Detection of Coherent Structures in 4-D Flows

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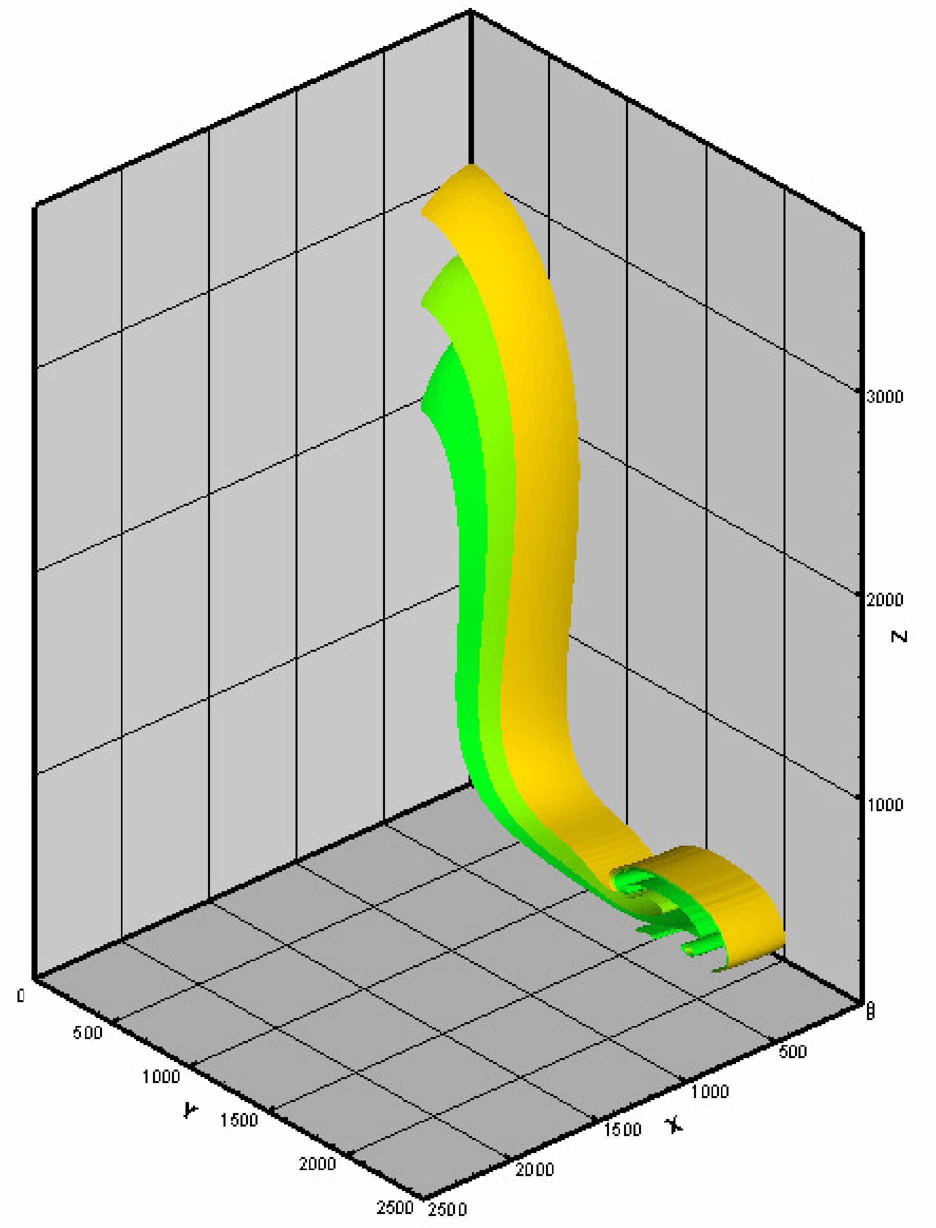
Current Work



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Approaches

- **Vorticity Magnitude**
  - Metcalf *et al.* (1985)
- **Q-Surfaces**
  - Hunt *et al.* (1988)
- **Kinematic Vorticity Number**
  - Trusdell (1953)
- **Complex Eigenvalues**
  - Jheong *et al.* (1995)
- **Others…**

Desired Features

- Galilean invariant
- Non-subjective
- Computationally practical
- Agrees with intuitive vortex region

Vorticity Magnitude

- **Intuitive approach**
  - Vorticity is a measure of the tendency for a region of fluid to rotate

$$\vec{\omega} = \vec{\nabla} \times \vec{v} \qquad \left|\vec{\omega}\right| = \sqrt{\omega_i^{\ 2}}$$

  - Wherever there is a velocity gradient (non-trivial cases) there will be vorticity
  - Problems near walls and in shear flows
  - Not absolute detection (subjective)

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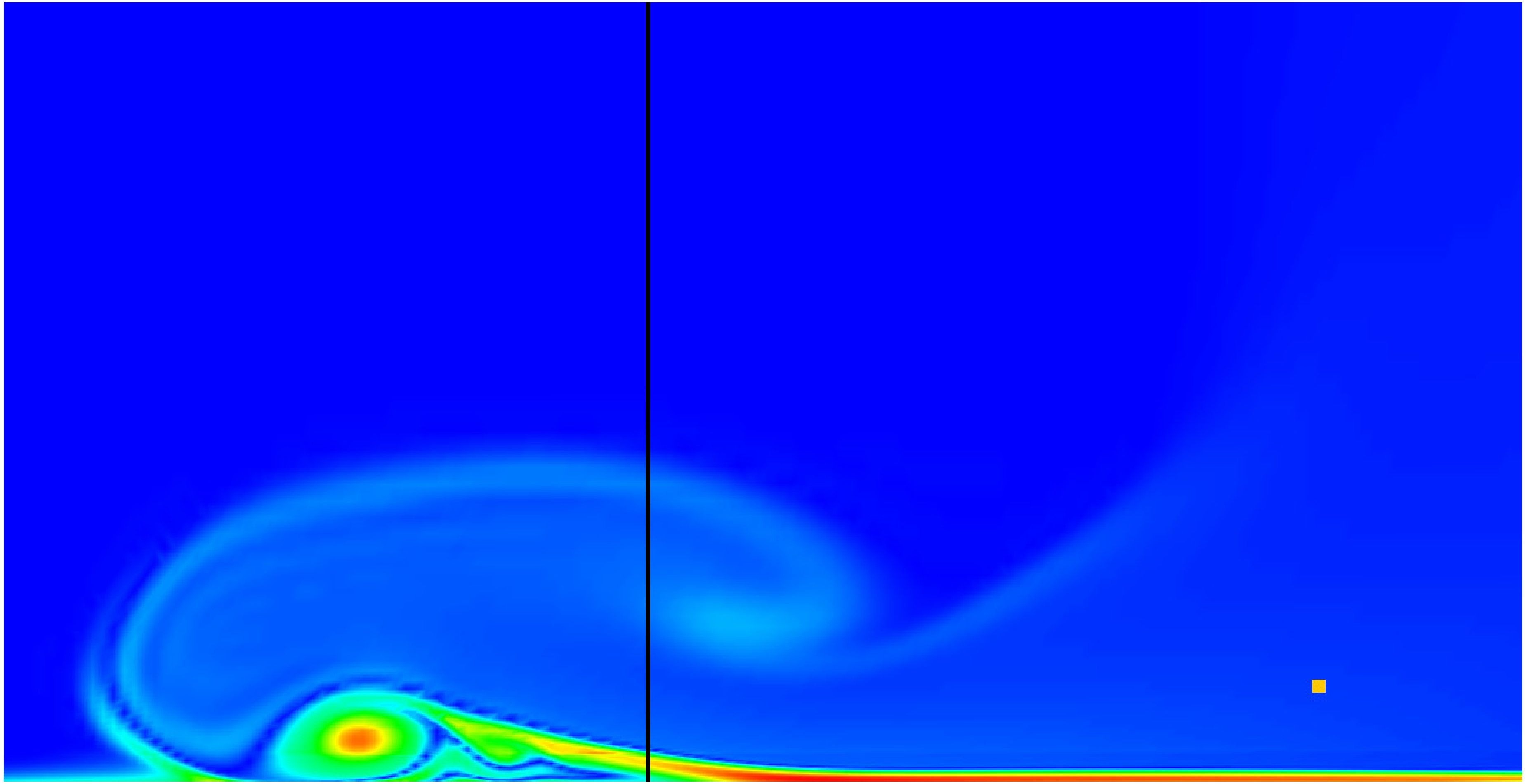
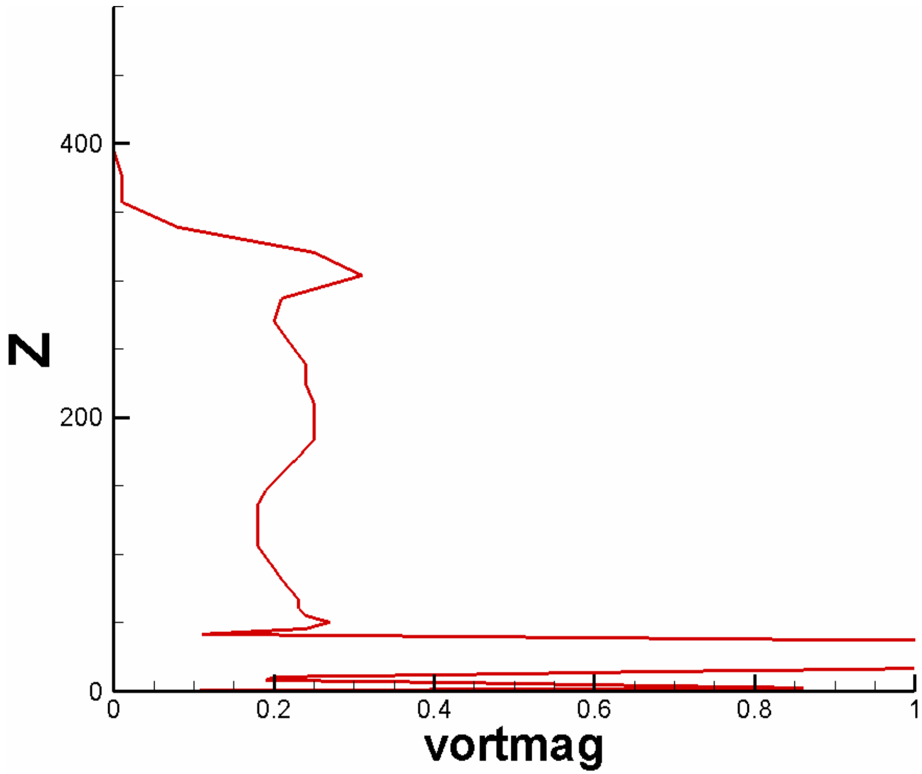
Vorticity Magnitude



Vorticity Magnitude

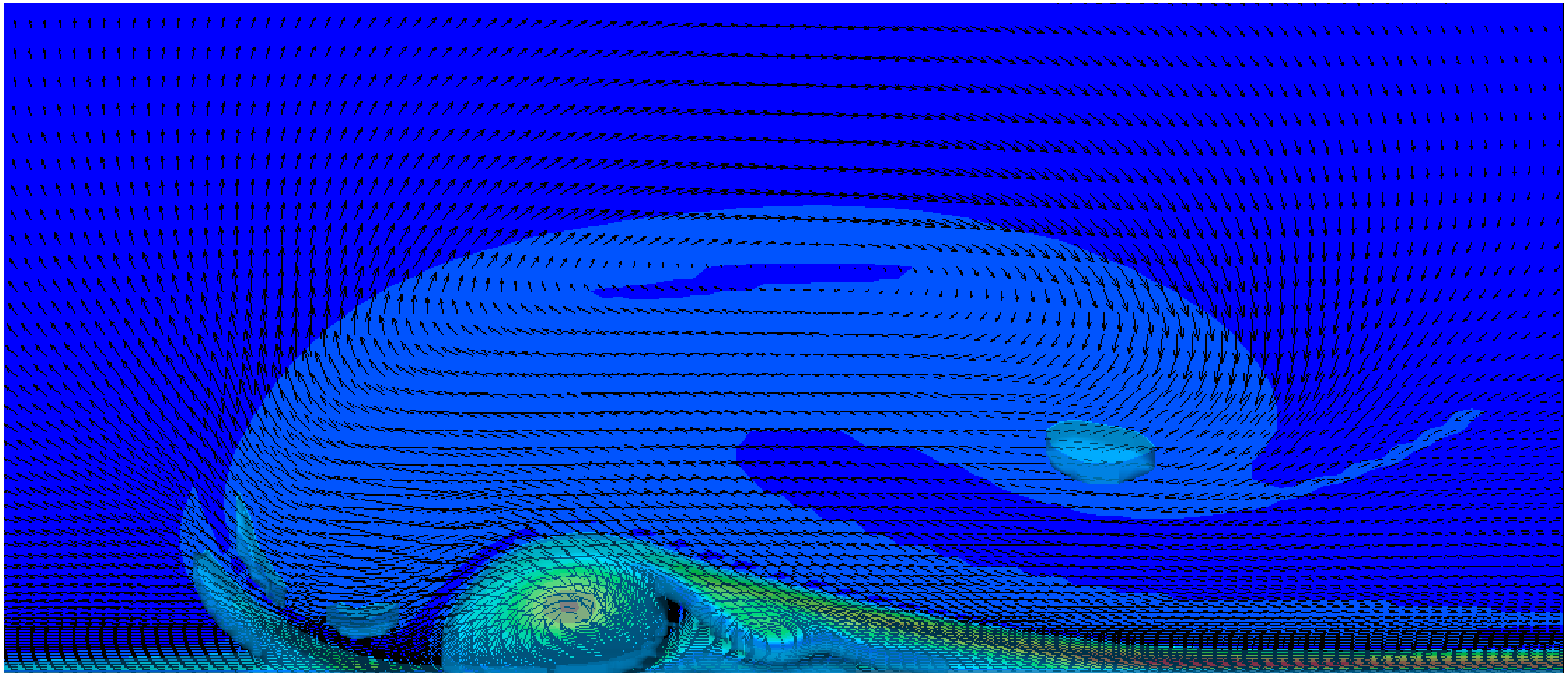


- Where does the shear layer end and the vortex begin?
- Is there a coherent vortex structure in the shear layer?
- Where does the top of the vortex end?
- Vorticity sheet is not necessarily a vortex sheet

Vorticity Magnitude

- **Vortex boundary**
  - In the downburst case the environment is quiescent (no shear, low vorticity)
  - Vortex structures are typically outside of the shear layer
  - We need to pick a suitable vorticity level to define a connected region (isosurface)

Vorticity Magnitude



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Q-Surfaces

- **Positive second invariant of velocity gradient**
  - **Balance between shear and local rotation rate**

$$Q = \frac{1}{2}\left(u_{i,i}{}^{2} - u_{i,j}u_{j,i}\right) = \frac{1}{2}\left(\|\Omega\|^{2} - \|S\|^{2}\right)$$

  - **If source of vorticity is shear, Q goes to zero**
  - **Vortex structure is outlined by Q = 0+**
  - **Inside of a vortex is anywhere Q > 0**

Kinematic Vorticity Number

- **Ratio of local rotation rate and shear**
  - **Same terms as Q-surfaces just defined slightly differently**

$$N_k = \frac{\|\Omega\|}{\|S\|}$$

  - **Gives the "quality" of rotation non-dimensionalized by the strain rate**
  - **$N_k = 1$ where $Q = 0$ for defining the vortex core**

Complex Eigenvalues

- **Complex eigenvalues from velocity gradient matrix**
  - **Suggests rotational form of streamlines**

$$\nabla\vec{v} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^{-1}$$

$$\nabla\vec{v} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 + \lambda_c i & \\ & & \lambda_2 - \lambda_c i \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^{-1}$$

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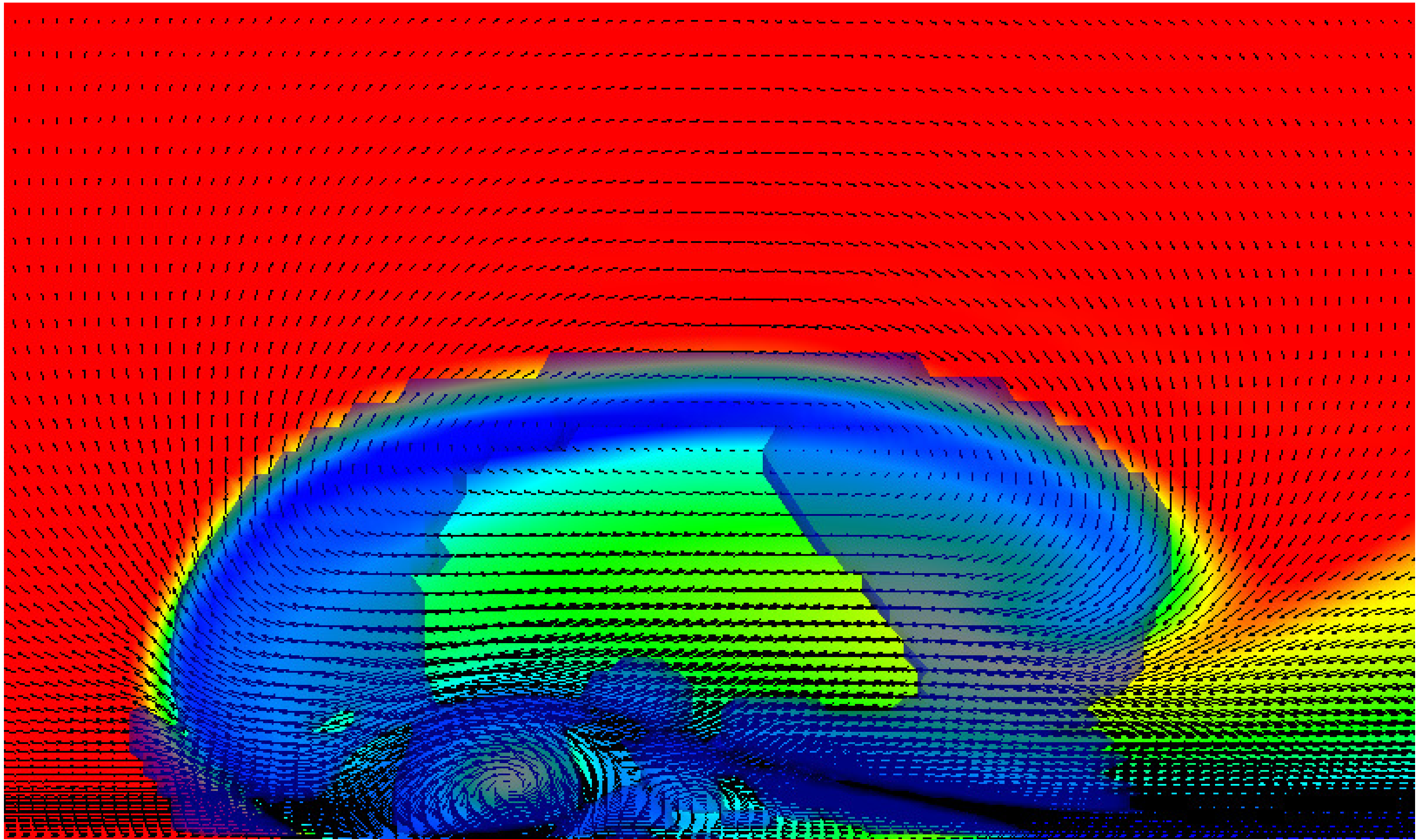
Complex Eigenvalues

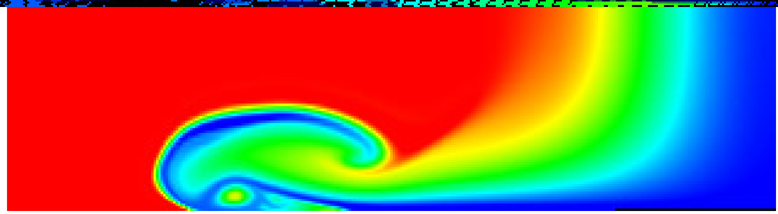
- Vortex is defined by a connected region of complex eigenvalues
- Intensity of rotation is defined by magnitude of imaginary component
- Vortex core is outlined by real to imaginary eigenvalue boundary

Technique Summary

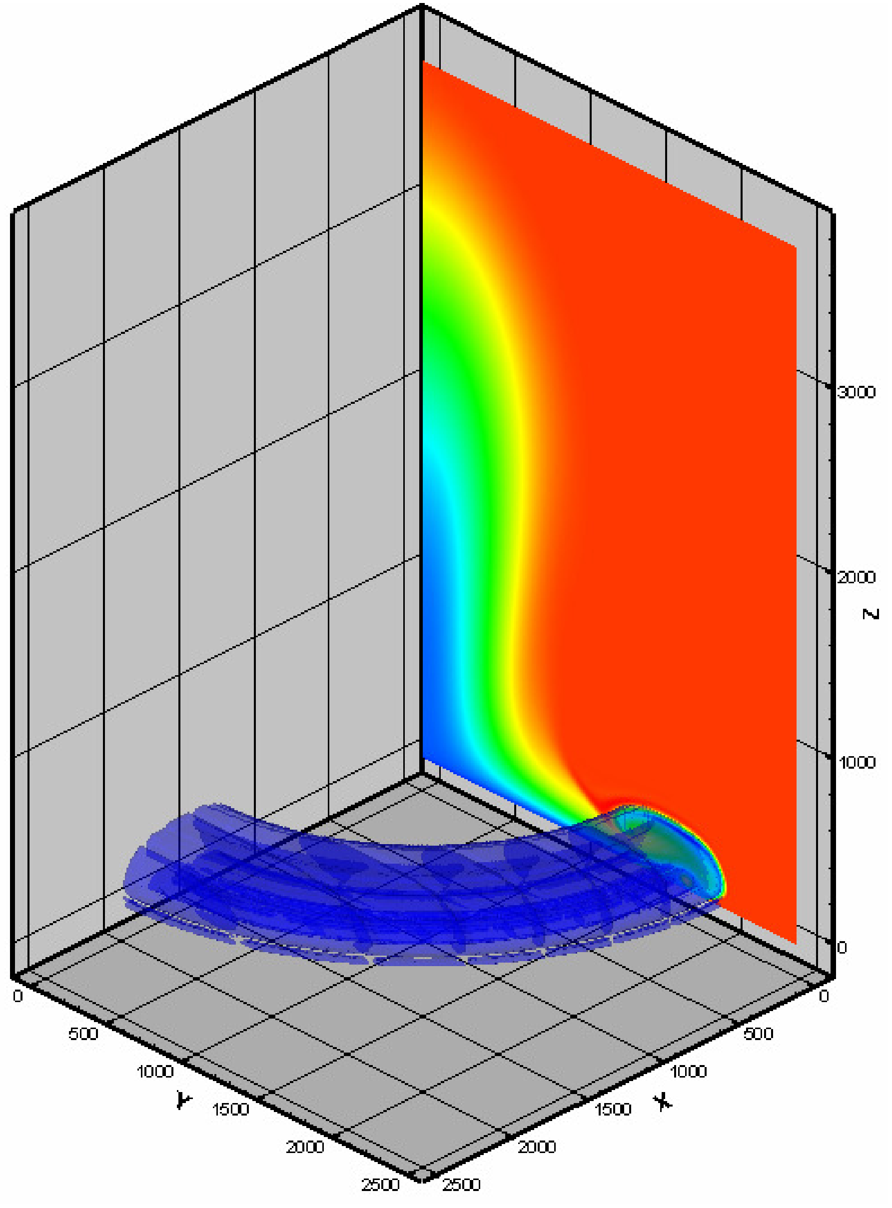
- Vorticity magnitude is not suitable
  - Subjective threshold value
- Q-Surfaces seem to meet all criteria
- Complex eigenvalue surfaces seem to meet al criteria
- What is the difference between Q-Surfaces and Complex Eigenvalues?

Comparison





Comparison



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Conclusions

- There are many different techniques
- Not “one size fits all”
- Vorticity magnitude generally does not work
- Complex Eigenvalue and Q-Surfaces give similar results (in my case)
  - Q-surface method is less restrictive

Others

- **Negative Eigenvalue of $S^2+\Omega^2$**
  - Zhou *et al.* (1999)
- **Lyapunov Exponents (Lagrangian)**
  - Green *et al.* (2007)
- **Negative gradient discriminant**
  - Chong *et al.* (1990)
- **And…**

Questions?

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