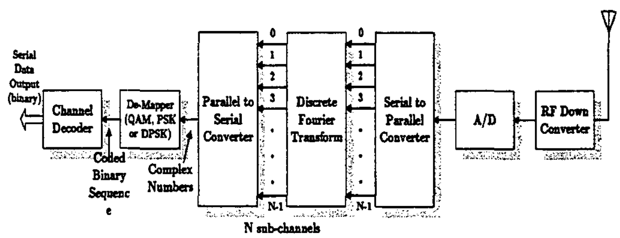

Fig. 2. OFDM Receiver

through a nonlinear device, such as a transmit power amplifier, the signal may suffer significant spectral spreading and in-band distortion [5]. It is desired to reduce PAPR because the power amplifiers used at the transmitter have a linear behavior upto a certain range. Beyond this range they become nonlinear causing signal distortion. Hence, the problem of Peak-to-Average Power Ratio (PAPR) has received widespread attention in recent years. Many methods have been suggested to alleviate the problem of PAPR. Among these, one of the suggestions is to employ block codes [6], [7], [8]. Yet another technique is to reduce PAPR through weighting of OFDM signal [9].

The intent of this paper is to explore the potential of some of the well known linear block codes in reducing PAPR. We introduce a scheme of biasing the codewords to reduce PAPR. In literature, block codes and weighting functions have been considered separately for the reduction of PAPR. Since weighted OFDM also reduces PAPR at the expense of BER, we investigate methods to compensate for the degradation of BER by employing these linear block codes. Hence, we address the problem of jointly reducing PAPR and BER using block codes and weighting functions.

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We consider a class of weighting functions such as Raised Cosine, Half Sine, Shannon, Gaussian etc. and investigate their interplay with linear block codes such as Hamming and BCH in order to jointly optimize PAPR and BER.

The paper is organized as follows: PAPR is introduced in the next section along with the proposed coding scheme. Weighting functions are described in section III while numerical results are presented in section IV. Paper is concluded in section V.

II. Coding Scheme

A typical OFDM symbol is represented by:

$$x(t) = \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \qquad 0 \leq t \leq NT \tag{1}$$

$X_n, \{n = 0, 1, \ldots, N-1\}$, are the outputs of the mapper which are ± 1 for BPSK. f_n is the frequency of nth carrier:

$$f_n = f_o + \frac{n}{t_s} \tag{2}$$

where f_o is the lowest frequency of carriers and t_s is OFDM symbol duration.

PAPR is defined as [4]:

$$PAPR = \frac{\max |x(t)|^2}{E\left[|x(t)|^2\right]} \tag{3}$$

We consider six linear block codes. These are Hamming(7,4) and (15,11), BCH(7,4), (15,11), (15,7) and (15,5). The first four codes can correct one bit errors each while BCH(15,7) and (15,5) can correct upto 2 and 3 errors, respectively [10]. The (7,4) codes will be employed for an eight carrier system while the (15,x) codes can be used for an eight (when using 4PSK) as well as sixteen (when using BPSK) carrier system. The strategy is shown in Fig. 3 and described below:

Using exhaustive search, a bias vector of n bits is chosen and added to all possible codewords (using modulo-2 addition) such that the maximum PAPR of all these codewords is minimized. For example, for a Hamming (7,4) code, choose a bias vector 0000001 and using modulo-2 addition, add it to all possible codewords. Compute PAPR for each codeword using (3). Store the maximum PAPR obtained using this bias vector. Choose another bias vector 0000010 and compute PAPR for all the codewords. Exhaust all the possibilities of choosing a bias vector such that all bits are zero except one. Retain the bias vector that gives the minimum PAPR. Next exhaust all the possibilities of having two bits as 1 in the bias vector and so on. If the PAPR does not improve further, we stop and choose the bias vector that has given the best PAPR. This approach is applied to all the six coding schemes and the best bias vectors found are shown in Table I.

For efficient FFT/IFFT implementation, it is desired that the number of carriers be in powers of two. Therefore, when using a (7,4) code, we add one redundant bit to all the biased codewords. Again we choose these bits such that the PAPR is reduced further. It should be noted that for a particular coding scheme, the redundant bit could be different for different code words depending upon which bit (1 or 0) gives a better PAPR. Once the bias vector and redundant bits are determined for a particular coding scheme, the new coding becomes (8,4) for a Hamming(7,4) or BCH(7,4) and (16,x) for a Hamming or BCH (15,x).

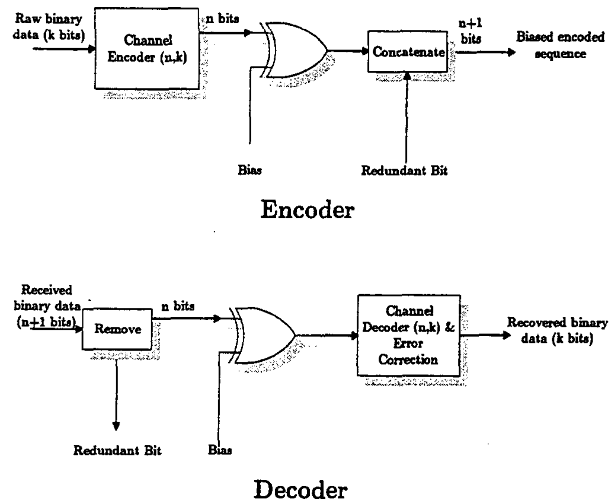

Fig. 3. Coding Scheme

TABLE I
Best Bias Vectors

Codes	Bias Vectors
Hamming (7,4)	0000001
BCH (7,4)	0000001
Hamming (15,11)	001000000000000
BCH (15,11)	001000000000000
BCH (15,7)	100000001010000
BCH (15,5)	100001001001000

At the decoder, the data on 8th or 16th carrier is discarded as it is redundant and the bias vector used at the encoder is added modulo-2 to the received codeword. Well-known techniques of error detection and correction for Hamming and BCH codes can now be applied to the received codewords to correct errors if any.

III. Weighting Functions

In [9], a novel scheme of using weighting functions to reduce PAPR has been introduced. In this scheme, each complex number is weighted by a real factor α_m, $m = 0, 1, \ldots, N-1$ before taking the IFFT. In such a case, the weighted OFDM signal becomes

$$x(t) = \sum_{n=0}^{N-1} X_n \alpha_n e^{j2\pi f_n t}, \qquad 0 \leq t \leq NT \tag{4}$$

In this paper, we consider the following six weighting functions:

Rectangular: This weighting function has a rectangular shape and is expressed by

$$\alpha_m = \begin{cases} A & 0 \leq m \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Bartlett: This weighting function has a triangular shape

$$\alpha_m = \begin{cases} A\left(1 - \frac{|m-\frac{N}{2}|}{\frac{N}{2}}\right) & 0 \leq m \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Gaussian: These factors are generated based on the Gaussian function, i.e.,

$$\alpha_m = \begin{cases} A \exp\left[-\frac{(m-\frac{N}{2})^2}{2\sigma^2}\right] & 0 \leq m \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where σ is the spread or standard deviation of the weighting factors around $N/2$. We have considered two values for σ, $N/8$ and $3N/16$.

Raised cosine: The shape of this function is described by

$$\alpha_m = \begin{cases} A \sin^2\left(\pi \frac{m}{N}\right) & 0 \leq m \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Half-sin: This weighting function is explained by

$$\alpha_m = \begin{cases} A \sin\left(\pi \frac{m}{N}\right) & 0 \leq m \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Shannon: The shape of these weighting factors is the sinc function, i.e.,

$$\alpha_m = \begin{cases} A \text{sinc}\left(\frac{2m-N}{N}\right) & 0 \leq m \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

The amplitude A in (5)-(10) is selected in such a way that the power of all weighting factors be constant, i.e.,

$$\sum_{m=0}^{N-1} \alpha_m^2 = 1 \tag{11}$$

IV. Numerical Results

Figs. 4 and 5 summarize achievable PAPR as a function of Hamming and BCH codes, with several weighting functions. It is observed that for 8 carriers both Hamming(7,4) and BCH(7,4) yield same PAPR without weightings. However, one obtains marginal improvements with weightings and PAPRs are identical. For Gaussian 1 ($\sigma = N/8$), Gaussian 2 ($\sigma = 3N/16$), Raised cosine and Shannon weightings PAPR deteriorates with these codes.

In case of 16-carrier BCH(15,5) coded and Gaussian 2 weighting, it is possible to achieve an improvement of nearly 6.2 dB relative to 16-carrier uncoded system without weighting. For identical weightings, both BCH(15,11) and Hamming(15,11) offer nearly the same performance. However, these systems are inferior to BCH(15,7) and BCH(15,5) by nearly 1.6 dB and 2.7 dB, respectively. It is observed that in case of BCH(15,7) system the best PAPR is achieved when either no weighting or rectangular weighting is employed.

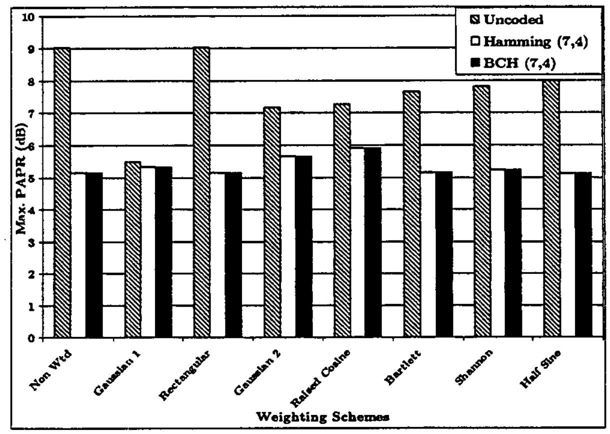

Fig. 4. Maximum PAPR for 8 carrier system

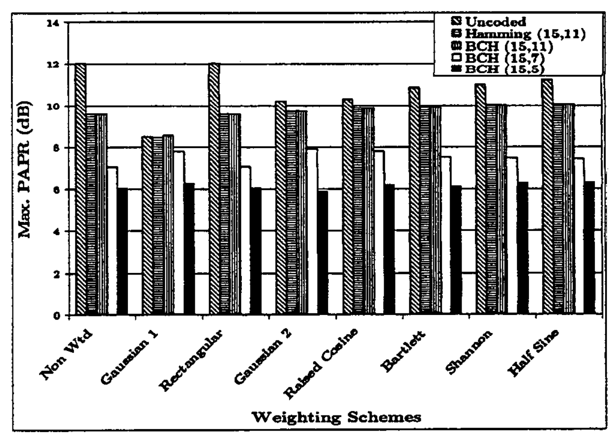

Fig. 5. Maximum PAPR for 16 carrier system

PAPR performance is often illustrated using complementary cumulative distribution function ($CCDF = Pr(PAPR > PAP_o)$) [4]. Figs. 6, 7 and 8 show CCDF plots for some of the schemes. Fig. 6 shows the CCDF for an 8 carrier coded and uncoded OFDM system. A 2 dB improvement is achieved when Hamming(7,4) or BCH(7,4) codes are used with biasing. Since in these cases, weightings

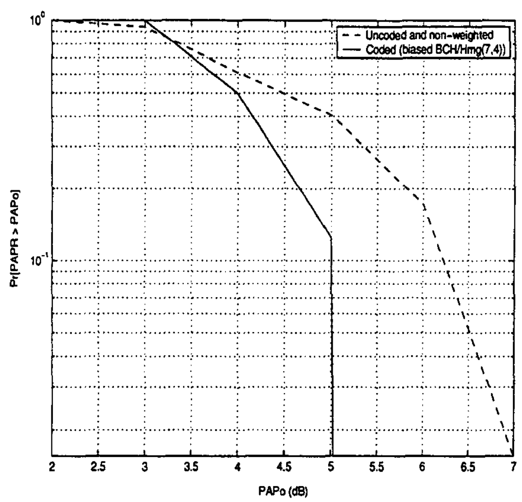

Fig. 6. Complementary cumulative distribution function for N=8 when biased Hamming(7,4) or BCH(7,4) is used

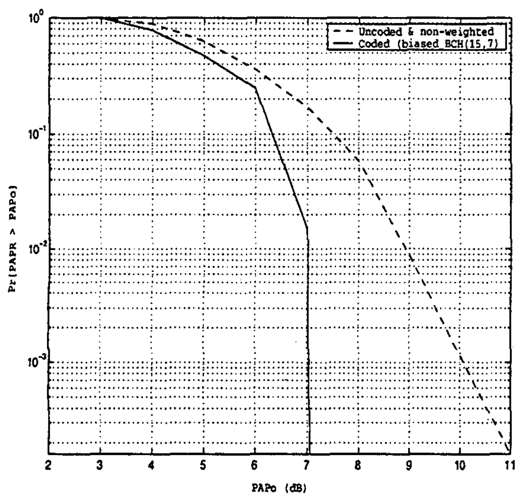

Fig. 7. Complementary cumulative distribution function for N=16 and biased BCH(15,7)

do not reduce PAPR any further, the plots do not show results for weighted OFDM. Fig. 7 shows the performance of a 16-carrier BCH(15,7) coded OFDM system. A reduction of 4 dB in PAPR is achieved in this case. Weighting does not play a significant role in reducing PAPR in this case as well. BCH(15,5) along with Gaussian 2 weighting gives the best performance for a 16 carrier OFDM system. Fig. 8 shows a gain of almost 6 dB when BCH(15,5) is used along with Gaussian weighting.

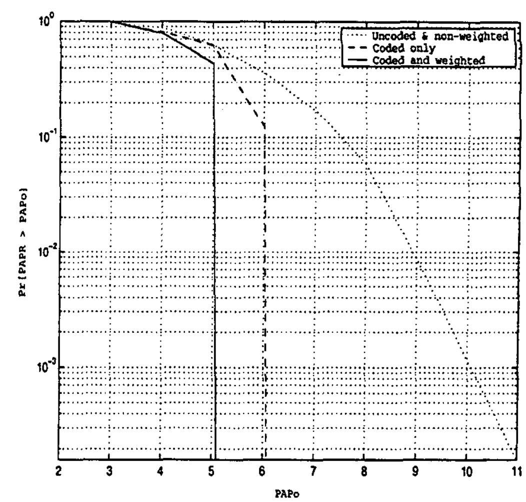

Fig. 8. Complementary cumulative distribution function for N=16 and biased BCH(15,5) when Gaussian function is used for weighting

V. Conclusions

A novel coding scheme based on Hamming and BCH codes is investigated in order to reduce PAPR in OFDM systems. It is shown that biased BCH(15,5) gives the lowest PAPR. A combination of these codes along with weighting functions is also considered. Biased BCH(15,5) with Gaussian weighting achieves a PAPR improvement of approximately 6 dB. With the proposed scheme, both PAPR reduction and BER improvement can be achieved with little overhead.

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