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PAPR Reduction of OFDM-CPM System Using Multiamplitude CPM Signals

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***Abstract*— Recently, a new class of orthogonal frequency division multiplexing-continuous phase modulation (OFDM-CPM) signals was introduced. In this paper, we propose and investigate the use of multiamplitude CPM signal mapper for reduction of PAPR. In this method, two signal points represent the same information and we choose the one that minimizes PAPR. The concept of partial transmit sequences (PTS) is also used to select the best signal points. Unlike other PAPR reduction schemes such as partial transmit sequences, our method does not require side information to be sent to the receiver. It is shown through simulations that the proposed scheme reduces PAPR of 128-carrier OFDM-CPM signals by more than 4 dB.**

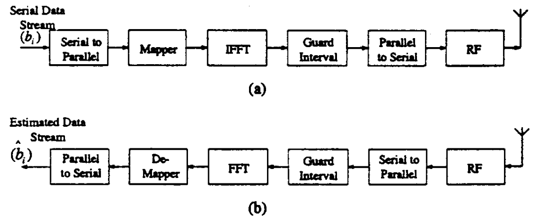

Fig. 1. OFDM transmitter and receiver

I. Introduction

OFDM is a good candidate for wireless multimedia communication by virtue of its excellent properties in frequency-selective fading environment [1], [2]. In OFDM, data is transmitted over several parallel low data rate channels. This provides data integrity due to fading, relative to methods that employ single channel for high data rate transmission. Among other benefits of OFDM is that it fully exploits the advantages of digital signal processing concepts [3].

A typical OFDM transmitter is shown in Fig. 1(a). A serial-to-parallel converter serially takes in the data stream and forms a parallel stream which is then sent to a mapper that outputs complex numbers. The mapper could be PSK, QAM, DPSK, DAPSK etc. Inverse fast Fourier transform (IFFT) is then applied to the parallel stream of complex numbers that results in orthogonal signals on the subchannels. In order to mitigate the effects of ISI, a guard interval is inserted at the transmitter and is removed at the receiver. The orthogonal signals are then converted back into a serial stream, up converted to desired carrier frequency and transmitted.

At the receiver (Fig.1(b)), the process described above is reversed. The received signal is down converted, passed through a serial-to-parallel converter, a guard interval remover, an FFT block, a de-mapper and finally a parallel-to-serial converter to eventually obtain the transmitted data sequence. In the absence of noise and fading, transmitted data is recovered without errors.

While in the literature OFDM-PSK, -QAM, -DPSK and -DAPSK have been considered [4]–[7], OFDM-CPM signals that use the concept of correlated phase states of a CPM signal have recently been introduced [8], [9]. One of the advantages of OFDM-CPM signals is that we can systematically introduce correlation amongst adjacent OFDM symbols by an appropriate choice of parameter h (in typical CPM signals h is modulation index). Furthermore, this correlation can be exploited in order to reduce the BER in such a system.

In OFDM, as the number of subcarriers increase, the effective waveform approaches that of a sample function from a Gaussian process [10]. This results in occasional peaks in the transmitted signal. The peak transmitted power of an OFDM signal may be up to N times the average power, with N the number of carriers [11]. Large peaks introduce an increased complexity of the analog-to-digital (A/D) and digital-to-analog (D/A) converters and degrade the efficiency of the RF power amplifier. Linear amplifiers that can handle the peak power are less efficient. Hard limiting of the transmitted signal generates intermodulation among the subcarriers and results in out-of-band radiation. Hence it is desired to reduce the peak power of OFDM signals. Peak-to-average power ratio (PAPR) is a good measure of these peaks. Worst possible PAPR of an N-carrier OFDM system is N. It is observed that highly correlated data sequences such as a sequence of all zeros or ones, or a sequence of alternate zeros and ones cause high signal peaks. A number of approaches have been suggested to alleviate the problem of PAPR. Some of these employ partial transmit sequences (PTS) [12]–[15], block codes [10],[16],[17], scrambling [18], weighting [19], [20], selective mapping [21], symbol or bit interleaving [22], phase optimization [13], and crest factor optimization [23].

We present a method that uses multiamplitude CPM signals and partial transmit sequences to reduce PAPR of OFDM-CPM signals. Unlike other PAPR reduction schemes our method does not require side information to be sent to the receiver. We show that gain of more than 4 dB in PAPR is possible with little complexity.

The paper is organized as follows. In Section II we review OFDM-CPM signaling scheme. Multiamplitude CPM signals are described in Section III while PAPR reduction algorithm is presented in Section IV. Simulation results are presented in Section V and the paper is concluded in Section VI.

II. OFDM-CPM Signaling Scheme

As shown in Fig. 2, serial bit stream b_i, $i = 0, 1, 2, \ldots$, with bit duration of T_b seconds is converted into blocks of N bits represented by $a_{k,p}$, $k = 0, 1, 2, \ldots$, and $p = 0, 1, 2, \ldots, N-1$, where N denotes the number of carriers and $a_{k,p} = \pm 1$. For example, $a_{0,p}$ would denote the first block of N bits and $a_{1,p}$ the second block of N bits and so on. The CPM mappers transform the incoming $a_{k,p}$ into appropriate complex numbers $c_{k,p}$ given by

$$c_{k,p} = \cos(\theta_{k,p}) + j\sin(\theta_{k,p}), \tag{1}$$

with

$$\theta_{k,p} = \begin{cases} a_{k,p}\pi h + \pi h \sum_{q=0}^{k-1} a_{q,p} + \phi; & k \geq 1 \\ a_{0,p}\pi h + \phi; & k = 0 \end{cases} \tag{2}$$

where parameter h defines the CPM mapper and ϕ represents the initial mapping point that is assumed zero without loss of generality. The angles $\theta_{k,p}$ depend not only on the current data but also on the past data. Fig. 3 shows values of $\theta_{k,p}$ as a function of time when $h = \frac{1}{2}$. Current value of θ is determined by adding $+\pi h$ (if data bit is a $+1$) or $-\pi h$ (if data bit is a -1) to the previous value of θ. The corresponding complex numbers lie on a circle.

The complex numbers from the output of CPM mappers are passed through pulse shaping filters $g(t)$, then modulated by orthogonal carriers and finally summed to give the transmitted OFDM symbol which is mathematically represented as

$$x(t) = \sum_{k}\sum_{p} c_{k,p}\, g(t-kT)\, e^{j\frac{2\pi}{T}pt}, 0 \leq t < \infty \tag{3}$$

where

$$g(t) = \begin{cases} \frac{1}{\sqrt{T}}; & -T_g \leq t \leq LT \\ 0; & \text{elsewhere.} \end{cases} \tag{4}$$

In (3) and (4) $T (= NT_b)$ is the OFDM symbol duration and T_g is the guard interval. If data is to be transmitted

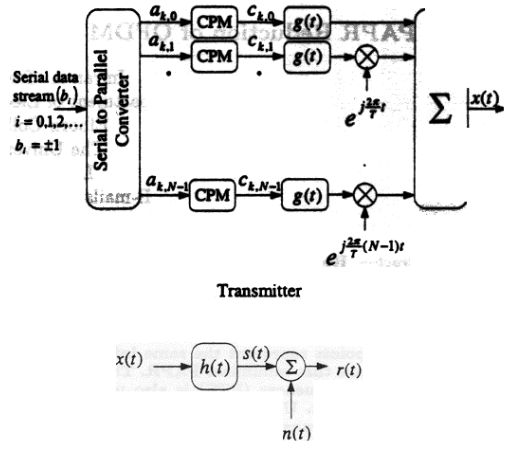

Fig. 2. OFDM-CPM Transmitter and Channel

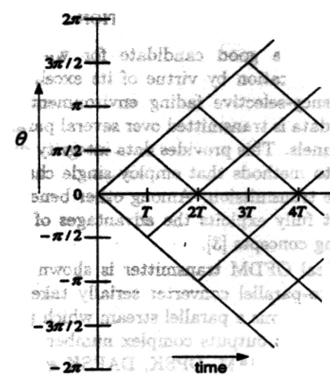

Fig. 3. Phase trellis for OFDM-CPM signaling

at the same information rate then sampling time T_b of the signal should be decreased to $T_b' = \left(\frac{LT}{LT+T_g}\right)T_b$. In (4) $L = 1$ for full response signaling. The parameters h and L can be chosen in various ways giving rise to some of the following possible OFDM-CPM signals.

A. Single-h OFDM-CPM Signals

In this case, the value of h remains constant for all OFDM symbols. By choosing h to be rational and $0 < h < 1$ it is possible to have finite number of points in the CPM constellation to reduce receiver complexity. If h

could be written as $2k/p$, where k and p are integers then p denotes the number of points in CPM constellation. In Fig. 4 is shown the constellation diagram of CPM mapper for $h = \frac{1}{2}$ (four constellation points) and $h = \frac{1}{4}$ (eight constellation points).

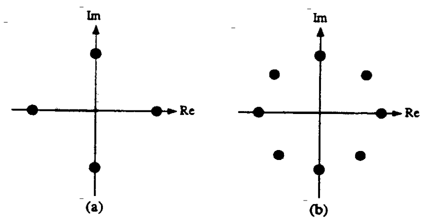

Fig. 4. Constellation diagram of CPM mapper for (a) $h = \frac{1}{2}$ and (b) $h = \frac{1}{4}$

B. Multi-h OFDM-CPM Signals

The value of h is cyclically chosen from a set of K values, $\{h_1, h_2, \ldots, h_K\}$. The value of h employed during the ith symbol is given by h_l, $l = i$ modulo K. By restricting h_l to include only multiples of $1/q$, q an integer, one obtains a property that all phases at times nT (n being observation interval) are some multiple of $2\pi/q$. A demodulator/decoder need only deal with transitions to these q phases [24].

For example, the complex numbers of a 4-carrier OFDM-CPM signal with $H_2 = \{\frac{2}{4}, \frac{1}{4}\}$ for first two blocks of data sequences are shown below (assuming initial mapping points to be $1 + j0$):

$$[a_{k,p}, a_{k+1,p}] \implies [c_{k,p}, c_{k+1,p}],$$

$$\begin{bmatrix} +1 & -1 \\ +1 & +1 \\ -1 & +1 \\ +1 & -1 \end{bmatrix} \implies \begin{bmatrix} +j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ +j & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ -j & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \\ +j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \end{bmatrix}.$$

C. Asymmetric OFDM-CPM Signals

While in multi-h OFDM-CPM signals h values are independent of data bits $a_{k,p} (= \pm 1)$, in asymmetric multi-h signals h is made a function of $a_{k,p}$. That is, the value of h during the ith symbol interval is chosen h_{+i} or h_{-i} accordingly as data is a $+1$ or -1 respectively. This gives additional flexibility to the designers to optimize system performance. For example, let $\{h_a, h_b, h_c\}$ be the set for a 3-h scheme. One way of implementing asymmetric signaling is to shift h_{-i} with respect to h_{+i} by one symbol interval as shown below.

i:	1	2	3	4	5	6	...
h_{+i}:	h_a	h_b	h_c	h_a	h_b	h_c	...
h_{-i}:	h_b	h_c	h_a	h_b	h_c	h_a	...

i.e., $h_{-i} = h_{+(i+1)}$ [25].

D. Partial Response OFDM-CPM Signals

In (4), by making $L > 1$ the pulse duration can be extended to more than one OFDM symbol. Using a value of $L = 2, 3, \ldots$ systematic correlation can be furthered amongst OFDM symbols which in turn can be exploited for improvement in system performance.

III. Multiamplitude CPM Signals

A typical OFDM symbol is represented by:

$$x(t) = \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \qquad 0 \le t \le NT_b \tag{5}$$

where $X_n, \{n = 0, 1, \ldots, N-1\}$, are the outputs of the signal mapper, f_n is the frequency of nth carrier, T_b is the original bit period and N is the number of orthogonal carriers.

PAPR is defined as [11]:

$$\mathrm{PAPR} = \frac{\max |x(t)|^2}{E\left[|x(t)|^2\right]} \tag{6}$$

In single carrier communications, multiamplitude CPM is a generalization of conventional CPM in which the signal amplitude is allowed to vary over a set of amplitude values while the phase of the signal is constrained to be continuous [26]. In this paper, we use two-component CPM constellations in the signal mapper to reduce PAPR where the signal amplitude is allowed to take one of the two possible values. Fig. 5 shows two-component CPM constellation for two values of h which are 1/2 and 2/3. Black dots on the inner circle indicate mapping points generated by CPM mappers while transparent dots on the outer circle are alternate signal points representing the same information as the black dots lying on the same axis. Hence, each black dot has an alternate transparent dot that could be chosen if it would reduce PAPR.

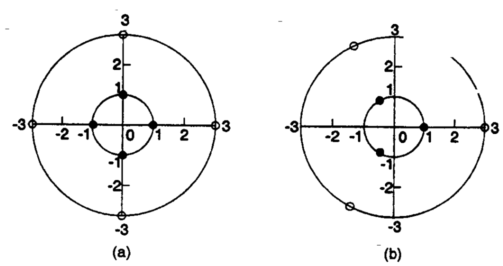

Fig. 5. Signal space diagram for two-component CPM with (a) $h = 1/2$ and (b) $h = 2/3$

At the receiver the signal is decoded based on the phase of complex numbers. This eliminates the need to send additional information to the receiver about the amplitudes of complex numbers.

IV. Proposed Algorithm

Similar to the suboptimal PTS approach presented in [11] the data block which is input to IFFT is partitioned into disjoint subblocks or clusters. Let this input vector be represented by $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$. Assuming that we wish to use M disjoint subblocks of equal size, partition the block $\mathbf{X}$ into M disjoint subblocks represented by $Y_m, m = 1, 2, \ldots, M$. In other words, all subcarrier positions in Y_m which are already represented in another subblock are set to zero. Mathematically,

$$\mathbf{X} = \sum_{m=1}^{M} Y_m \tag{7}$$

Next, we need to determine which of the M subblocks should have their amplitudes expanded as shown in Fig. 5. Let a_m be a factor which is 3 if the amplitude of subblock m is to be expanded and 1 if it is to remain the same. Then,

$$\mathbf{X}' = \sum_{m=1}^{M} a_m Y_m, \tag{8}$$

and in the time domain,

$$x' = \sum_{m=1}^{M} a_m . \mathrm{IFFT}\{Y_m\} = \sum_{m=1}^{M} a_m x_m. \tag{9}$$

In (9), the linearity of IFFT is exploited.

To begin, assume that $a_m = 1$ for all m and compute x' using (9) and then calculate the resulting PAPR. Expand the amplitudes of elements of the first block by setting a_1 equal to 3 and recompute PAPR. If the new PAPR is less than that in the previous step, retain the new amplitudes otherwise a_1 goes back to 1. The algorithm continues until all the possible M blocks are explored in this fashion.

V. Simulation Results

Figs. 6 and 7 show complementary cumulative distribution function (CCDF) of a 128-carrier OFDM-CPM system with $h = 1/2$ and $h = 2/3$ respectively when the proposed algorithm is used. The simulation was run for 10,000 OFDM blocks and the transmitted signal was oversampled by a factor of four which is sufficient to capture the peaks. It can be seen that the PAPR for $h = 1/2$ is marginally better than for $h = 2/3$ when using the proposed algorithm. We investigated PAPR performance for a number of h values and observed that the PAPR for various values of h differs only slightly. This gives additional advantage as the system designer need not worry about the PAPR performance while choosing h for an OFDM-CPM system. For $h = 1/2$, the unmodified OFDM signal has a PAPR of 11.24 dB for 0.1% of the blocks. By using the proposed scheme with 8 clusters (i.e., 16 symbols in each cluster), the 0.1% PAPR reduces to 8.78 dB—a gain of 2.46 dB. As we increase the number of clusters (which corresponds to decreasing the number of symbols in each cluster), PAPR reduces even further and for 32 clusters, the 0.1% PAPR is 7.20 dB—a gain of 4.04 dB.

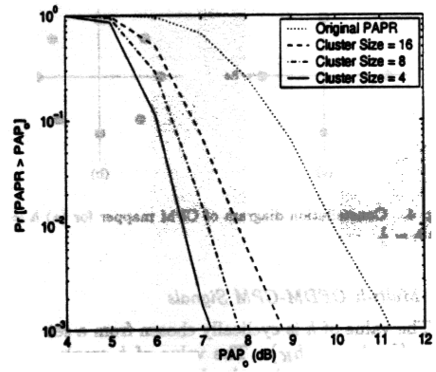

Fig. 6. Complementary cumulative distribution function of a 128-carrier OFDM-CPM system for various number of clusters with $h = \frac{1}{2}$

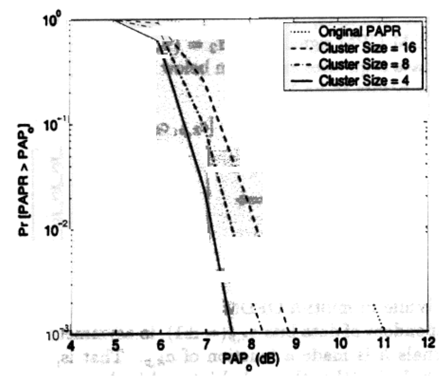

Fig. 7. Complementary cumulative distribution function of a 128-carrier OFDM-CPM system for various number of clusters with $h = \frac{2}{3}$

Although we have demonstrated the usefulness of this algorithm for OFDM-CPM signals, the algorithm can also be used for typical OFDM signals with PSK or QAM mappers. The PSK constellation can be expanded the same way as CPM constellation is expanded to obtain multiamplitude constellation. Then the proposed algorithm can be applied for reduction of PAPR. Similarly, QAM constellation can be expanded as proposed in [27] and the proposed algorithm can be used for reduction of PAPR.

VI. Conclusions

PAPR of OFDM-CPM signals can be reduced by more than 4 dB when multiamplitude CPM signals are used along with partial transmit sequences. The complexity and overhead of proposed algorithm is minimal and no additional information need to be transmitted. The proposed algorithm is also applicable to OFDM-PSK and -QAM signals for reduction of PAPR.

References

[1] J.A.C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Communications Magazine*, pp. 5–14, May 1990.

[2] Leonard J. Cimini Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. on Comm.*, vol. 33, no. 7, pp. 665–675, July 1985.

[3] S.B. Weinstein and Paul M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," *IEEE Trans. on Comm. Tech.*, vol. 19, no. 5, pp. 628–634, October 1971.

[4] Minoru Okada, Shinsuke Hara, and Norihiko Morinaga, "Bit error rate performances of orthogonal multicarrier modulation radio transmission systems," *IEICE Trans. Commun.*, vol. E76-B, no. 2, pp. 113–119, February 1993.

[5] Yun Hee Kim, Iickho Song, Hong Gil Kim, Taejoo Chang, and Hyung Myung Kim, "Performance analysis of a coded OFDM system in time-varying multipath Rayleigh fading channels," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 5, pp. 1610–1615, September 1999.

[6] Thomas May, Hermann Rohling, and Volker Engels, "Performance analysis of Viterbi decoding for 64-DAPSK and 64-QAM modulated OFDM signals," *IEEE Trans. on Comm.*, vol. 46, no. 2, pp. 182–190, February 1998.

[7] Jun Lu, Tjeng Thiang Tjhung, Fumiyuki Adachi, and Cheng Li Huang, "BER performance of OFDM-MDPSK system in frequency-selective Rician fading with diversity reception," *IEEE Transactions on Vehicular Technology*, vol. 49, no. 4, pp. 1216–1225, July 2000.

[8] Imran A. Tasadduq and Raveendra K. Rao, "Detection of OFDM-CPM signals over multipath channels," *IEEE International Conference on Communications, ICC'2002*, 28 April-02 May 2002.

[9] Imran A. Tasadduq and Raveendra K. Rao, "OFDM-CPM signals," *Electronics Letters*, vol. 38, no. 2, pp. 80–81, 17 January 2002.

[10] Hideki Ochiai and Hideki Imai, "Block coding scheme based on complementary sequences for multicarrier signals," *IEICE Trans. Fundamentals*, vol. E80-A, no. 11, pp. 2136–2143, November 1997.

[11] Leonard J. Cimini Jr. and Nelson R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," *IEEE Communications Letters*, vol. 4, no. 3, pp. 86–88, March 2000.

[12] S.H. Müller and J.B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Electronic Letters*, vol. 33, no. 5, pp. 368–369, 27 February 1997.

[13] C. Tellambura, "Phase optimisation criterion for reducing peak-to-average power ratio in OFDM," *Electronics Letters*, vol. 34, no. 2, pp. 169–170, January 1998.

[14] G.R. Hill, M. Faulkner, and J. Singh, "Reducing the peak-to-average power ratio in OFDM by cyclically shifting partial transmit sequences," *Electronics Letters*, vol. 36, no. 6, pp. 560–561, March 2000.

[15] A.D.S. Jayalath and C. Tellambura, "Adaptive PTS approach for reduction of peak-to-average power ratio of OFDM signal," *Electronics Letters*, vol. 36, no. 14, pp. 1226–1228, July 2000.

[16] A.E. Jones, T.A. Wilkinson, and S.K. Barton, "Block coding scheme for reduction of peak to mean envelope power ratio of mulitcarrier transmission schemes," *Electronic Letters*, vol. 30, no. 25, pp. 2098–2099, 20 September 1994.

[17] J.A.Davis and J. Jedwab, "Peak-to-mean power control and error correction for OFDM transmission using Golay sequences and Reed-Muller codes," *Electronic Letters*, vol. 33, no. 4, pp. 267–268, 13 February 1997.

[18] P. Van Eetvelt, G. Wade, and M. Tomlinson, "Peak to average power reduction for OFDM schemes by selective scrambling," *Electronic Letters*, vol. 32, no. 21, pp. 1963–1964, 10 October 1996.

[19] Homayoun Nikookar and Ramjee Prasad, "Weighted OFDM for wireless mulitpath channels," *IEICE Transactions on Communications*, vol. E83-B, no. 8, pp. 1864–1872, August 2000.

[20] Imran A. Tasadduq and Raveendra K. Rao, "Weighted OFDM with block codes for wireless communication," *2001 IEEE Pacific Rim Conference on Communcations, Computers and Signal Processing (PACRIM 2001), Victoria, B.C.*, vol. 2, pp. 441–444, August 2001.

[21] R.W. Bäuml, R.F.H. Fischer, and J.B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," *Electronic Letters*, vol. 32, no. 22, pp. 2056–2057, 24 October 1996.

[22] A.D.S. Jayalath and C. Tellambura, "Reducing the peak-to-average power ratio of orthogonal frequency division multiplexing signal through bit or symbol interleaving," *Electronic Letters*, vol. 36, no. 13, pp. 1161–1163, 22 June 2000.

[23] M. Friese, "Multicarrier modulation with low peak-to-average power ratio," *Electronics Letters*, vol. 32, no. 8, pp. 713–714, 11 April 1996.

[24] John B. Anderson and Desmond P. Taylor, "A bandwidth-efficient class of signal-space codes," *IEEE Transactions on Information Theory*, vol. 24, no. 6, pp. 703–712, November 1978.

[25] Hong-Kuang Hwang, Lin-Shan Lee, and Sin-Horng Chen, "Multi-h phase-coded modulations with asymmetric modulation indexes," *IEEE Journal on Selected Areas in Communications*, vol. 7, no. 9, pp. 1450–1460, December 1989.

[26] John G. Proakis, *Digital Communications*, McGraw Hill Inc., 2001.

[27] Chan-Soo Hwang, "Peak power reduction method for multicarrier transmission," *Electronics Letters*, vol. 37, no. 17, pp. 1075–1077, August 2001.