Detection of OFDM-CPM Signals over Multipath Channels

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***Abstract*—A class of Orthogonal Frequency Division Multiplexing - Continuous Phase Modulation (OFDM-CPM) signals is introduced in which binary data sequence is mapped to complex symbols using the concept of correlated phase states of a CPM signal. Canonical optimum and suboptimum multiple-symbol-observation OFDM-CPM receivers are derived. Multipath channel with AWGN is assumed. The receivers are analyzed for bit-error-rate (BER) performance in terms of high- and low-SNR bounds. These bounds are illustrated as a function of parameter h, time delay and attenuation level. It is shown through numerical computation that OFDM-CPM system outperforms a similar OFDM-PSK system in a two-ray multipath model. Moreover, an OFDM-CPM system gives the best error performance when the parameter h is 0.5 with observation interval of 2.**

I. Introduction

OFDM is a good candidate for wireless multimedia communication by virtue of its excellent properties in frequency-selective fading environment [1], [2]. In OFDM, we transmit data over several parallel low data rate channels. This provides data integrity due to fading, relative to modulation methods that employ single channel for high data rate transmission. Among other benefits of OFDM is that it fully exploits the advantages of digital signal processing concepts [3].

A typical OFDM transmitter works as follows: Serial encoded data is sent to a mapper that outputs a complex number. A serial-to-parallel converter serially takes in the complex numbers and forms a parallel stream by increasing their time period. This stream has as many complex numbers as the number of subchannels. Inverse Fast Fourier Transform (IFFT) is then applied to the parallel stream that results in orthogonal signals on the subchannels. The orthogonal signals are then converted back into a serial stream and after up converting the signal to desired carrier frequency the signal is transmitted.

While in the literature OFDM-PSK, QAM, DPSK and DAPSK have been considered [4]-[7], OFDM-CPM signals that use the concept of correlated phase states of a CPM signal have not been considered to-date. One of the advantages of OFDM-CPM signals is that we can systematically introduce correlation amongst adjacent OFDM symbols by an appropriate choice of parameter h (in typical CPM signals h is modulation index). Furthermore, this correlation can be exploited in order to reduce the BER in such a system. Thus in the paper we address the detection problem of observing n OFDM-CPM symbols and arriving at an optimum decision on one of the symbols. The channel is modeled as multipath and AWGN.

The paper is organized as follows: In section II we describe OFDM-CPM signaling scheme. We address the detection problem in section III and arrive at the receiver structure. In section IV high and low SNR bounds on the performance of the optimum OFDM-CPM receiver are derived. These bounds are illustrated through numerical computation in section V. The paper is concluded in section VI.

II. OFDM-CPM Signaling Scheme

As shown in Fig. 1, serial bit stream b_i, $i = 0, 1, 2, \ldots$, with bit duration of T_b seconds is converted into blocks of N bits represented by $a_{k,p}$, $k = 0, 1, 2, \ldots$, and $p = 0, 1, 2, \ldots, N-1$, where N denotes the number of carriers and $a_{k,p} = \pm 1$. For example, $a_{0,p}$ would denote the first block of N bits and $a_{1,p}$ the second block of N bits and so on. The CPM mappers transform the incoming $a_{k,p}$ into appropriate complex numbers $c_{k,p}$ given by

$$c_{k,p} = \cos(\theta_{k,p}) + j\sin(\theta_{k,p}), \tag{1}$$

with

$$\theta_{k,p} = a_{k,p}\pi h + \pi h \sum_{q=0}^{k-1} a_{q,p} + \phi; \tag{2}$$

where h, $0 < h < 1$, is a parameter and ϕ represents the initial mapping point that is assumed zero without loss of generality. It is evident from (2) that the angles $\theta_{k,p}$ depend not only on the current data but also on the past data. Fig. 2 shows all the possible values of $\theta_{k,p}$ as a function of time when $h = 0.5$. Current value of θ is determined by adding $+\pi h$ (if data bit is a $+1$) or $-\pi h$ (if data bit is a -1) to the previous value of θ.

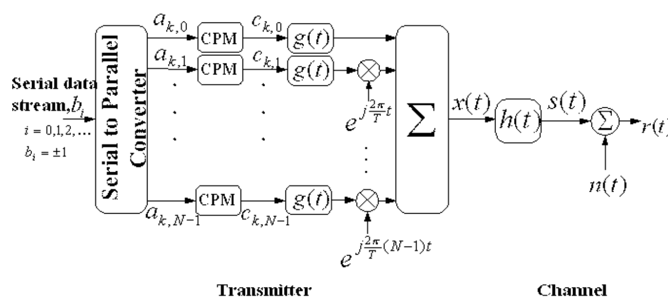

Fig. 1. OFDM-CPM Transmitter and Channel

The corresponding complex numbers lie on a circle. In Fig. 3 we show the constellation diagram of CPM mapper for $h = 0.5$ and $h = 0.25$. The corresponding complex numbers of a 4-carrier OFDM-CPM signal with $h = 0.5$ for two consecutive blocks of data sequences are shown below:

$$[a_{k,p}, a_{k+1,p}] \implies [c_{k,p}, c_{k+1,p}],$$
$$\begin{bmatrix} +1 & -1 \\ +1 & +1 \\ -1 & +1 \\ +1 & -1 \end{bmatrix} \implies \begin{bmatrix} +j & 1 \\ +j & -1 \\ -j & 1 \\ +j & 1 \end{bmatrix}.$$

The complex numbers from the output of CPM mappers are passed through pulse shaping filters $g(t)$, then modulated by orthogonal carriers and finally summed to give the transmitted OFDM symbol which is mathematically represented as

$$x(t) = \sum_k \sum_p c_{k,p} g(t - kT) e^{j\frac{2\pi}{T}pt}, 0 \leq t < \infty \qquad (3)$$

where

$$g(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t \leq T \\ 0 & \text{elsewhere.} \end{cases}$$

In (3), $T(= NT_b)$ is the OFDM symbol duration.

III. The Detection Problem

With reference to Fig. 1, we can model the received signal as

$$\left.\begin{aligned} r(t) &= x(t) * h(t) + n(t) \\ &= s(t) + n(t) \end{aligned}\right\} 0 \leq t \leq nT, \qquad (4)$$

where $h(t)$ is the channel impulse response, $n(t)$ is AWGN with a double sided power spectral density of $\frac{N_o}{2}$ and * denotes convolution. $s(t)$ is given by [8]

$$s(t) = \sum_{k=0}^{n-1} \sum_{p=0}^{N-1} c_{k,p} \psi_p(t - kT), \qquad (5)$$

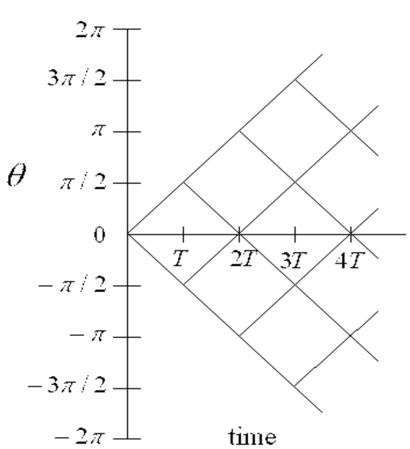

Fig. 2. Phase trellis for OFDM-CPM signaling

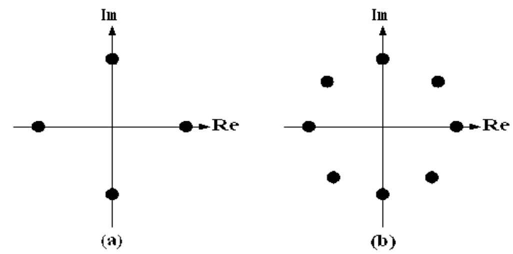

Fig. 3. Constellation diagram of CPM mapper for (a) $h = 0.5$ and (b) $h = 0.25$

where

$$\psi_p(t) = g(t) e^{j\frac{2\pi}{T}pt} * h(t). \qquad (6)$$

$s(t)$ can be written as $s(t, A_0, [A_1, \ldots, A_{n-1}])$ where A_k is a vector $[c_{k,0}, c_{k,1}, \ldots, c_{k,N-1}]^T$ of complex numbers which in turn relates to the input data sequence $[a_{k,0}, a_{k,1}, \ldots, a_{k,N-1}]^T$. Thus there are 2^N possible A_ks. The detection problem is to observe $r(t)$ over nT seconds and arrive at an optimum estimate of A_0 (given by $\hat{A}_0$) transmitted during $0 \leq t \leq T$ seconds. For the case of N carriers and n observation intervals, the detection problem is the composite hypothesis testing problem stated below [9]:

$$H_i : r(t) = s\left(t, A_0^i, [A_1, \ldots, A_{n-1}]\right) + n(t), \qquad (7)$$

where $i = 0, 1, 2, \ldots, 2^N - 1$ and $0 \leq t \leq nT$. A_k^i denotes the ith of the 2^N possible A_ks. The solution to this problem is the likelihood ratio test (LRT) which can be expressed as

$$\Lambda_i = \int_B \Pr(R \mid H_i, B) \Pr(B)\, dB, \qquad (8)$$

where

$$\begin{aligned} B &= [A_1, A_2, \ldots, A_{n-1}], & (9) \\ \int_B dB &= \int_{A_1}\int_{A_2} \cdots \int_{A_{n-1}} dA_1 dA_2 \ldots dA_{n-1}, & (10) \\ \Pr(B) &= \Pr(A_1)\Pr(A_2)\ldots\Pr(A_{n-1}), & (11) \\ \Pr(A_l) &= \frac{1}{v}\sum_{i=0}^{v-1} \delta\left(A_l - A_l^i\right) & (12) \end{aligned}$$

and $v = 2^N$. Substituting (9)-(12) in (8) it can be shown that the likelihood functions can be written as

$$\begin{aligned} \Lambda_i = \sum_{d=0}^{z-1} \exp\Bigg[&\frac{2}{N_o}\int_0^{nT} \operatorname{Re}\left\{r(t) s^*\left(t, A_0^i, B^d\right)\right\} dt \\ &- \frac{1}{N_o}\int_0^{nT} \left|s\left(t, A_0^i, B^d\right)\right|^2 dt \Bigg], \end{aligned} \qquad (13)$$

where $z = 2^{N(n-1)}$ and B^d is the dth of the z possible realizations. The corresponding receiver structure is shown in Fig. 4 where $K_{i,d}$ represents the portion in (13) that is independent of the received signal $r(t)$.

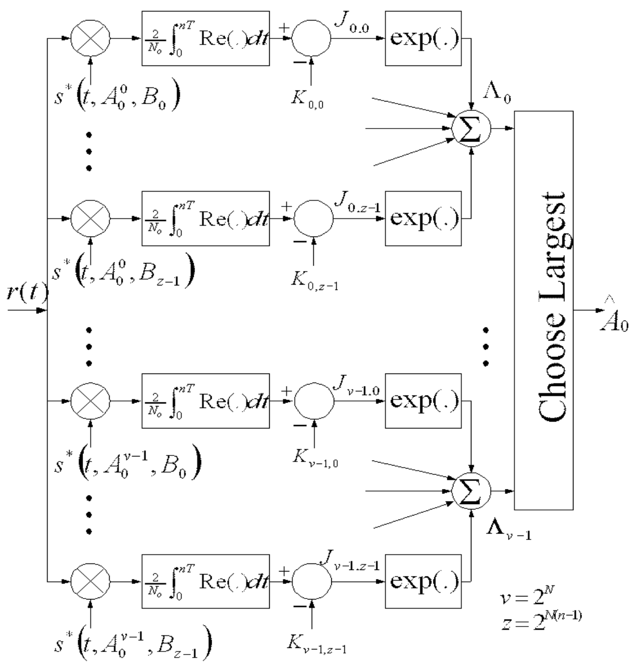

Fig. 4. OFDM-CPM receiver

IV. Performance of OFDM-CPM Receiver

It is too complex to analytically compute the performance of the receiver shown in Fig. 4. Therefore, we derive performance bounds at low and high SNRs. These bounds taken as a single bound will be a good performance bound at all values of SNR [10].

A. Low SNR Approximation

At low SNR the likelihood function can be approximated by

$$\Lambda_i \approx 1 + \frac{2}{N_o}\int_0^{nT} \mathrm{Re}\left\{ r(t) \sum_{d=0}^{z-1} s^*\left(t, A_0^i, B^d\right)\right\} dt - \frac{1}{N_o}\int_0^{nT} \sum_{d=0}^{z-1} \left|s\left(t, A_0^i, B^d\right)\right|^2 dt; \quad (14)$$

the receiver dictated by (14) is given as

$$\Lambda_i = 2\int_0^{nT} \mathrm{Re}\left\{ r(t) \sum_{d=0}^{z-1} s^*\left(t, A_0^i, B^d\right)\right\} dt - \int_0^{nT} \sum_{d=0}^{z-1} \left|s\left(t, A_0^i, B^d\right)\right|^2 dt. \quad (15)$$

Let hypothesis j and a data sequence B^ν be true, then the received signal $r(t)$ is given by

$$r(t) = s\left(t, A_0^j, B^\nu\right) + n(t). \quad (16)$$

Since Λ_is are Gaussian, in order to determine the error performance of the receiver we need to find the conditional means and variances of Λ_is. Then the probability of symbol error will be given by

$$\mathrm{Ps} = \frac{1}{z}\sum_{\nu=0}^{z-1}\left(1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) \mathrm{x} \prod_{\substack{i=0 \\ i\neq j}}^{v-1}\left(1 - Q\left[\frac{\sigma_y u + \mu_y^\nu - \mu_{\Lambda_i}^\nu}{\sigma_{\Lambda_i}}\right]\right) du\right), \quad (17)$$

where $y = [\Lambda_j \mid H_j]$ and μ_y^ν, $\mu_{\Lambda_i}^\nu$, σ_y^2 and $\sigma_{\Lambda_i}^2$ are the conditional means and variances of y and Λ_i given by

$$\begin{aligned}
\mu_y^\nu &= 2\mathrm{Re}\left[\sum_{d=0}^{z-1}\sum_{k=0}^{n-1}\sum_{k'=0}^{n-1} \mathbf{C}_{k'}^{j,d^H} R(k'-k)\mathbf{C}_k^{j,\nu}\right] - \sum_{d=0}^{z-1}\sum_{k=0}^{n-1}\sum_{k'=0}^{n-1} \mathbf{C}_{k'}^{j,d^H} R(k'-k)\mathbf{C}_k^{j,d}, \\
\mu_{\Lambda_i}^\nu &= 2\mathrm{Re}\left[\sum_{d=0}^{z-1}\sum_{k=0}^{n-1}\sum_{k'=0}^{n-1} \mathbf{C}_{k'}^{i,d^H} R\left(k'-k\right)\mathbf{C}_k^{j,\nu}\right] - \sum_{d=0}^{z-1}\sum_{k=0}^{n-1}\sum_{k'=0}^{n-1} \mathbf{C}_{k'}^{i,d^H} R(k'-k)\mathbf{C}_k^{i,d}, \\
\sigma_y^2 &= N_o \sum_{d=0}^{z-1}\sum_{k=0}^{n-1}\sum_{d'=0}^{z-1}\sum_{k'=0}^{n-1} \mathbf{C}_k^{j,d^H} R\left(k'-k\right)\mathbf{C}_{k'}^{j,d'}, \\
\sigma_{\Lambda_i}^2 &= N_o \sum_{d=0}^{z-1}\sum_{k=0}^{n-1}\sum_{d'=0}^{z-1}\sum_{k'=0}^{n-1} \mathbf{C}_k^{i,d^H} R\left(k'-k\right)\mathbf{C}_{k'}^{i,d'}.
\end{aligned}$$

In these expressions R is a matrix of cross correlations whose ath row and bth column is given by

$$R_{a,b}(k'-k) = \int_0^{nT} \psi_b(t-kT)\psi_a^*(t-k'T)dt. \quad (18)$$

$\mathbf{C}_k^{d,i}$ are vectors of complex numbers defined as

$$\begin{aligned}
\mathbf{C}_k^{\nu,j} &= \left[c_{k,0}^{\nu,j}, c_{k,1}^{\nu,j}, \ldots, c_{k,N-1}^{\nu,j}\right]^T, \\
\mathbf{C}_{k'}^{d,i} &= \left[c_{k',0}^{d,i}, c_{k',1}^{d,i}, \ldots, c_{k',N-1}^{d,i}\right]^T
\end{aligned}$$

and $\mathbf{C}^H = (\mathbf{C}^*)^T$.

B. High SNR Approximation

The optimum receiver of (13) can be written as

$$\Lambda_i = \sum_{d=0}^{z-1} \exp\left(J_{i,d}\right), \quad (19)$$

where

$$J_{i,d} = \frac{2}{N_o}\int_0^{nT} \mathrm{Re}\left\{r(t)s^*\left(t, A_0^i, B^d\right)\right\} dt - \frac{1}{N_o}\int_0^{nT} \left|s\left(t, A_0^i, B^d\right)\right|^2 dt. \quad (20)$$

At high SNR it can be shown that

$$\sum_{d=0}^{z-1} \exp\left(J_{i,d}\right) \approx \exp\left(\widetilde{J}_i\right), \quad (21)$$

where

$$\widetilde{J}_i = \max\left\{J_{i,d}; d = 0, 1, 2, \ldots, z-1\right\}.$$

Since the function exp(.) is monotonic, the high SNR sub-optimum receiver computes $J_{i,d}, i = 0, 1, 2, \ldots, v-1; d = 0, 1, 2, \ldots, z-1$ and a decision is made based on the largest of these. Upper bound on the error rate of such a receiver can be union bounded which is given by

$$\mathrm{Ps} \leq \frac{1}{vz}\sum_{i=0}^{v-1}\sum_{l=0}^{z-1}\sum_{d=0}^{z-1} \mathrm{Pr}\left(J_{0,l} \leq J_{i,d} | s\left(t, A_0^0, B^l\right)\right), \quad (22)$$

where $\mathrm{Pr}\left(J_{0,l} \leq J_{i,d} | s\left(t, A_0^0, B^l\right)\right)$ can be shown to be

$$\mathrm{Pr}\left(J_{0,l} \leq J_{i,d} | s\left(t, A_0^0, B^l\right)\right) = Q\left[\sqrt{\frac{E_{0,l} + E_{i,d} - 2\rho_{0,l}^{i,d}}{N_o}}\right] \quad (23)$$

and

$$E_{0,l} = \sum_{k=0}^{n-1}\sum_{k'=0}^{n-1} \mathbf{C}_{k'}^{0,l^H} R(k'-k)\mathbf{C}_k^{0,l},$$

$$E_{i,d} = \sum_{k=0}^{n-1}\sum_{k'=0}^{n-1} \mathbf{C}_{k'}^{i,d^H} R(k'-k)\mathbf{C}_k^{i,d},$$

$$\rho_{0,l}^{i,d} = \mathrm{Re}\left\{\sum_{k=0}^{n-1}\sum_{k'=0}^{n-1} \mathbf{C}_{k'}^{i,d^H} R(k'-k)\mathbf{C}_k^{0,l}\right\}.$$

V. Numerical Results

We now demonstrate some of the results based on the theory presented in previous sections. For all the results a two path channel is assumed having impulse response given by

$$h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau), \quad (24)$$

where α_1 and α_2 are the levels of direct and delayed paths and τ is the delay. In all the results that we present, observation interval is two symbols and initial mapping points are assumed zero. Further, in Figs. 5-7, $\alpha_1/\alpha_2 = 4$ dB and $\tau/T = 0.1$. The performance of OFDM-CPM system is evaluated for three values of h which are 0.25, 0.5 and 0.715. Constellation diagrams for $h = 0.25$ and $h = 0.5$ are shown in Fig. 3. $h = 0.715$ is chosen because it gives optimum error performance in a single-carrier CPM system for a 5-bit observation interval. In the paper we derived expressions for OFDM symbol error rate from which BER can be approximated by using the approach given in [11].

Fig. 5 shows the BER performance of an 8-carrier OFDM-CPM system with $h = 0.5$. The composite upper bound is constructed by taking the smaller of the high SNR bound and the low SNR bound. The BER performance of an 8-carrier OFDM-BPSK system with symbol by symbol detection is also shown in this figure. For the case of OFDM-CPM, significant improvement in performance is observed because OFDM-CPM symbols are correlated and multiple-symbol-observation receiver is employed. This receiver exploits the extra information and as a result BER improves.

Fig. 6 shows a plot of composite upper bound of an 8-carrier OFDM-CPM system for various values of h. Best performance is achieved when $h = 0.5$. This is due to the fact that signal constellation has four points and the distance between signals is the maximum. When $h = 0.25$, there are eight constellation points and as a result the distance between signals decreases which deteriorates the performance. BER for $h = 0.715$ falls somewhere in between $h = 0.25$ and $h = 0.5$.

BER performance of an OFDM-CPM system as a function of τ/T is shown in Fig. 7. SNR is fixed at 10 dB and $\alpha_1/\alpha_2 = 4$ dB. Performance is sensitive to delay for τ/T less than 0.1. However, BER stabilizes when τ/T goes beyond 0.1. This is because delayed and direct waves are synchronized at some point adding the overall power. This keeps the BER stable. However, we anticipate that the performance will deteriorate for higher values of τ/T when the ratio α_1/α_2 is larger because ICI and ISI will dominate. It is seen that the proposed system performs better than OFDM-PSK and the best performance is given by $h = 0.5$.

Fig. 8 shows a plot of composite bound as a function of α_1/α_2 for an 8-carrier OFDM-CPM system with $h = 0.25, 0.5$ and 0.715 and SNR=10 dB. For low levels of α_1/α_2, BER is high because of the ICI and ISI introduced by the delayed wave. As α_1/α_2 increases, system performance improves because the

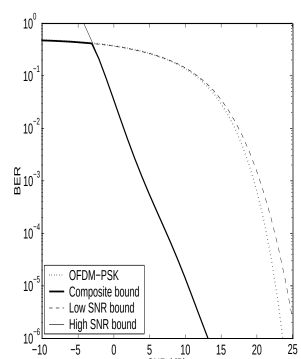

Fig. 5. BER of an 8-carrier OFDM-CPM system with $h = 0.5$, $\alpha_1/\alpha_2 = 4$ dB and $\tau/T = 0.1$

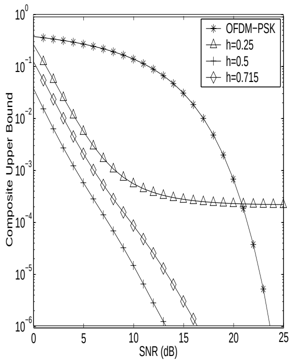

Fig. 6. Composite bound of an 8-carrier OFDM-CPM system for various values of h with $\alpha_1/\alpha_2 = 4$ dB and $\tau/T = 0.1$

errors are predominantly due to Gaussian noise.

VI. Conclusions

We introduced a new class of OFDM-CPM signals and derived optimum and suboptimum multiple-symbol-observation receivers assuming multipath and AWGN. The performance was compared with a similar OFDM-PSK system and it was shown that OFDM-CPM outperformed OFDM-PSK. The implementation and performance evaluation of the proposed receivers requires very high computational power and that is why we could not go beyond 8 carriers and two observation intervals. However, their performance is useful as a reference on what is possible to attain.

The receiver complexity can be reduced by choosing rational values for h that guarantees finite number of points on constellation. Suboptimum algorithms such as MLSE (Maximum Likelihood Sequence Estimation) using Viterbi algorithm can

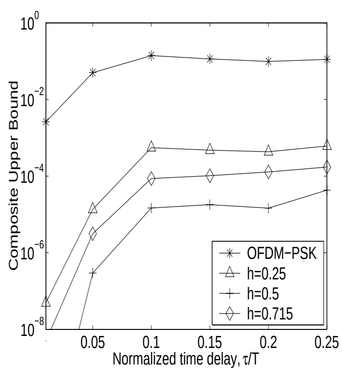

Fig. 7. Composite bound of an 8-carrier OFDM-CPM system for various values of τ/T with $\alpha_1/\alpha_2 = 4$ dB and SNR = 10 dB

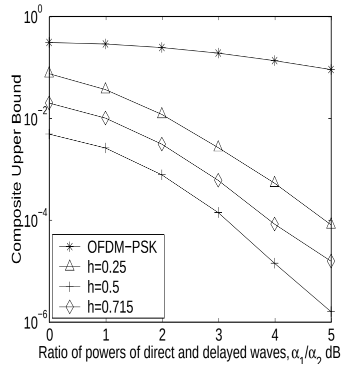

Fig. 8. Composite bound of an 8-carrier OFDM-CPM system as a function of α_1/α_2 with SNR = 10 dB and $\tau/T = 0.1$

also be investigated for detection of OFDM-CPM signals. Furthermore, we are investigating the performance of an FFT-based receiver for an OFDM-CPM system. This will allow us to go for higher number of carriers.

References

[1] J.A.C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Communications Magazine*, pp. 5–14, May 1990.

[2] Leonard J. Cimini Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. on Comm.*, vol. 33, no. 7, pp. 665–675, July 1985.

[3] S.B. Weinstein and Paul M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," *IEEE Trans. on Comm. Tech.*, vol. 19, no. 5, pp. 628–634, October 1971.

[4] Minoru Okada, Shinsuke Hara, and Norihiko Morinaga, "Bit error rate performances of orthogonal multicarrier modulation radio transmission systems," *IEICE Trans. Commun.*, vol. E76-B, no. 2, pp. 113–119, February 1993.

[5] Yun Hee Kim, Iickho Song, Hong Gil Kim, Taejoo Chang, and Hyung Myung Kim, "Performance analysis of a coded OFDM system in time-varying multipath Rayleigh fading channels," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 5, pp. 1610–1615, September 1999.

[6] Thomas May, Hermann Rohling, and Volker Engels, "Performance analysis of Viterbi decoding for 64-DAPSK and 64-QAM modulated OFDM signals," *IEEE Trans. on Comm.*, vol. 46, no. 2, pp. 182–190, February 1998.

[7] Jun Lu, Tjeng Thiang Tjhung, Fumiyuki Adachi, and Cheng Li Huang, "BER performance of OFDM-MDPSK system in frequency-selective Rician fading with diversity reception," *IEEE Transactions on Vehicular Technology*, vol. 49, no. 4, pp. 1216–1225, July 2000.

[8] Anders Vahlin and Nils Holte, "Maximum-likelihood sequence estimation for OFDM," *Signal Processing in Telecommunications: Proc. of the 7th Int'l Thyrrhenian Workshop on Digital Communcations, Viareggio, Italy/Ezio Biglieri and Marco Luise, eds.*, pp. 207–219, September 10-14 1995.

[9] Anthony D. Whalen, *Detection of signals in noise*, Academic Press, 1971.

[10] William P. Osborne and Michael B. Luntz, "Coherent and noncoherent detection of CPFSK," *IEEE Trans. on Comm.*, vol. 22, no. 8, pp. 1023–1036, August 1974.

[11] John G. Proakis, *Digital Communications*, McGraw Hill Inc., 2001.