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Frequency dependent dynamic properties from resonant column and cyclic triaxial tests

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Abstract

The resonant column (RC) and cyclic triaxial (CT) devices are commonly used for the measurement of soils' dynamic properties. The results of these tests do not agree when extrapolated to similar strain levels. The main objectives of this paper are to evaluate the effect of excitation frequency on the dynamic properties of soils, and to provide a methodology to reconcile shear modulus values obtained from RC and CT tests. The effect of frequency on the dynamic properties is evaluated using the new non-resonance (NR) method in the RC device and CT tests. Sand specimens with varying percentages of bentonite–water mixture and a clay specimen are tested. The results obtained from RC tests utilizing the NR method indicate significant change in shear modulus with frequency. The extrapolation of shear modulus from the conventional RC results to shear strains used in CT is significantly overestimated. The extrapolations improved when the results were corrected for frequency effect inferred from the NR method.

Keywords: Resonant column; Dynamic properties; Non-resonance methods; Viscoelastic behavior

1. Introduction

The variation of dynamic properties of soils (shear modulus and damping ratio) as a function of shear strain and loading frequency are crucial input for solving geotechnical problems

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involving dynamic loadings (e.g. earthquake, wind and machine vibrations). Several methods are used to measure these dynamic properties using different devices, which have certain limitations in the attainable strain levels and range of excitation frequencies. Moreover modes of excitation, boundary conditions, and assumptions in the analysis affect the results. Therefore, test results from different devices do not always yield similar values.

Cyclic triaxial (CT) and resonant column (RC) devices are widely used to evaluate dynamic properties of soils following relevant ASTM standard tests. They operate at a wide range of strain levels ($10^{-5}\% < \gamma < 10^{0}\%$) and excitation frequencies (0.2 Hz $< f <$ 170 Hz). Resonant column tests are commonly performed at frequencies larger than 30 Hz and low-strain levels, whereas in CT testing frequencies between 1 and 5 Hz and higher strain levels are typically used. The overlapping range of shear strains in RC and CT tests is commonly restricted to $\gamma = 0.03$–0.08%.

Soils and geomaterials have the ability to store and dissipate energy. At low shear strain levels, the theory of viscoelasticity describes the phenomenon of energy dissipation in soils through wave propagation [1,2]. One important property of a linear viscoelastic medium is that dispersion and shear modulus are not independent and are related by the Kramers–Kronig equations [2]. Thus, the comparison between the results from resonant column and CT tests should be done at similar frequencies.

Dynamic soil properties in resonant column tests are evaluated at the resonant frequency of the specimen (ASTM D4105-92). The conventional analysis of RC tests assumes an elastic system with zero damping for the evaluation of shear modulus. The damping ratio is computed independently from the shear modulus using the free vibration, half-power bandwidth method, or transfer function method. Frequency dependent dynamic properties are difficult to evaluate using resonance tests as they would require significant changes in the geometry of the specimen.

Non-resonance (NR) methods have been proposed to evaluate dynamic properties in resonant column tests [1–3] and at different levels of shear strains [1]. This study compares the results from the newly proposed NR tests and more conventional CT tests at low frequencies (RC $f = 3$ Hz, CT $f = 1$ Hz) at different shear strain levels. Results from the conventional resonant column tests are also presented.

The cyclic triaxial test is an ASTM standard [4] test to evaluate the dynamic properties of soils at frequencies usually below 2 Hz and shear strain levels from 0.001% to 1%. Unlike the resonant column or torsional shear devices, the cyclic loads are axial instead of torsional. Consequently, the axial strain is measured and the elastic Young's modulus is evaluated. The corresponding shear modulus and shear strain are computed by assuming the Poisson's ratio of the soil.

Sand specimens with varying percentages of bentonite–water mixture and a clay specimen are tested. The results obtained from RC tests utilizing the NR method indicate significant change in dynamic properties with frequency. Even dry sands exhibit some viscoelastic behavior, especially in shear modulus, at low-strain levels. The extrapolation of shear modulus from the conventional resonant column results to shear strains used in CT is significantly overestimated. The extrapolated values improved when the results were corrected for frequency effect.

2. Background

In resonant column testing, a soil specimen is excited under torsional excitation at different frequencies to measure the transfer function between the applied force (current)

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and the measured response (acceleration). The measured data is fitted with the theoretical transfer function of a single-degree-of-freedom system to determine the resonant frequency and the damping ratio of the material [5]. The shear wave velocity (V_S) is related to the resonant frequency (ω_o) by:

$$V_S = \sqrt{\omega_o{}^2\left[\left(I/3 + I_o\right)H\right]/[J_P\rho]} \tag{1}$$

where J_P and I are the area polar moment of inertia and the mass polar moment of inertia of the specimen, respectively; H and ρ are the height and mass density of the specimen, and I_o is the mass polar moment of inertia of the driving system.

In conventional RC tests, the shear modulus (G) and the damping ratio (ξ) of the material are computed independently; whereas in NR tests, the dynamic properties at different frequencies are computed simultaneously. The NR method relates the applied torque and the resulting rotation angle at the top of the specimen [1–3]. Using the correspondence principle [6], the damping ratio is incorporated in the imaginary component of the complex shear modulus. The transfer function between the applied torque and the induced angle of twist (Fig. 1a) is given by

$$\frac{T_o(\omega)e^{-i\varphi(\omega)}}{\phi(\omega)} = J_P\rho\omega^2 H\frac{1}{[\sqrt{\rho\omega^2H^2/G_S^*(\omega)}\tan(\sqrt{\rho\omega^2H^2/G_S^*(\omega)})]} - I_o\omega^2 \tag{2}$$

where $T_o(\omega)$ is the amplitude of torque, $\varphi(\omega)$ is the amplitude of twist angle, $\phi(\omega)$ is the phase lag between the torque and the twist angle, J_P is the polar moment of inertia, ρ is the mass density, H is the height of the specimen, ω is the angular frequency, I_o is the mass polar moment of inertia of the driving plate, and $G_s^*(\omega)$ is the complex shear modulus given in terms of real and imaginary components ($G_R(\omega)$ and $G_I(\omega)$)

$$G_s^*(\omega) = G_R(\omega) + iG_I(\omega) \tag{3}$$

Eq. (3) indicates that the storage modulus ($G_R(\omega)$) and the loss modulus ($G_I(\omega)$) are function of frequency.

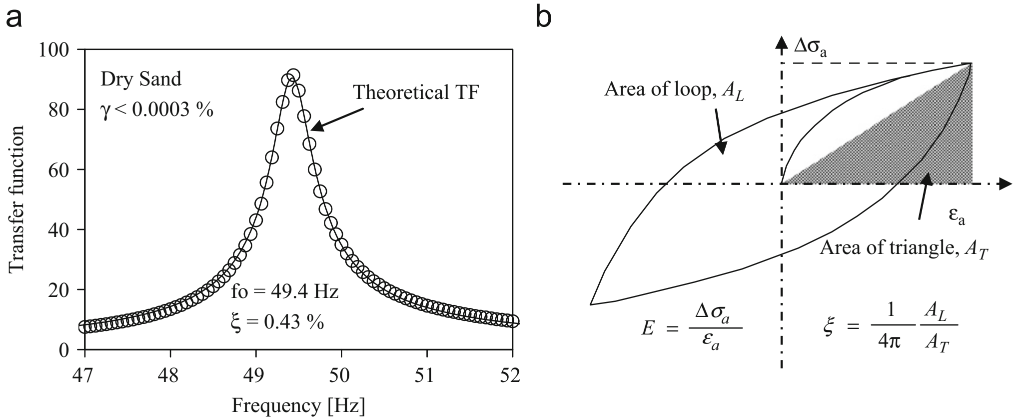

Fig. 1. Typical measurements in RC and CT tests: (a) measured and theoretical transfer functions (RC) and (b) hysteretic loop in CT.

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A transfer function (TF) method proposed by [1] considers the damping ratio (ξ) to be frequency independent and the transfer function between the applied torque and twist angle in this case is

$$\frac{T_o(\omega)e^{-i\varphi(\omega)}}{\phi(\omega)} = \left(I_o + \frac{I}{3}\right)\omega_o^2\left[1-\left(\frac{\omega}{\omega_o}\right)^2 + i2\xi\left(\frac{\omega}{\omega_o}\right)\right] \tag{4}$$

where I is the mass polar moment of inertia of the specimen. The NR and the TF methods yield similar results for dry sands tested at different confinements and shear strain levels [1].

Cyclic triaxial tests are either performed in strain or stress controlled modes. In either case, the analysis of the results is similar. The deviator stress is plotted against the axial strain and a hysteretic loop is generated. The phase lag between the applied stress and the resulting strain represents the ability of the specimen to store and dissipate energy in a finite period of time. A typical hysteretic loop is presented in Fig. 1(b). The equations for the computation of elastic modulus (E) and damping ratio (ξ) are shown in figure. The shear modulus is computed from the Young's modulus in CT as

$$G = \frac{E}{2(1+v)} \tag{5}$$

Several studies have examined the effect of frequency on the dynamic properties of soils [e.g. 7–11]. In general, these studies indicate that the dynamic properties of cohesionless soils do not exhibit frequency dependency, whereas cohesive soils exhibit a moderate increase in shear modulus and a decrease in damping ratio with frequency at frequencies up to 1 Hz.

No significant variation in the shear modulus of dry and saturated sands in torsional shear and resonant column tests at frequencies between 0.1 and 100 Hz was reported [8]. Similarly, experiments on dry sands at different shear strain levels using torsional shear tests at 1 Hz, and resonant column tests at frequencies between 45 and 95 Hz, with no obvious variation of dynamic properties with frequency [9]. Similarly no change in the dynamic properties of sand between loading rates of 0.1–2 Hz were reported by [12].

The validity of a frequency independent damping in soils and geomaterials, however, has been argued by different authors [13–15]. Most soils are neither purely elastic nor viscous but viscoelastic; therefore, the evaluation of frequency effects on the dynamic properties is desirable.

Previous studies indicated an increase in shear modulus with frequency of clayey soils [10], kaolin [16], sandy lean clay [17], and stiff clays [7] in the frequency range 0.001 Hz $< f <$ 200 Hz. Most of these studies found smaller increase in shear modulus at frequencies smaller than 1 Hz, and a larger increase at frequencies greater than 10 Hz. These studies also showed that damping ratio increased with frequency at frequencies greater than 1 Hz.

Simple shear tests on different sands and clay specimens were performed [18]. Their results indicated that shear modulus increased with the increase in frequency for all soils. No increase in shear modulus of undisturbed Bangkok clays tested in cyclic triaxial apparatus at frequencies up to 1 Hz was observed [19]. However, a small decrease in damping ratio was observed.

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Cyclic triaxial tests on Touyora sand and over-consolidated clay were performed [20]; they found that the shear modulus increased slightly with frequency. It was observed that the Young's modulus of a silty sand increased faster with frequency as the axial strain level was increased [21]. At small strains ($\gamma < 10^{-5}$), the increase with frequency was insignificant and the behavior was almost elastic.

2.1. Sand and bentonite mixtures

The masses of dry sand (M_S), bentonite (M_B), and water (M_W) required for the sand–bentonite mixtures are computed using phase relationships. The volume of sand (V_{OS}) and the void ratio of the sand-matrix were kept approximately constant for all specimens; whereas, the amount of bentonite (fines) and water is varied to achieve different degrees of bentonite–water saturation. The degree of bentonite–water saturation (D_{BW}) of the sand-matrix is computed by

$$D_{\mathrm{BW}} = \frac{M_{\mathrm{BW}}}{\rho_{\mathrm{BW}}} \frac{1}{V_{\mathrm{OS}} e} \tag{6}$$

where M_{BW} is the mass of the bentonite–water mixture, ρ_{BW} is its bulk density, determined experimentally, and e is the void ratio of sand skeleton. The mass density (ρ_s) of the sand-matrix is given by

$$\rho_s = \frac{\rho_b}{(1 + B_C)(1 + w)} \tag{7}$$

where ρ_b is the bulk mass density of the mixture, B_C is the bentonite content (M_B/M_S), w is the water content ($M_W/(M_S+M_B)$).

3. Experimental setup and experimental program

The shear modulus and damping ratio of sand–bentonite mixtures were measured using both resonant column and cyclic triaxial devices. In all specimens, the water–bentonite ratio per weight was constant (155%). Four degrees of bentonite–water saturation were used (D_{BW} = 0%, 11%, 31%, and 32%). A modified Stoke resonant column was used to measure the dynamic properties employing the resonance method (RC) and non-resonance (NR) method. The measurements were obtained at different shear strain levels at a constant low confinement to increase the maximum shear strain during testing σ_o = 33 kPa. In the NR method, low-strain dynamic properties were measured at different frequencies from f = 1.5 Hz to the resonant frequency (f_o); a frequency of f = 3 Hz is used to measure dynamic properties at different shear strains. Five specimens were tested in resonant column and another five were tested in cyclic triaxial device. Large-strain measurements were performed after completing low-strain measurements in all specimens.

Cyclic triaxial tests were performed first at a constant confinement σ_o = 33 kPa, at frequency f = 1 Hz and different shear strain levels. After testing the specimens at different strain levels, frequency dependent dynamic properties were measured by testing another set of specimens at frequencies from f = 0.2 to 2.5 Hz (for constant shear strain value), at a constant confinement σ_o = 33 kPa and a constant shear strain, $\gamma \approx 0.034\%$. This strain level represents the average of the range of shear strains yielding consistent results in the CT device.

3.1. Resonant column setup

Fig. 2 illustrates the typical instrumentation used in the RC device. The applied torque is computed from the measured current through the coils $I_C(\omega)$, and the distance from the magnets from the center of the specimen (r_m). The measured magnetic coil factor (B_L) and the resistance (R) of the coil circuit are used to convert current amplitude to torque amplitude as

$$T_o(\omega) = B_L \frac{I_C(\omega)}{R} r_m \tag{8}$$

The angular rotation of the specimen is measured with proximity probes. The phase lag between the torque and rotation is computed using the cross-power Fourier spectrum of the current $I_C(\omega)$ and the response of the proximity probes. The shear modulus and damping ratio as a function of frequency are computed using Eq. (2) or Eq. (4). For the conventional resonance method (RC), the acceleration is measured with an accelerometer mounted on the driving plate.

3.2. Cyclic triaxial setup

Sand–bentonite specimens were tested in a cyclic triaxial device (Wykeham Farrance 28-WF4100). The device is instrumented with LVDT, a load cell, pore pressure and a cell pressure transducer and a computer controlled data acquisition system. Typical dimensions of cylindrical specimens tested in the cyclic triaxial device are $70 \times 140\ \text{mm}^2$.

A Poisson's ratio $\nu = 0.2$ is selected as the mean value for sand specimens [22] for the calculation of shear modulus (Eq. (5)). A variation of 20% in Poisson's ratio (i.e. $\nu = 0.16$ to 0.24) had a small effect on the computed shear modulus ($\pm 3.5\%$ change).

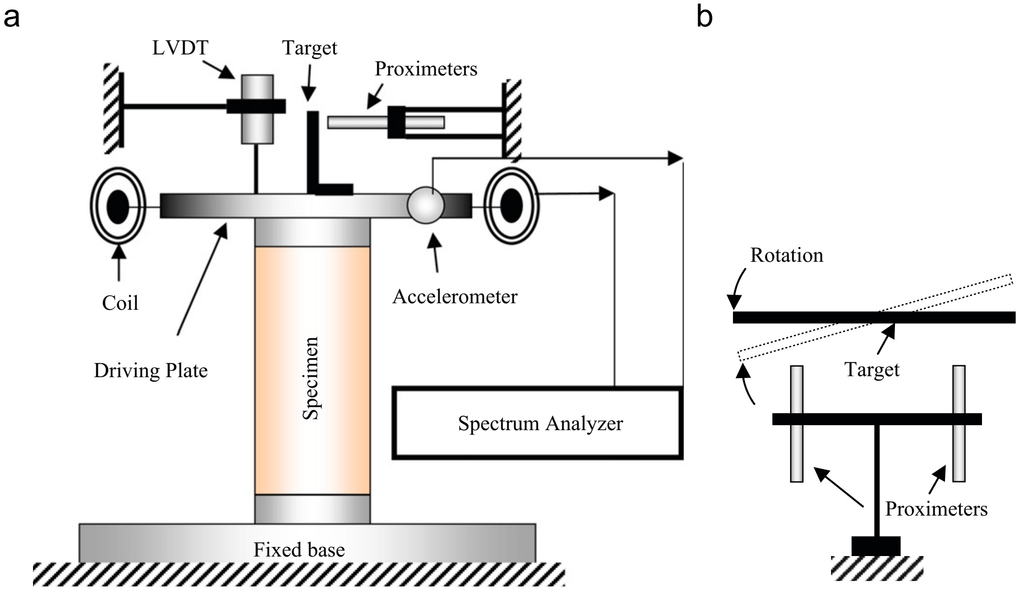

Fig. 2. Typical instrumentation for low frequency resonant column measurements.

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4. Sample preparation

First the sand and bentonite–water mixtures were mixed thoroughly in a pan, and then the mixture is compacted in five layers of desired density. Finally, specimen dimensions were noted before closing the chambers. Specimens of dry sand with four different values of degree of bentonite–water saturation were prepared ($D_{BW}=0\%$, 11%, 21%, and 32%). In all cases, voids ratio of sand skeleton was approximately equal to $e\approx 0.71$.

All specimens were prepared under vacuum (−20 kPa). Dry sand specimens were prepared by pouring the sand through a funnel in a mould. Slightly lower densities were achieved, than desired, by maintaining a constant funnel height above the sand surface. The specimen mould was then tapped gently to achieve the required height and density for the sand specimen only. The sand is silica sand with sub-rounded grains (10% passing $D_{10}=0.12$, coefficient of uniformity $C_u=1.8$, and coefficient of curvature $C_C=1.04$). The friction angle (from direct shear tests) $\phi=32.5^\circ$. The minimum and maximum void ratios ($e_{max}=0.75$ and $e_{min}=0.53$) were measured in accordance to ASTM D4253 and D4254. The clay had a plasticity index of 35, which indicates medium to high plasticity.

5. Results and discussion

Fig. 3 presents the dynamic properties from NR and NTF methods for a calibration aluminum probe. Results from the standard RC tests are also shown in this figure. The dynamic properties at different frequencies are almost constant and in good agreement with the results from RC test results.

Fig. 4(a) presents typical phase angles of the input excitation current (proportional to the applied torque) and the response of proximity probes for dry sand ($f=1.5$ Hz, $\gamma<10^{-6}$). The ratio between the torque and rotation angle was computed from the spectral magnitudes (Eqs. (2) and (4)). Fig. 4(b) presents the same plots for sand–bentonite mixture ($D_{BW}=32\%$, $f=1.5$ Hz, $\gamma<10^{-6}$). The lag between excitation and response was difficult to resolve in time domain; whereas, it is precisely measured in the frequency domain (phase

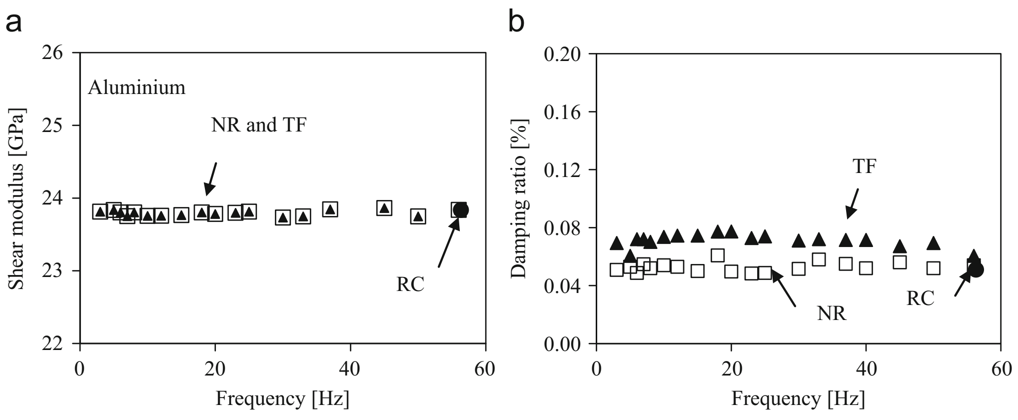

Fig. 3. Dynamic properties as a function of frequency from non-resonance method (NR) and standard resonant column tests ($\gamma<10^{-6}$) for a calibration aluminum probe ($f_o=57$ Hz): (a) shear modulus (minimum $Vs=2948$ m/s) and (b) damping ratio.

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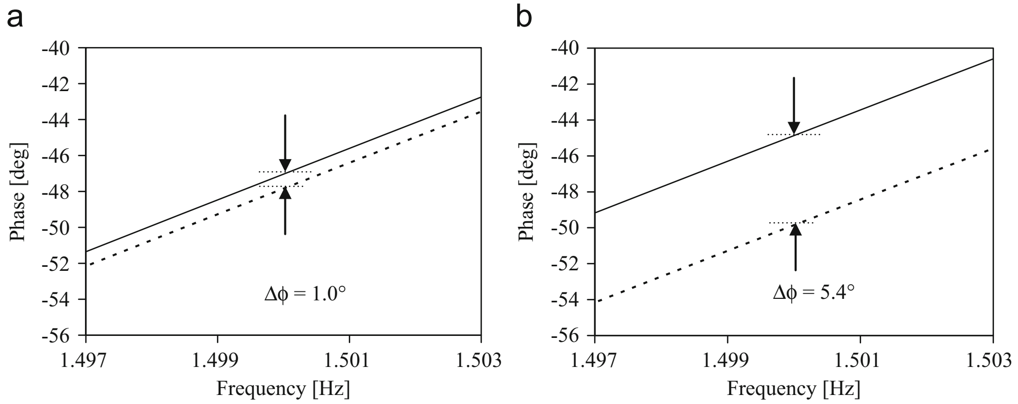

Fig. 4. Phase angles for the input (current) and the output response of proximity probes ($f = 1.5$ Hz, $\gamma < 10^{-4}$%): (a) saturation of bentonite paste $D_{BW} = 0\%$ and (b) saturation of bentonite paste $D_{BW} = 32\%$.

resolution of the spectrum analyzer is 0.001 rad). The phase difference for $D_{BW} = 32\%$ specimen was $\Delta\phi = 5.4°$, which corresponded to a time delay of $\Delta t = 8.35$ ms.

Fig. 5 presents the frequency dependent dynamic properties of sand–bentonite mixtures and the clay specimen. The results were obtained for an isotropic confinement of $\sigma_o = 33$ kPa and low-strain level excitations ($\gamma < 10^{-6}$). The dynamic properties were evaluated using the non-resonance (NR) method (Eq. (2)). Log scale was used for the frequency to show all the measurements in one plot. The shear modulus increases linearly with frequency (normal scale) and approaches the values obtained from RC tests, whereas the damping ratio tends to increase slightly with frequency. For the clay specimen, the damping ratio actually decreased. Log scale was used to enhance the variation at low frequencies.

The largest increase in shear modulus and damping ratio within the studied frequency range ($f = 1.5$–51 Hz) are 17% and 8.4%, respectively, for the maximum degree of bentonite–water saturation ($D_{BW} = 32\%$). The increase in shear modulus was 0.16 MPa/Hz.

These results suggest the significant influence of the viscous properties of the fines on the dynamic properties of sands. However, for the dry sand specimen, the variations in shear modulus and damping ratio are 11% and 0%, which suggest viscoelastic behavior of particulate materials at low shear strain levels ($\gamma < 10^{-4}$%). This behavior of sand is still under investigations and further tests are being performed. The clay specimen shows an increase in shear modulus of 17% and a decrease of 16% in damping ratio within the same frequency range (1.5–50 Hz).

Fig. 6 presents the measured dynamic properties of sand–bentonite specimens using the CT device. The average shear strain that provided repeatable measurements was 0.034%. The dynamic properties were almost constant in this frequency range except for the specimen with $D_{BW} = 32\%$ and 21%, which showed an increase of 0.6 MPa/Hz in a shear modulus with frequency. Although significant for small frequency range, rate of increase in shear modulus at broader frequency range (up to 50 Hz) is anticipated to decrease. The results of large strain testing from RC tests indicate convergence of shear modulus values to CT results as the shear strain increases and resonant frequencies decrease in RC tests (Fig. 7).

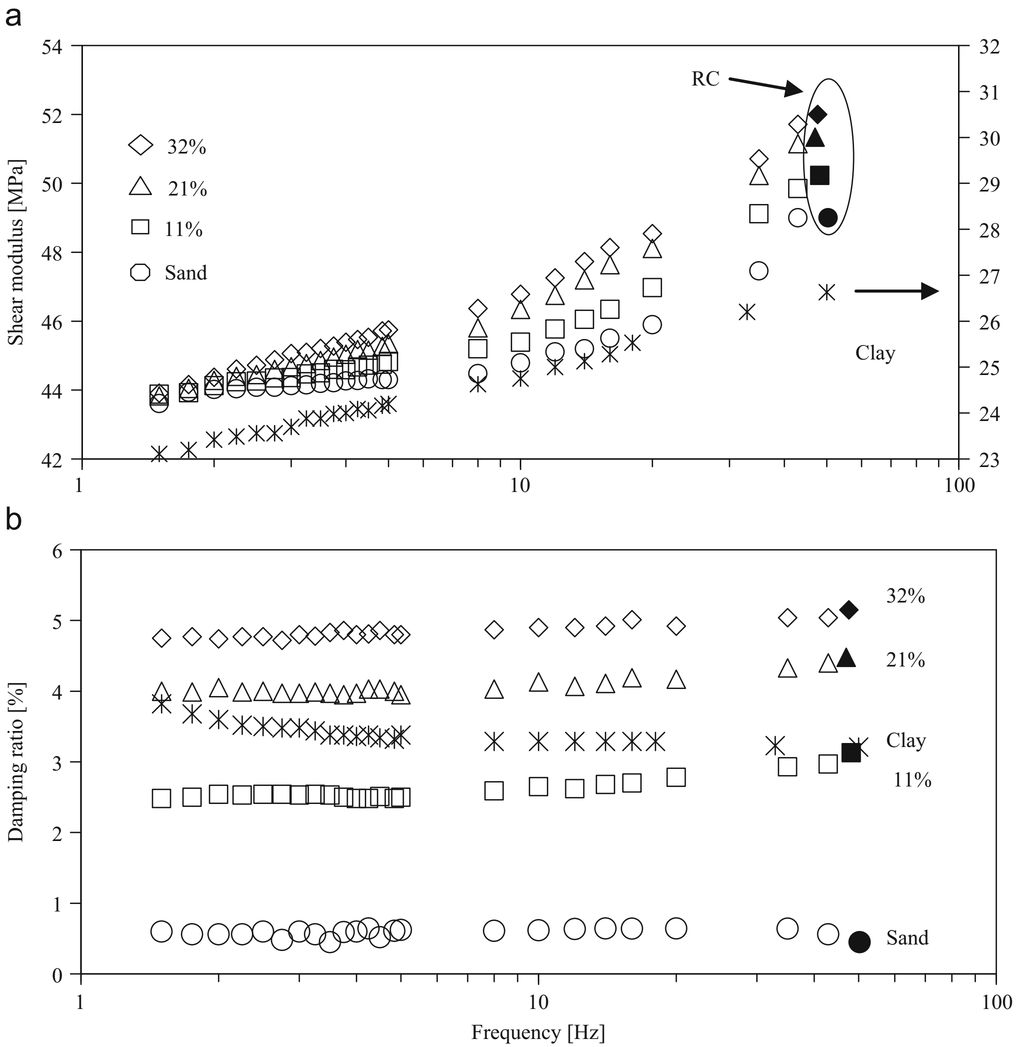

Fig. 5. Variation of dynamic properties with frequency for different sand–bentonite mixtures (saturation D_{BW} = 32%) and a clay specimen using the NR method ($\sigma_o \approx 33$ kPa, $\gamma < 10^{-4}$%): (a) shear modulus and (b) damping ratio.

Fig. 7 presents the comparison of the shear modulus as a function of shear strain level from RC tests (24 Hz $< f <$ 51 Hz), NR tests (f = 3 Hz) and CT tests (f = 1 Hz, $\sigma_o \approx 33$ kPa). The results are presented for sand–bentonite saturation levels of D_{BW} = 0% and 32%. The dynamic properties from CT tests were evaluated at shear strains close to γ = 0.034%.

The CT results show better agreement with NR test results than with RC test results as NR tests are performed at a frequency (f = 3 Hz) close to the frequency of excitation in CT tests (f = 1 Hz). The RC tests require higher frequencies than NR and CT tests, thus the measured shear modulus tends to be larger. The difference between RC and NR test results

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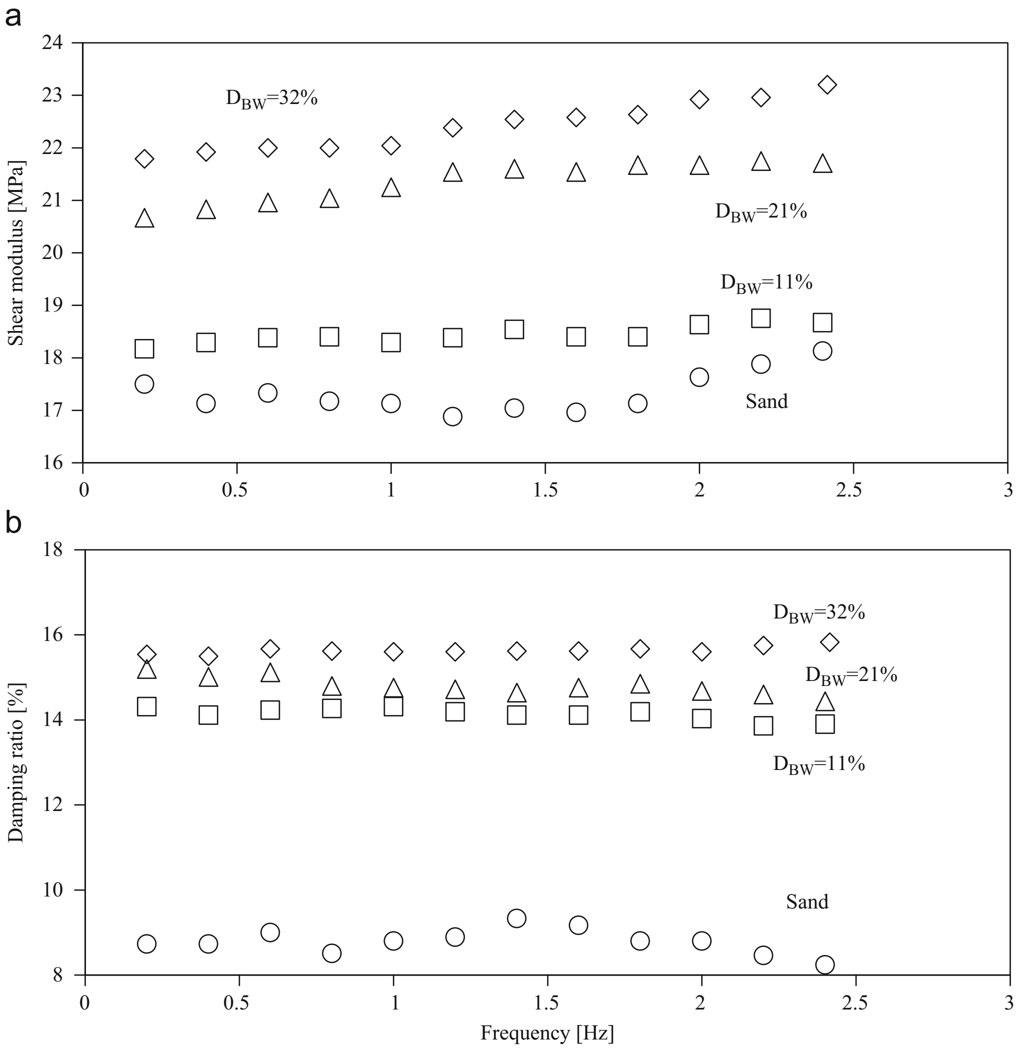

Fig. 6. Variation of dynamic properties with frequency for different sand–bentonite mixtures from CT tests ($\sigma_o \approx 33$ kPa, $0.031 < \gamma < 0.034\%$): (a) shear modulus and (b) damping ratio.

tends to decrease with the increase in shear strain because resonant frequencies decrease with the increase in shear strain level.

6. Model for frequency dependent shear modulus

Tests on synthetic sand specimens in resonant column, torsional shear, free–free resonant column, and cyclic resilient modulus testing apparatus were performed by [23]. A model was suggested to predict the modulus at a given frequency. The model was based on results from different devices and suggested a linear increase in shear modulus with the logarithm of frequency. This model is modified and simplified in this study using results

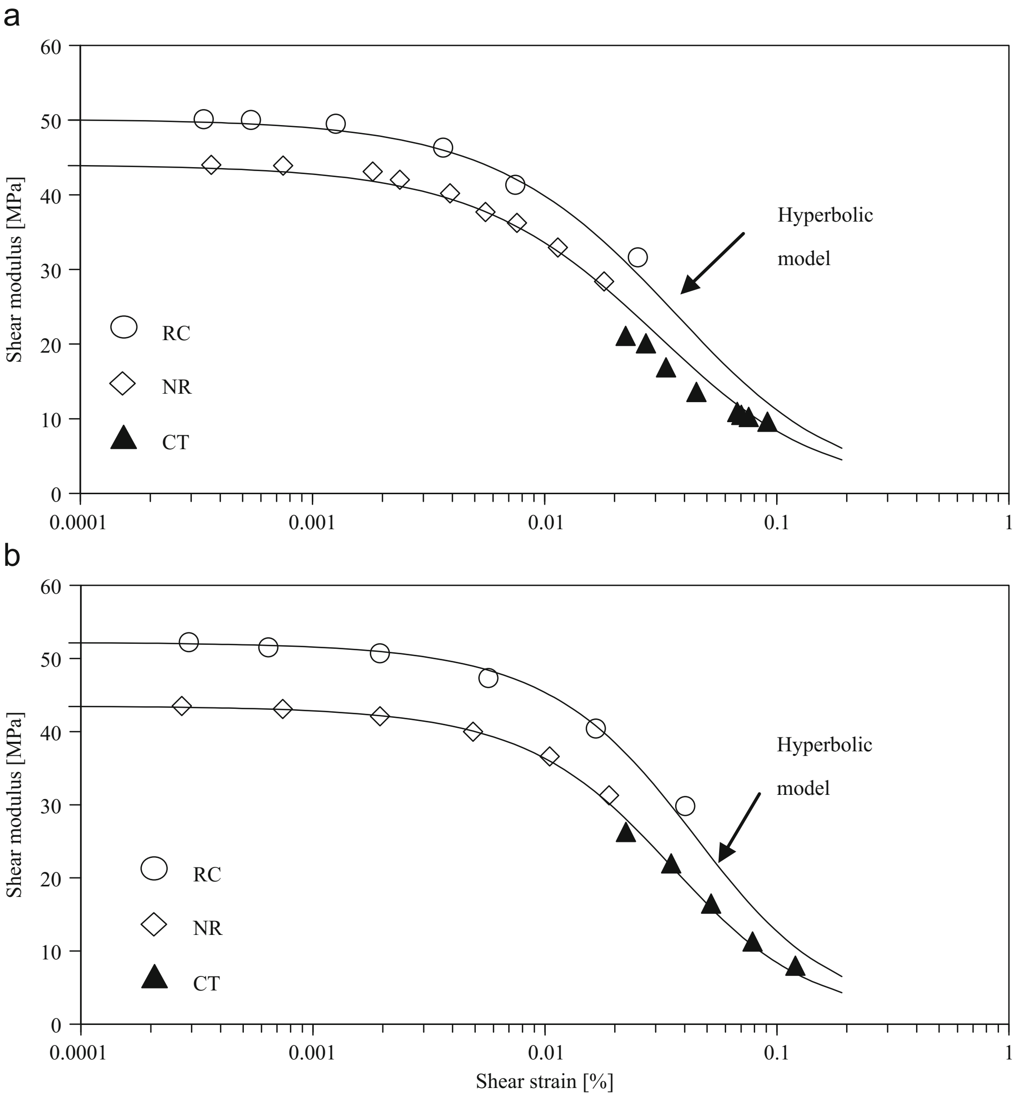

Fig. 7. Dynamic properties for the sand–bentonite specimen for (a) $D_{BW} = 0\%$ and (b) $D_{BW} = 32\%$ from RC tests (37 Hz $< f <$ 50 Hz), NR tests ($f = 3$ Hz) and CT tests ($f = 1$ Hz) at $\sigma_o \approx 33$ kPa.

from the resonant column device. The variation of the shear modulus with frequency is measured using the non-resonant method [1]. The results from this study indicate a linear increase in shear modulus with frequency that can be given by

$$G(f_2) = \frac{1 + \alpha f_2}{1 + \alpha f_1} G(f_1) \tag{9}$$

where α is the slope of the regression line of normalized shear modulus versus frequency, $G(f_1)$ is the measured shear modulus at frequency (f_1), and $G(f_2)$ is the predicted shear modulus at frequency (f_2).

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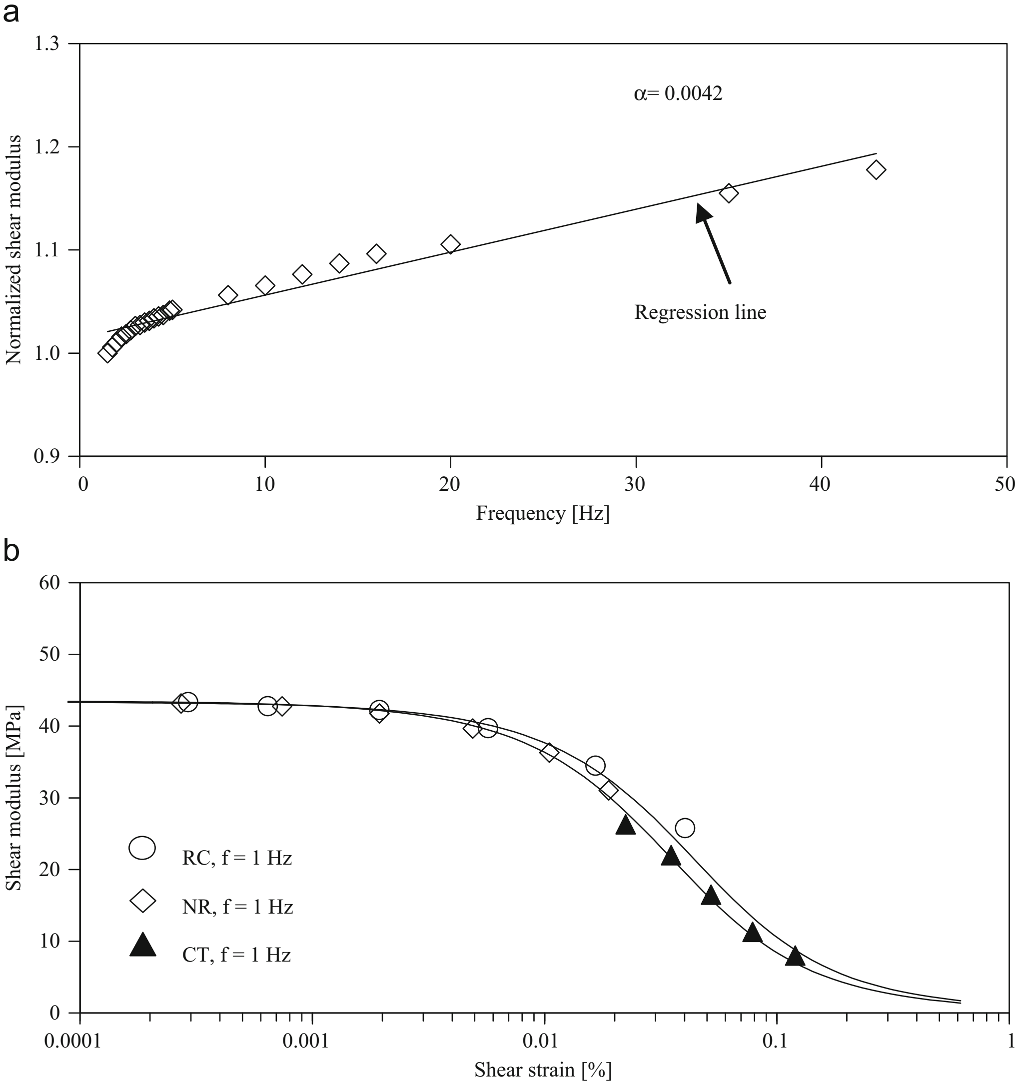

Fig. 8. (a) Curve fitting of the low-strain results from NR tests ($D_{BW}=32\%$, $\gamma<10^{-6}$) and (b) corrected dynamic properties from RC tests (50 Hz$<f<$33 Hz), and NR tests ($f=1$ Hz) at $\sigma_o\approx 33$ kPa.

Eq. (9) is used to curve fit the variation of shear modulus for the sand–bentonite specimen of $D_{BW}=32\%$ using $f_1=1$ Hz. The normalized variation of shear modulus with frequency is presented in Fig. 8(a). The corrected properties are presented in Fig. 8(b). The overestimation of 20% (Fig. 7) at low-strain levels is reduced to 0.5%. Thus, the shear modulus measured at any strain level using the conventional resonance method is overestimated with respect to the corresponding results in CT tests; though at large strain levels the overestimation of shear moduli decreases because of smaller difference between frequencies. Similar results were obtained for sand and clay specimens.

7. Conclusions

The dynamic properties for sand–bentonite specimens and a clay specimen were measured as a function of frequency using the resonant column and the cyclic triaxial devices. The non-resonance (NR) method was used to measure the dynamic properties in the RC device at frequencies close to the excitation frequencies used in the CT tests.

The frequency domain analysis provided better precision for the analyses of the NR method results for the computation of time lag between two signals. The time delay was very small ($\Delta t < 8$ ms) to resolve precisely. In addition, the excitation function at low-strain levels was not a perfect sinusoidal function, thus noise affected the time signals even if there is enough resolution in the time domain.

The results from RC and CT tests indicate an increase in dynamic shear modulus with frequency. The observed increase in shear modulus was significantly larger in RC tests because of the lower shear strain levels. The standard resonance method predicted larger dynamic shear modulus than the NR method because of the higher frequencies involved in the RC measurements; however, the difference decreases with the increase in shear strain level because of reduction in resonant frequencies. Consequently, the extrapolation of the results to the strain levels used in CT tests tends to overestimate the shear modulus.

Dry sands exhibited viscoelastic behavior in shear modulus at low-strain levels, although the behavior was not significant. Further tests are being performed to validate and understand the results.

The presence of plastic fines increased the viscoelasticity of sands. The percentage increase in shear modulus for sand–bentonite mixture with 32% of bentonite–water saturation was similar to that for pure clay (18% for $D_{BW} = 32\%$ and 16% for clay). The bentonite content for $D_{BW} = 32\%$ is $B_C = 12\%$.

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