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Technical Note

Estimation of total circulation within a plume in a crosswind

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Abstract

The present paper describes an approach which may be adopted for determination of the total circulation associated with the large-scale streamwise vorticity within a plume deflected by a crosswind. The method requires a much smaller experimental data set than would otherwise be necessary in order to define the variation of circulation within the plume with distance from the source. Such data may be used to validate numerical models for the prediction of the near-field flow regime close to a pollutant source. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Aerodynamics; Vorticity; Near-field behaviour

1. Introduction

The behaviour of a plume issuing vertically and normally into a crosswind may be well-defined far from the source by using similarity profiles for wind speeds and pollutant concentrations. Typically, simple or modified forms of Gaussian distributions for concentrations may be utilised, whereby only the initial source strength and location, a representative wind speed and the vertical and lateral spreads of the plume (defined by the standard deviations of the Gaussian profile) are required to give the full distribution. However, the near-field behaviour, very close to the source, is not so clearly defined because this region is characterised by the rapid development of the plume as it is deflected by the crosswind. Here, there are significant spatial variations in wind speed such that similarity profiles cannot be used. Because of this complexity current prediction models for atmospheric pollution concentrations do not consider the detailed behaviour of the plume close to the source. Nevertheless, in the future it may be necessary to be able to fully describe the fluid dynamics in these regions, for example in assessing the exposure of pedestrians to individual vehicle exhausts within urban street canyons or the detailed effects of building ventilation systems on nearby people.

The development of prediction capabilities for these flows will inevitably rely upon some form of turbulence modelling which, in turn, will require validation using complete experimental data sets obtained from laboratory and field tests. In order to ensure the quality of the predictions it will be necessary to define some appropriate gross parameters, derived from the flow field data, against which to check the results. It is well known that the near-field behaviour of a plume or jet in a crosswind is characterised by large-scale vorticity in the form of a dominant pair of streamwise contrarotating vortices, as shown in Fig. 1 and described by Fric and Roshko (1989) and Margason (1993). Hence, an appropriate gross parameter for defining this large-scale behaviour is the total circulation associated with one of these vortices, from which the variation with distance from the source may be assessed and comparisons made between the experimental data and the numerical predictions. This quantity is quite important because although turbulence models have been used to predict the vorticity distributions and total circulation, they presently tend to under-predict the lateral diffusion of the streamwise vorticity and over-predict the total circulation, for example Sakellariou (1993). Clearly, further comparisons between complete flow field data sets, such as those obtained by the present authors, Savory et al. (1996), and numerical predictions are required for a wide range of initial conditions in order

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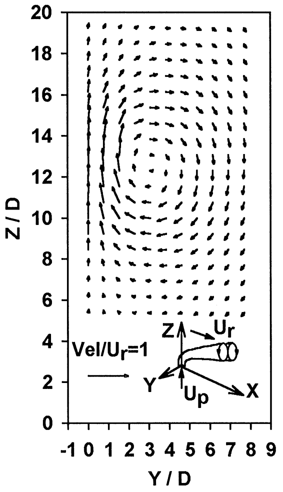

Fig. 1. Typical example of projected velocity vectors in a YZ plane across the plume ($X/D = 20$, $\alpha = 8$).

to improve our modelling capabilities. Unfortunately, there is a drawback in terms of the large amount of experimental data required in any lateral plane across the plume in order to compute a single value of circulation. This restriction may be a difficulty for laboratory studies but would certainly be prohibitive for field experiments. The present paper describes an approach which seeks to overcome this problem by permitting the reliable computation of total circulation at any location down wind of the source using a more limited data set.

2. Experimental details

The principal wind tunnel data set upon which the present analysis is based has been fully described in an earlier paper, Savory et al. (1996). Essentially, the velocity vector distributions have been measured in detail within non-buoyant plumes issuing normally from a circular nozzle (of diameter $D = 28.5$ mm) located flush within in a wind tunnel ground plane. The crossflow velocity (U_r) was maintained at 12 m s^{-1} whilst three different initial plume velocities were studied of 48, 72 and 96 m s^{-1}, giving velocity ratios of $\alpha = 4$, 6 and 8. All of the measurements were taken with a five-hole pressure probe. It is recognised that this source configuration is very simple and idealised. Nevertheless, it does provide a well-specified boundary condition for the analysis. Furthermore, the research of Endo and Nakamura (1970) shows that the basic fluid dynamics of this arrangement is similar to that which occurs when the source is a nozzle raised above the ground plane in the form of a chimney. During the present work further experiments have been carried out in another wind tunnel facility, using a similar source nozzle arrangement but with a smaller diameter of $D = 13$ mm. In this case the crossflow velocity and plume velocity were fixed at 6.6 m s^{-1} and 53 m s^{-1}, respectively, giving a velocity ratio of $\alpha = 8$. Measurements were taken in YZ planes across the plume, at downstream locations from $X/D = 7.5$ to 25 in 2.5D increments, using a crossed hot-wire anemometer probe with the method described by Cutler and Bradshaw (1991).

3. Discussion

The circulation (k) associated with one of the contrarotating vortices may be computed from

$$k = \int_A \omega_x \, \mathrm{d}A$$

where the vorticity

$$\omega_x = \frac{\partial V}{\partial Z} - \frac{\partial W}{\partial Y},$$

is integrated over the area (A) in the YZ plane where ω_x is non-zero. It is evident that a significantly large VW velocity component data set is required in order to adequately resolve the vorticity distribution and, thereby, the total circulation. However, the circulation may also be determined from

$$k = \oint_S V_S \, \mathrm{d}S$$

where V_S is the tangential velocity along a line (S) bounding the region of vorticity. As the length of S is increased to encompass all of the vortex the integral asymptotes towards the value of circulation. Hence, in principle, only the velocity vector component along the sides of a single "box" enclosing the vortex is required to yield the circulation. Unfortunately, as S increases V_S decreases towards zero, such that the resulting integral becomes dominated by small measurement errors in V_S and the integrated value does not asymptote satisfactorily. Both the analysis of Thompson (1971) and the present authors fail to yield accurate results for circulation using this method and so an approach for dealing with this problem will now be outlined.

Consider the four sides of a rectangular "box" enclosing the vortex in a YZ plane where one side comprises the Z, or vertical, axis. Fig. 1 shows the projected velocity vectors (Vel) associated with this vortex, with the arrows scaled by the magnitude of the velocity normalised by the freestream value. It is evident that the vertical (W) velocity component along this side is quite substantial through the plume, whereas the relevant components along the other three sides become very small as the box size increases. As $S \to \infty$ then $V_S \to 0$ with the resulting product and its integral having a finite but non-zero value that is difficult to determine experimentally.

However, along the Z-axis the velocity component ($V_S = W$) is very well-defined, giving $W = 0$ at $Z = 0$ and a rapid asymptote of W towards zero above the plume. Hence, it may be reasonable to use this dominant contribution to the circulation as a measure of the overall total circulation associated with the vortex. On this basis one may perform the simple integral

$$k_c = \int_0^\infty W \, dZ$$

to obtain a "centre-line" value for the circulation (k_c). Although integration between these limits might include a contribution to the W-component velocity beneath the main body of the plume arising from any growth of the ground plane boundary layer it is considered that this would be negligible in most practical cases. Using the complete planes of available experimental data Fig. 2 shows a comparison between the values of circulation obtained by integrating the entire vorticity distribution (K) and those obtained from the line integral along the centre-line (Z-axis) only (K_c). In each case non-dimensional values are shown, defined as; $K = k/(U_r D)$ and $K_c = k_c/(U_r D)$. Considering first the main data set, comprising three different velocity ratios, it may be seen that there is very good correlation between the two methods of computing the circulation such that

$$K = 1.2014 K_c.$$

with a correlation coefficient of 0.996. This indicates that the centre-line contributes some 83% of the total circulation derived from a full contour integral. The quality of the correlation suggests that, provided some full-plane velocity data are available for a given configuration in order to provide a "calibration" of the total circulation, the value at all other downstream locations may be obtained from simple centre-line integrals.

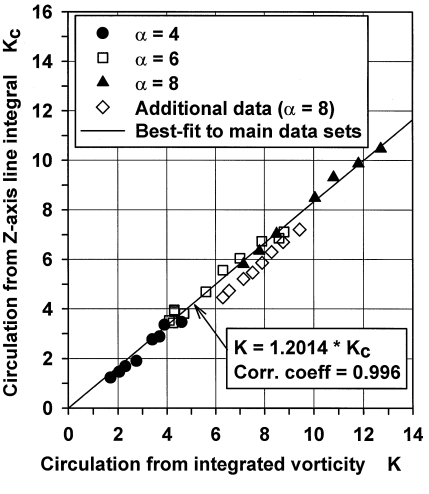

Fig. 2. Comparison of total circulation associated with one vortex computed from integration of vorticity and from line integral along vertical centre-line.

In the case of the additional data set there is, again, a good correlation between the circulation values obtained using the two different methods although the results do not fall on the same line as before. Some discrepancies between the data sets obtained from the two experiments is to be expected because of the differences in boundary conditions. Firstly, there is a difference in the thickness (δ) of the approaching boundary layer on the ground plane upstream of the nozzle. For the main set of measurements this gave $\delta/D = 3.58$, whilst for the hot-wire experiments the value was $\delta/D = 2.30$. Certainly, it has been shown by Sugiyama and Kawase (1985) that the effect of the boundary layer on the plume development can be quite significant, especially for low velocity ratios where the plume path tends to be closer to the ground. This is because the initial distribution of vorticity (with the ground plane boundary layer and nozzle wall being the sources) plays a role in its subsequent redistribution. The mechanisms by which the ground plane boundary layer is entrained into the plume have been well-described by Fric and Roshko (1989) from flow visualisation studies. In addition, the wind tunnel sectional area blockage effect, which may slightly influence the growth of the plume, is different in the two cases. In the main experiments the section area was 37.44D (width) × 48.14D (height), whilst for the hot-wire tests the dimensions were 47.69D (width) × 57.69D (height). Hence, the blockage in the latter experiments was some 65% of that in the former tests. Nevertheless, the available data presented here indicates that, provided at least one (preferably two) complete YZ planes of velocity information are obtained for a given configuration, the complete profile of circulation decay with distance from the source may be determined from a series of centre-line profile measurements of the vertical velocity component only.

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