PAPR Reduction using Combined Coding, Weighting, and Mapping in OFDM Systems[1]

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Abstract— In this paper we present methods for reduction of Peak-to-Average Power Ratio (PAPR) in multi-carrier Orthogonal Frequency Division Multiplexing (OFDM) through the joint use of coding, weighting, and signal mapping. Simple and easy-to-implement coding schemes have been identified that when employed with suitable weighting functions can reduce PAPR considerably. Indeed it is shown that PAPR can be controlled by appropriate choice of coding scheme and weighting function for a given signal mapping. A thorough numerical investigation using computer simulations is presented and schemes that offer considerable reduction in PAPR are identified as a function of number of sub-carriers in OFDM systems.

I. Introduction

The demand for mobile communications and computing combined with increased growth for Internet access is growing rapidly. The key to realizing this potential is the development and deployment of high performance radio systems that support high data rates with wide area coverage. In conventional single-carrier communication systems for high data rates the effect of multipath propagation increases the equalization cost due to short symbol durations and relatively long channel delay times. OFDM [1]-[2] is an alternative technique for high bit rate transmission in a radio environment in which data transmission is parallel; unlike in the single-carrier system where it is serial. In such a system, the deleterious effect of fading is spread over many bits; therefore, instead of a few adjacent bits completely destroyed by the fading, it is more likely that several bits only be slightly affected by the channel. Furthermore, when the data is transmitted at high bit rates over mobile radio channels, there exists ISI due to the channel impulse response extending over several symbol durations. In such a situation OFDM can be gainfully employed to mitigate the effects of ISI. A technique to reduce ISI is to increase the number of sub-carriers by reducing the bandwidth of each sub-channel while maintaining the overall bandwidth. In OFDM systems ISI can be eliminated using guard interval. Among the other advantages of OFDM are: a) Spectral efficiency due to overlapping sub-channels; b) realization of modulation and demodulation using FFT techniques; c) Less sensitivity to sample timing offsets than single carrier systems; d) Superior performance over frequency-selective channels via the use of channel coding and interleaving; and e) protection against co-channel interference.

In recent years OFDM systems are included in digital audio/video broadcasting (DAB/DVB) standards in Europe [3]-[4], while the discrete multi-tone (DMT) (its wire line counterpart in USA) has been applied to high-speed digital sub-carrier (DSL) modems over twisted pair [5]-[6]. OFDM is also a candidate proposed for digital cable TV systems [7] and local area mobile wireless networks such as IEEE 802.11a, the MMAC, and the HIPERLAN/2 [8]-[9]. Multi-carrier hybrids with DS-CDMA have also been developed for wideband cellular communications.

In OFDM systems, the transmitted signal occasionally exhibits very high peaks. Thus an inherent difficulty is the fact that they have a very large PAPR. The principal drawback of OFDM is that the peak-transmitted power may be up to N times the average power, with N, the number of sub-carriers [10]. A large PAPR introduces an increased complexity of the analog-to-digital and digital-to-analog converters and degrade the effi-

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ciency of RF power amplifier. Linear amplifiers that can handle the peak power are less efficient. Hard limiting of the transmitted signal generates intermodulation among the sub-carriers and results in out-of-band radiation. In the literature [2],[10]–[23] various techniques have been proposed and investigated to overcome the problem of large peaks in the transmitted OFDM signals. Recently, weighting functions [12]-[13] have been considered for reduction of PAPR. Appropriate choice of coding and weighting thus still remains an open option in order to jointly optimize PAPR and bit error rate (BER). While in the literature, coding and weighting have been considered separately for reduction of PAPR, our objective of this paper is, therefore, to investigate the combined influence of coding and weighting on PAPR in OFDM systems. It is shown that PAPR can be controlled by appropriate choice of coding scheme and weighting function for a given signal mapping used in OFDM system. While weighting can be used to reduce PAPR, it increases the probability of bit error. However, by choosing coding and weighting it is possible to reduce PAPR and at the same time one can control bit error rate. The objective therefore is to investigate the problem of PAPR by examining the influence of coding, in particular, parity bit and horizontal/vertical parity bit codings on PAPR. These codings have not been examined in the literature. Next, we consider novel weighting functions in an attempt to reduce PAPR. Finally, A thorough numerical investigation using computer simulations is presented and schemes that offer considerable reduction in PAPR are identified as a function of number of sub-carriers in OFDM systems.

In Section II we briefly describe a typical OFDM system. In section III we address the problem of controlling PAPR in a OFDM system by jointly using coding and weighting and arrive at expressions for its computation. In Section IV the coding and weighting functions considered are briefly explained. Section V is devoted to numerical results and discussion of techniques for controlling PAPR. The paper is concluded in Section VI.

II. Typical OFDM System

The block diagram of a typical OFDM system is shown in Fig. 1 . The data source is assumed to be a sequence of discrete digits:

$$\{a_0, a_1, a_2, ...\}$$

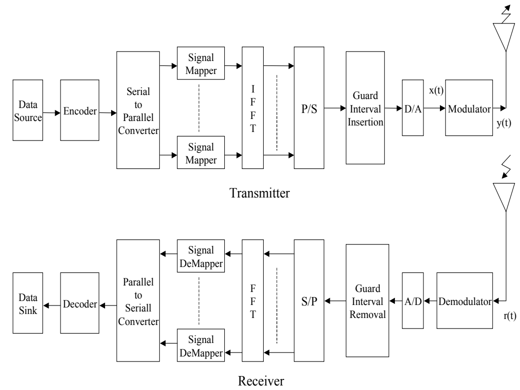

Fig. 1. Block diagram of a typical OFDM system.

where,

$$a_i = 1 \quad or \quad 0, \qquad i = 0, 1, 2, 3, ...$$

and

$$P(a_i = 1) = P(a_i = 0) = \frac{1}{2}$$

The output of the encoder is fed to a serial-to-parallel converter that partitions the input data arriving at the rate R into N parallel information symbols each at a reduced data rate of $\frac{R}{N}$. The number of bits in each of the N output sequences of a serial-to-parallel converter is determined by the constellation of the signal mapper. For example, when a BPSK mapper is used, each output sequence carries one bit of information. For QPSK and 16-QAM, each channel carries 2 and 4 bits of information. The output of the signal mapper can be thought of as a discrete complex signal which during any arbitrary OFDM symbol duration is a vector of N complex numbers given by:

$$C = [C_0, C_1, ..., C_{N-1}] \tag{1}$$

This discrete signal representing the data is fed to an N-point IFFT block to obtain a transformed discrete signal given by:

$$D = [D_0, D_1, ..., D_{N-1}] \tag{2}$$

The relationship between the input and the output discrete signals of IFFT block is given by the well known Discrete Time Fourier Series (DTFS). That is,

$$\begin{aligned} D_n &= A'_n + jB'_n \\ &= \sum_{m=0}^{N-1} C_m e^{j\frac{2\pi}{N}mn}; n = 0, 1, ..., N-1 \end{aligned} \tag{3}$$

where it is assumed that the input sequence C is periodic with period equal to N.

$$x(t) = \sum_{m=0}^{N-1} C_m e^{j2\pi f_m t} \tag{4}$$

where $f_m = \frac{m}{NT_b}$ and $t = nT_b$. T_b is the original duration of bit and f_m is the frequency of the mth carrier. It can be shown that the discrete sequence $D = [D_0, D_1, ..., D_{N-1}]$ can be obtained by sampling $x(t)$ at $t = nT_b, n = 0, 1, ..., N-1$. That is ,

$$x(nT_b) = D_n = \sum_{m=0}^{N-1} C_m e^{j2\pi f_m nT_b} \tag{5}$$

The complex continuous-time modulating signal given in (4) can be written as:

$$x(t) = m_I(t) + jm_Q(t) \tag{6}$$

where

$$m_I(t) = Re\{x(t)\} \tag{7}$$

and

$$m_Q(t) = Im\{x(t)\} \tag{8}$$

The relationship between discrete and continuous signals given by (3) and (4) is illustrated in Figs.2 and 3. The signals $m_I(t)$ and $m_Q(t)$ can be written as:

$$m_I(t) = \sum_{m=0}^{N-1} (a_m \cos 2\pi f_m t - b_m \sin 2\pi f_m t) \tag{9}$$

and

$$m_Q(t) = \sum_{m=0}^{N-1} (a_m \sin 2\pi f_m t + b_m \cos 2\pi f_m t) \tag{10}$$

where $C_m = a_m + jb_m$, with a_m and b_m representing the in-phase and quadrature components, respectively. The modulating signal given by (6) is then modulated and transmitted using in-phase and quadrature-phase processing using a carrier frequency f_c. The modulator structure is shown in Fig.4 and the modulated signal can be mathematically written as:

$$y(t) = m_I(t) \cos 2\pi f_c t - m_Q(t) \sin 2\pi f_c t \tag{11}$$

Thus the objective at the receiver is to recover $m_I(t)$ and $m_Q(t)$ from $y(t)$, assuming a noiseless system. This can be accomplished using the in-phase and quadrature-phase detector shown in Fig.5. The demodulated baseband signals $m_I(t)$

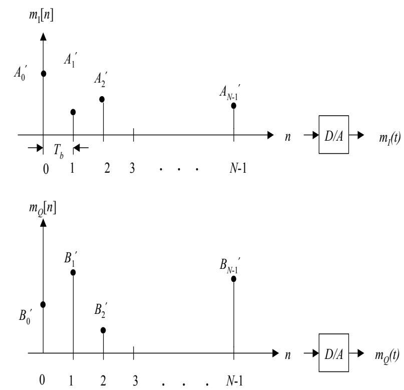

Fig. 2. Discrete-time signals at the output of the parallel-to-serial converter

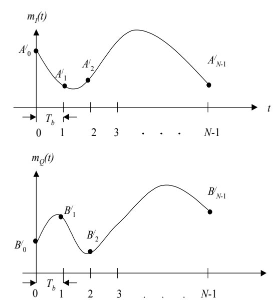

Fig. 3. Continuous-time signals at the output of the D/A converter

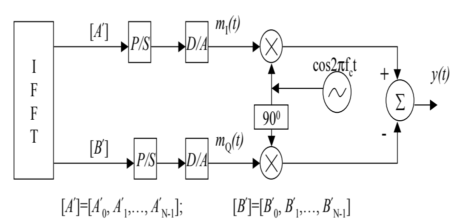

Fig. 4. In-phase and quadrature-phase modulator

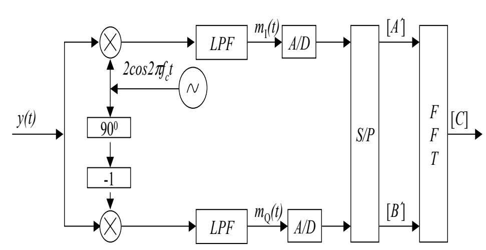

Fig. 5. In-phase and quadrature-phase detector

and $m_Q(t)$ are sampled and fed to the S/P converter which produces a vector of complex numbers $[D_0, D_1, ..., D_{N-1}]$. The complex numbers

$[C_0, C_1, ..., C_{N-1}]$ can then be recovered using FFT operation and thus the transmitted data sequence using the signal de-mapper.

III. Formulation of PAPR Problem

The transmitted OFDM signal may be modeled as (from (11)):

$$y(t) = m_I(t)\cos 2\pi f_c t - m_Q(t)\sin 2\pi f_c t \quad (12)$$

where $m_I(t)$ and $m_Q(t)$ are given by (9) and (10), respectively. The complex modulating signal is given by:

$$x(t) = m_I(t) + jm_Q(t) = \sum_{m=0}^{N-1} C_m e^{j2\pi \frac{m}{T} t} \quad (13)$$

The transmitted signal in (12) can be written as:

$$\begin{aligned} y(t) &= \sqrt{m_I^2(t) + m_Q^2(t)} \\ &\times \cos\left[2\pi f_c t + \tan^{-1}\left(\frac{m_Q(t)}{m_I(t)}\right)\right] \end{aligned} \quad (14)$$

The envelope of the transmitted signal is thus given by

$$\text{Envelope of } y(t) = \tilde{y}(t) = \sqrt{m_I^2(t) + m_Q^2(t)} \quad (15)$$

The PAPR of the envelope of the transmitted signal can then be written as:

$$\text{PAPR} = \frac{\max\{[\tilde{y}(t)]^2\}}{E\{[\tilde{y}(t)]^2\}} = \frac{\max\{|x(t)|^2\}}{E\{|x(t)|^2\}} \quad (16)$$

where $x(t)$ denotes the complex modulating signal. It is noted that the instantaneous power of the envelope of the transmitted OFDM signal is given by:

$$p(t) = [\tilde{y}(t)]^2 = |x(t)|^2 \quad (17)$$

The computation of PAPR in (16) is approximated using discrete-time PAPR, which is obtained from the samples of the OFDM signal. The sampling rate is the Nyquist rate or a multiple of it (oversampling). An important question is how large the oversampling factor should be in order for the approximation to be fairly accurate. The reasons for discrete-time approach for computation of PAPR are: i) most systems are implemented in discrete-time; ii) computation in continuous-time is too complex as closed-form expression is difficult to arrive at; and iii) when oversampling rate of six times that of Nyquist rate is used, it closely approximates continuous-time PAPR. Therefore, our approach is to employ discrete-time approximation of continuous-time PAPR by using a sampling rate that is six times the Nyquist rate. The discrete-time computation of PAPR is approached by sampling the envelope of $y(t)$. Which is equivalent to sampling the complex modulating signal. That is,

$$\tilde{y}(t = t') = |x(t = t')| \quad (18)$$

When Nyquist rate of N samples over the OFDM symbol interval is taken of $\tilde{y}(t)$, we obtain:

$$\begin{aligned} |D_n| &= |x(t = nT_b)| \\ &= |\sum_{m=0}^{N-1} C_m e^{j2\pi \frac{m}{N} n}| \end{aligned} \quad (19)$$

where $n = 0, 1, ..., N-1$. Using this discrete-time approach PAPR in (16) becomes

$$\text{PAPR} = \frac{\max\{|D_n|^2, \quad n = 0, 1, ..., N-1\}}{\frac{1}{N}\sum_{n=0}^{N-1} |D_n|^2} \quad (20)$$

Using (20) PAPR can be computed easily, since $\{D_n; n = 0, 1, ..., N-1\}$ is the output of IFFT block in response to the information carrying input sequence $\{C_m; m = 0, 1, ..., N-1\}$. In arriving at the PAPR using (20) we have used an N-point approximation of the continuous-time envelope of OFDM signal. It is noted that the N-point approximation may not reveal the real peak power of the envelope. A better approximation of PAPR can be obtained by using higher sampling rate than the Nyquist rate N, which is explained next.

Consider using a sampling rate that is two times the Nyquist rate in which case the input to the IFFT block would consist of the original sequence $[C]$ and an additional N zeros appended to it. Denoting this new sequence as $[C'] = [C'_0, ..., C'_{N-1}, C'_N, ..., C'_{2N-1}]$, it is noted that

$$C'_m = \begin{cases} C_m, & m = 0, 1, ..., N-1 \\ 0, & m = N, N+1, ..., 2N-1 \end{cases} \quad (21)$$

The $2N$-point IFFT of $[C']$ then is given by $[D'] = [D'_0, ..., D'_{N-1}, D'_N, ..., D'_{2N-1}]$, where

$$D'_n = \sum_{m=0}^{2N-1} C'_m e^{j2\pi(\frac{m}{2N})n} \quad (22)$$

The sequence of $2N$ samples $[D'_n; n = 0, 1, ..., 2N-1]$ at the output of the IFFT block, after passing through parallel-to-serial and digital-to-analog converter can be written as:

$$x^{'}(t) = \sum_{m=0}^{2N-1} C_{m}^{'} e^{j2\pi f_{m}^{'} t} \tag{23}$$

where

$$f_{m}^{'} = \frac{m}{2N\frac{T_b}{2}} \tag{24}$$

It is noted that by employing a sampling rate that is twice that of Nyquist rate it is possible to take into account peaks of the envelope that lie between the adjacent samples. By increasing the sampling rate further the discrete-time PAPR becomes close to that of continuous-time PAPR. The expression of PAPR when the sampling rate is L times the Nyquist rate N can be written as:

$$\mathrm{PAPR} = \frac{\max\{|D_{n}^{'}|^2, \quad n = 0, 1, ..., LN - 1\}}{\frac{1}{LN}\sum_{n=0}^{N-1} |D_{n}^{'}|^2} \tag{25}$$

IV. Coding and Weighting

In this Section we briefly describe the coding and weighting functions used in this paper.

A. Coding

In the literature there exist a number of methods for encoding, such as block codes, convolutional codes, etc. In this paper we consider a class of block codes and, in particular, single parity check and horizontal and vertical parity check codes. Hence we describe these two classes of codes and list their properties [24].

1) Single Parity Check Codes: In single parity check coding a single bit is appended, called a parity check, to a string of data bits. This parity check bit has the value 1 if the number of 1's in the bit string is odd, and has the value 0 otherwise. This type of coding is referred to as even single parity check coding. While in even single-parity check coding a block of N bits is mapped to $N+1$ bits with the final bit the modulo 2 sum of the N data bits, in odd single parity check coding the final check bit added has a value 0 if the number of 1's in the data bit string is odd, and a value 1 if it is even. The rate of the code is $\frac{K}{K+1}$, where K is the length of the data sequence. While it is well known that single parity check bit coding is inadequate from the viewpoint of reliable detection of errors [25], the objective in this paper is to investigate this type of coding from the viewpoint of PAPR reduction. At the transmitting end single-parity check coding can be easily implemented using modulo-2 arithmetic and at the receiving end the decoding consists of testing whether the modulo-2 sum of the code word bits yields a zero result (for even parity) or one result (for odd parity). The bit rate at the output of a single parity bit encoder is given by:

$$R = \frac{K+1}{K} R_b \quad bits/\sec \tag{26}$$

2) Horizontal and Vertical Parity Check Code: Another simple and intuitive approach of encoding is to arrange a string of data bits in a two dimensional array with one parity check for each row and one for each column. The parity check in the lower right corner can be viewed as a parity check on the row parity checks, on column parity checks, or on the data array. These horizontal and vertical parity checks are also called rectangular codes and product codes. In general the incoming serial data is arranged in the form of a matrix of M rows and N columns and this matrix is mapped to $(M+1) \times (N+1)$ matrix that contains appended check bits. Thus the rate of a rectangular code is given by

$$\frac{k}{n} = \frac{MN}{(M+1)(N+1)} \tag{27}$$

The bit rate at the output of a rectangular encoder is given by:

$$R = \frac{(M+1)(N+1)}{MN} R_b \quad bits/\sec \tag{28}$$

Rectangular code has error correcting capability besides error detection properties. A major weakness of this scheme is that it may fail to detect rather short bursts of errors.

B. Weighting

In order to introduce weighting for each subcarrier in (13), we employ discrete weighting function given by $w_m, m = 0, 1, 2, ..., N-1$, in (13). Thus the weighted complex OFDM modulating signal can be written as:

$$x_w(t) = \sum_{m=0}^{N-1} C_m w_m e^{j2\pi f_m t} \tag{29}$$

Denoting the product $C_m w_m$ by C_{mw} the transmitted weighted OFDM signal can be written as

$$y_w(t) = m_{Iw}(t)\cos 2\pi f_c t - m_{Qw}(t)\sin 2\pi f_c t \tag{30}$$

where

$$m_{Iw}(t) = Re\{x_w(t)\} \quad (31)$$

$$m_{Qw}(t) = Im\{x_w(t)\} \quad (32)$$

and

$$x_w(t) = m_{Iw}(t) + jm_{Qw}(t) \quad (33)$$

The weighted transmitted OFDM signal $y_w(t)$ can be written as

$$\begin{aligned} y_w(t) &= \sqrt{m_{Iw}^2(t) + m_{Qw}^2(t)} \\ &\times \cos\left[2\pi f_c t + \tan^{-1}\left(\frac{m_{Qw}(t)}{m_{Iw}(t)}\right)\right] \end{aligned} \quad (34)$$

The envelope of the weighted transmitted signal is therefore given by

$$\text{Envelope of } y_w(t) = \tilde{y_w}(t) = \sqrt{m_{Iw}^2(t) + m_{Qw}^2(t)} \quad (35)$$

The PAPR of the envelope is given by:

$$\text{PAPR} = \frac{\max\{[\tilde{y_w}(t)]^2\}}{E\{[\tilde{y_w}(t)]^2\}} = \frac{\max\{|x_w(t)|^2\}}{E\{|x_w(t)|^2\}} \quad (36)$$

where $x_w(t)$ denotes the complex modulating signal. The instantaneous power associated with this envelope is given by :

$$p_w(t) = [\tilde{y_w}(t)]^2 = |x_w(t)|^2 \quad (37)$$

Again we approach the computation of PAPR in (36) using discrete samples of $y_w(t)$. When Nyquist rate is employed, using developments carried out for the non-weighted OFDM signals from (12) to (20), the PAPR can be written as

$$\text{PAPR} = \frac{\max\{|D_{nw}|^2; n = 0, 1, ..., N-1\}}{\frac{1}{N}\sum_{m=0}^{N-1}|D_{nw}|^2} \quad (38)$$

where

$$D_{nw} = \sum_{m=0}^{N-1} C_{mw} e^{j2\pi(\frac{m}{N})n} \quad (39)$$

The set of complex number $\{D_{nw}, n = 0, 1, .., N-1\}$ is the output of IFFT block in response to the set of weighted complex numbers $\{C_{nw}, n = 0, 1, .., N-1\}$. Also, it can be shown that

$$\begin{aligned} |D_{nw}| &= \tilde{y}_w(t = nT_b) = |x_w(t = nT_b)| \\ &= \left|\sum_{m=0}^{N-1} C_{mw} e^{j2\pi(\frac{m}{N})} n\right| \end{aligned} \quad (40)$$

where $n = 0, 1, ..., N-1$. The PAPR of the weighted OFDM signal given above can be simplified by first finding an expression for its instantaneous power. That is

$$\begin{aligned} p_w(t) &= |x_w(t)|^2 \\ &= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(C_m w_m)(C_n w_n)^* e^{j2\pi\frac{(m-n)}{T}t} \end{aligned} \quad (41)$$

which can be simplified and written as:

$$\begin{aligned} p_w(t) &= \sum_{m=0}^{N-1}|C_m w_m|^2 \\ &+ \sum_{m=0}^{N-1}\sum_{n=0, n\neq m}^{N-1}\Big[(C_m w_m)(C_n w_n)^* \\ &\times \; e^{j2\pi\frac{(m-n)}{T}t}\Big] \end{aligned} \quad (42)$$

The average power is nothing but the expectation of $p_w(t)$ which is given by:

$$\begin{aligned} E\{p_w(t)\} &= \sum_{m=0}^{N-1}|C_m|^2|w_m|^2 \\ &+ \sum_{m=0}^{N-1}\sum_{n=0, n\neq m}^{N-1}\Big[E\{C_m C_n^*\} w_m w_n^* \\ &\times \; e^{j2\pi\frac{(m-n)}{T}t}\Big] \end{aligned} \quad (43)$$

since the symbols on different carriers are assumed independent

$$E\{C_m C_n^*\} = E\{C_m\}E\{C_n^*\} \quad (44)$$

using (44) in (43) we obtain:

$$E\{p_w(t)\} = \sum_{m=0}^{N-1}|w_m|^2 \quad (45)$$

for PSK and QPSK. In order to compare the performance of the OFDM system for various weighting functions we normalize

$$\sum_{m=0}^{N-1}|w_m|^2 = 1 \quad (46)$$

Next, we describe the various weighting functions used in the paper:
Bartlett: This function has a triangular shape for $0 \leq n \leq N-1, i.e.,$

$$\omega_{1,n} = \begin{cases} A\left[.01 + \left(1 - \frac{|n - \frac{N}{2}|}{\frac{N}{2}}\right)\right]; 0 \leq n \leq N-1 \\ 0 \quad otherwise \end{cases} \quad (47)$$

Gaussian: Based on the Gaussian function these factors are generated, i.e.,

$$\omega_{2,n} = \begin{cases} A \exp\left[-\frac{\left(n-\frac{M}{2}\right)^2}{2s^2}\right]; 0 \leq n \leq N-1 \\ 0 \quad otherwise \end{cases} \tag{48}$$

where s is the standard deviation of the weighting factors around $N/2$.

Shannon: The shape of this weighting function is the sinc function i.e.

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

and written as

$$\omega_{3,n} = \begin{cases} A \ \left[.01 + \mathrm{sinc}\left(\frac{2n-N}{N}\right)\right]; 0 \leq n \leq N-1 \\ 0 \quad otherwise \end{cases} \tag{49}$$

Half-sine: This weighting function is described by:

$$\omega_{4,n} = \begin{cases} A \ \left[.01 + \sin\left(\pi\frac{n}{N}\right)\right]; 0 \leq n \leq N-1 \\ 0 \quad otherwise \end{cases} \tag{50}$$

Raised Cosine: The shape of this weighting function in the interval [0, N-1] is described by 1-cos(2πn/N) or equivalently by

$$\omega_{5,n} = \begin{cases} A \ \left[.01 + \sin^2\left(\pi\frac{n}{N}\right)\right]; 0 \leq n \leq N-1 \\ 0 \quad otherwise \end{cases} \tag{51}$$

Chebyshev: The shape of this weighting function is exponentially increasing;

$$\omega_{6,n} = \begin{cases} A \ \left[.01 + \sin^{-1}\left(h\frac{n}{N}\right)\right]; & \begin{matrix} 0 \leq n \leq N-1 \\ h = .1, .2, ..., .8 \end{matrix} \\ 0 \quad otherwise \end{cases} \tag{52}$$

Trapezoid: The shape of this weighting function is trapezoidal form;

$$\omega_{7,n} = \begin{cases} A\left[.01 + \frac{n}{\frac{N}{4}-1}\right]; & 0 \leq n \leq \frac{N}{4}-1 \\ 1.01A; & \frac{N}{4} \leq n \leq \frac{3N}{4}-1 \\ A\left[.01 + \frac{4(N-n-1)}{N-4}\right]; & \frac{3N}{4} \leq n \leq N-1 \end{cases} \tag{53}$$

The above functions are sketched in Figs. 7 and 8. It should be noted that the weighting factors are discrete. However, for simplicity in Figs. 7 and 8, the weighting functions are plotted continuous. For performance comparison of OFDM systems, the amplitude A in (47)-(53) is selected in such a way that the power of all weighting factors is constant, i.e.

$$\sum_{n=0}^{N-1} \omega_{i,n}^2 = 1 \qquad i = 1, 2, ..., 7 \tag{54}$$

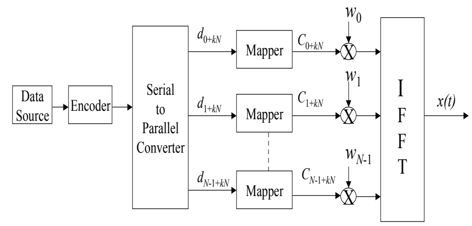

Fig. 6. Generation of weighted and coded OFDM Signal.

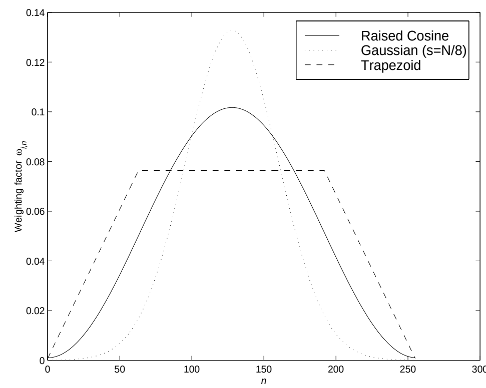

Fig. 7. Different waveforms for the weighting factors ω_n of the OFDM signal ($N = 256$).

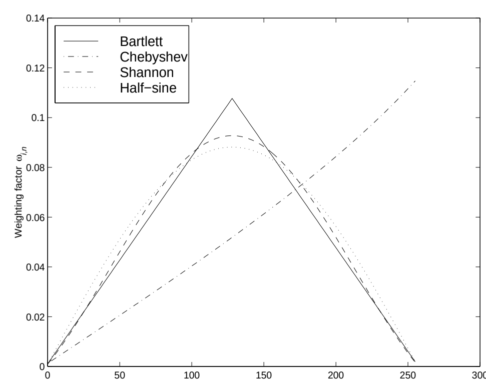

Fig. 8. Different waveforms for the weighting factors ω_n of the OFDM signal ($N = 256$).

In (47) - (54) we have used the notation $\omega_{i,n}$ to denote the nth weighting coefficient for the ith weighting function. In general when $|C_m|^2 = 1$, $m = 0, 1, 2, ..., N-1$, the average power can be set to unity by setting (43) to unity through appropriate choice of $[w_m]$ for a given $[C_m, \quad m = 0, 1, 2, ..., N-1]$. Thus far, we have described weighted OFDM and the various weighting functions. Coding can be introduced into these signals prior to serial-to-parallel conversion of the incoming message bits. A block diagram for generation of coded and weighted OFDM signal is shown in Fig.6.

V. Numerical Results and Discussions

In order to illustrate the influence of weighting and coding on PAPR of OFDM signals, we compute the Complementary Cumulative Distribution Function (CCDF). This is a plot of Prob.$[PAPR > PAPR_0]$ vs $PAPR_0$. To generate this plot, for a specific OFDM signaling situation, $50,000$ random OFDM blocks are generated and for each block PAPR is computed and thus CCDF is plotted. In all cases the function in (13) has been over sampled by a factor of 6 to accurately determine PAPR. The number of subcarriers N are 8, 16, 32, 64, and 128. It is noted that PAPR is expressed in dB in all illustrations. All computations have been carried out using MATLAB. Next, we illustrate numerical results for various cases of weighting and coding and offer comments on achievable improvement in PAPR.

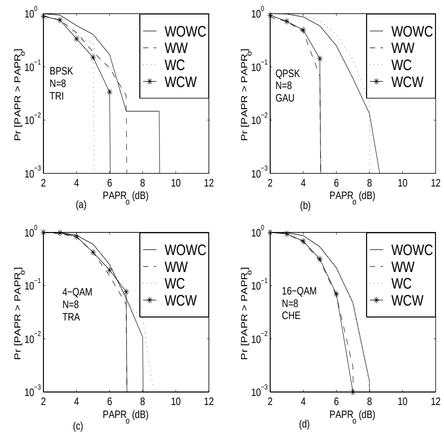

Fig. 9. CCDF for an 8-subcarriers odd parity coded OFDM system as a function mapper and weightings

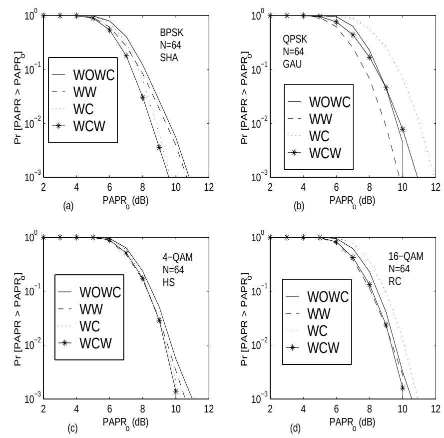

Fig. 10. CCDF for an 64-subcarriers odd parity coded OFDM system as a function mapper and weighting

A. Odd Parity Coding and Weightings

In Figs. 9 and 10 sample CCDFs have been plotted. For example, in Fig.9(a) are shown CCDFs for OFDM with BPSK signal mapper with number of subcarriers equal to 8, odd parity bit coding, and the Bartlett weighting function. Also shown in the same figure are CCDFs for the following cases: i) CCDF without weighting and coding (WOWC); ii) CCDF with only weighting (WW); iii) CCDF with only coding (WC); and iv) CCDF with coding and weighting (WCW). The CCDF of WCW can be compared with other CCDFs in order to estimate the possible improvements. For easy understanding and comparison the legends and abbreviations shown in Fig. 9(a) have been maintained throughout the paper. Consider the plot of WOWC in Fig. 9(a). This refers to OFDM system without weighting and coding. This plot has been arrived at by examining 50,000 random OFDM symbols. It is noted that the highest value of PAPR in these OFDM symbols is nearly 9 dB. The probability that the PAPR exceeds or equal to 9 dB is nearly 10^{-2}. Although, CCDF has been shown as a continuous function of PAPR, there are only finite number of possible values of PAPR. Next, consider the effect of weighting on PAPR by considering the WW plot in the same figure. The highest value of PAPR when Bartlett (TRI) weighting is employed is no greater than 7 dB, which implies an improvement of 2 dB relative to the case when no weighting/coding is employed. When only odd parity coding is employed the highest value of PAPR is slightly greater than 5 dB, which means a gain of nearly 4dB relative to the case when no weighting/coding is employed. Finally when both the coding and weighting are employed the highest value of PAPR turns out to be nearly 6 dB resulting in a gain of 3 dB relative to without coding and weighting case. It is noted that when coding and weighting are jointly employed the relative degradation in PAPR relative to OFDM system with only coding is less than 1 dB.

In Fig. 9(b) CCDFs shown are for OFDM signals with QPSK signal mapper, number of carriers equal to 8 and Gaussian weighting function. For QPSK signal mapper odd parity coding does not really help in improving the PAPR. The improvement in PAPR in coding is less than 0.5 dB. However, with weighting a gain of nearly 3.25 dB can be achieved. The improvement when coding and weighting are simultaneously employed is also nearly 3.25 dB.

In Fig. 9(c) and 9(d) are shown CCDFs for 4-QAM (with Trapezoidal (TRA) weighting) and 16-QAM (with Chebyshev (CHE) weighting) respectively. While in the former case coding deteriorates PAPR, the latter maintains the same performance relative to the cases without weighting and coding. It is noted that in both systems PAPR can be improved by employing coding and weighting. The achievable improvements in both OFDM systems is nearly a dB each.

In Figs. 10(a) to (d) are shown CCDFs for BPSK, QPSK, 4-QAM, and 16-QAM OFDM systems. In the case of QPSK and the number of subcarriers equal to 64 when Gaussian weighting is employed, only marginal improvement can be achieved relative to WOWC. However when only odd parity coding is employed in this OFDM system, PAPR deteriorates by less than a dB. In the OFDM systems illustrated in Figs. 9(a), (c), and (d), PAPR improves when coding and weightings are jointly employed.

All the computations that have been performed for odd parity codings have been summarized in Tables I to VII. In these tables gains in PAPR have been tabulated as a function of the number of subcarriers and signal mappers. The gains are relative to OFDM systems without coding and weightings. Next, we offer important observations based on these tables.

It is observed from Table I that for Bartlett weighting and odd parity coding regardless of the signal mapping, in general, as the number of subcarriers increases in the OFDM system the improvement in PAPR decreases. The best improvement is achieved with number of subcarriers equal to 8. For BPSK signal mapper the best improvement is achieved with number of subcarriers equal to 8. For QPSK, with number of subcarriers equal to 8 the best improvement is achieved with either weighting only or with weighting and coding. Coding really does not improve performance in the case of QPSK. Thus, it is wise to deploy weighting. With 4-QAM and 16-QAM OFDM systems one would be required to employ weighting to reduce PAPR. In general, when an OFDM system is designed coding could be used to combat channel noise, while weighting could be employed to reduce PAPR.

In Table II, investigation of PAPR for odd parity coding and Gaussian weighting function is tabulated. For BPSK mapper, it is possible to employ either coding or Gaussian weighting or both to achieve an improvement in PAPR of nearly 5 dB. When a 16-QAM signal mapping is used, the best improvement in PAPR is achieved by employing jointly weighting and coding. The gain achieved is equal to 3.15 dB. With 4-QAM it is possible employ a larger number of subcarriers and yet achieve improvement in PAPR of nearly 1.5 dB.

In Tables III to VII achievable gains in PAPR for odd parity coding as a function of weighting functions, number of subcarriers and signal mappings are tabulated. Chebyshev and raised cosine weightings are worth considerations, as with these weightings and odd parity coding it is possible to achieve 5 dB and 3 dB improvements with BPSK mapping. As the number of subcarriers increases PAPR deteriorates for 4-QAM and 16-QAM OFDM systems. For large number of subcarriers it is wise to use only weighting to have an advantage in PAPR.

TABLE I

Gains as a function of modulations/subcarriers for Bartlett weighting and odd parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	1.96	4.00	3.00	1.70	0.70	1.70
16	0.40	2.00	3.00	0.75	-0.60	0.60
32	-0.40	1.00	1.40	0.50	-0.60	0.30
64	0.25	1.40	1.65	0.00	-2.00	-1.00
128	0.09	1.00	1.08	0.30	-0.95	0.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.00	-0.75	1.00	1.00	0.00	1.00
16	0.75	0.75	0.75	0.50	0.00	0.50
32	0.50	0.25	0.00	0.30	0.00	0,40
64	0.60	1.10	1.10	0.60	-0.40	0.60
128	0.30	0.20	1.00	0.00	-1.60	1.00

TABLE II

Gains as a function of modulations/subcarriers for Gaussain weighting and odd parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	5.02	5.02	5.02	3.81	0.75	3.83
16	2.10	2.10	4.20	2.60	-0.50	1.82
32	0.25	1.00	3.00	1.42	-0.61	1.42
64	0.20	1.25	2.20	0.21	-1.82	-0.85
128	0.75	1.20	1.25	0.58	-1.21	-0.21
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	3.12	-0.75	3.12	2.25	0.00	3.15
16	2.75	0.72	2.75	1.51	0.00	1.35
32	1.00	0.25	1.00	0.78	0.05	0.95
64	1.45	1.00	1.00	1.05	-0.45	1.61
128	0.54	0.25	1.45	0.95	-1.55	0.95

TABLE III

Gains as a function of modulations/subcarriers for Shannon weighting and odd parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.15	4.21	3.22	1.62	0.61	1.62
16	0.25	2.21	3.22	0.65	-0.55	0.61
32	-0.45	1.15	1.48	0.55	-0.58	0.35
64	0.15	1.20	1.25	0.91	-0.81	0.91
128	0.11	1.21	1.26	0.45	-0.98	0.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.10	-0.75	1.10	1.02	0.00	1.02
16	0.63	0.63	0.63	0.28	0.00	0.48
32	0.41	0.15	0.00	0.31	0.02	0.49
64	0.51	1.05	1.05	0.61	-0.42	0.00
128	0.21	0.21	0.62	0.02	-1.62	1.02

TABLE IV

Gains as a function of modulations/Subcarriers for half-sine weighting and odd parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	1.12	4.05	3.21	0.65	0.65	0.85
16	0.11	2.11	3.15	0.51	-0.52	0.51
32	-0.51	1.12	1.45	0.58	-0.60	-0.32
64	0.05	1.22	1.25	0.00	-1.98	-1.05
128	0.05	1.05	1.21	0.46	-1.10	0.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	0.26	-0.65	0.24	1.00	0.00	1.00
16	0.86	0.86	0.86	0.29	0.00	0.21
32	0.41	0.35	0.00	0.25	0.05	0.41
64	0.48	1.05	1.05	0.18	-0.05	0.21
128	0.00	0.25	0.41	0.05	-1.52	-1.15

TABLE V

Gains as a function of modulations/subcarriers for raised-cosine weighting and odd parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	4.02	3.10	1.75	0.71	1.95
16	0.21	2.10	3.25	1.10	-0.55	0.58
32	-0.38	1.20	1.62	0.72	-0.61	0.38
64	0.05	1.21	1.55	0.00	-1.95	-1.00
128	0.10	1.21	1.21	0.42	-1.25	0.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.25	-0.71	1.25	1.20	0.00	1.10
16	1.41	0.98	1.10	1.00	0.00	1.00
32	1.00	0.21	0.31	0.22	0.05	1.00
64	0.72	1.10	1.10	0.00	-0.45	0.65
128	0.38	0.35	1.00	0.05	-1.55	-0.65

B. Even Parity Coding and Weightings

In Figs. 11 and 12 CCDFs are shown as a function of signal mapping, number of subcarriers, weightings and even parity coding. In Fig. 11(a), is plotted CCDF for BPSK, number of subcarriers equal to 8 and Bartlett weighting. It is noted that even parity coding does not provide any advantage at all. The maximum PAPR with coding and without coding is nearly 9 dB. The probability with which this value occurs is nearly 0.1, which implies that 1 in 10 OFDM blocks have PAPR of 9 dB. Hence, even parity coding is not a wise choice. On the other hand odd parity coding is a good choice (refer Fig. 9(a)) as far as improvement in PAPR is concerned. In the case of QPSK even parity coding, in fact, deteriorates the PAPR performance, as can be observed in Fig. 11(b). A similar observation can be made in respect of 16-QAM, Fig. 11(d). In Fig. 12(a) CCDFs are shown for Shannon weighting and even parity coding. It is noted that, BPSK with number

TABLE VI

Gains as a function of modulations/subcarriers for Chebyshev weighting and odd parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	4.10	5.00	1.72.	0.71	1.72
16	0.31	2.00	3.45	0.51	-0.55	0.51
32	-0.15	1.00	2.00	0.55	-0.60	0.35
64	0.10	1.20	1.50	0.00	-1.75	1.20
128	0.20	1.10	1.10	0.40	-1.00	-1.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.00	-0.85	1.00	1.00	0.00	1.00
16	0.91	0.91	0.91	1.00	0.00	0.48
32	0.35	0.35	1.00	0.45	0.05	1.00
64	1.00	1.00	1.00	0.60	-0.41	0.60
128	0.00	0.42	0.61	0.05	-1.60	-0.85

TABLE VII

Gains as a function of modulations/Subcarriers for trapezoidal weighting and odd parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	4.00	2.00	1.61	0.60	1.61
16	0.00	2.00	3.00	0.60	-0.62	-0.31
32	-0.75	1.15	1.35	0.41	-0.61	-0.58
64	0.00	1.20	1.50	0.00	-1.95	-1.00
128	0.00	1.25	1.35	0.05	-1.00	-0.50
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.00	-0.71	1.00	1.00	0.00	1.00
16	0.80	0.80	1.00	0.00	0.00	0.15
32	0.20	0.25	0.05	0.40	0.00	1.00
64	0.40	1.00	1.00	0.00	-0.40	-0.40
128	0.00	0.30	0.25	0.00	-1.50	-1.00

of subcarriers equal to 16, even parity coding alone can provide an improvement of nearly 1 dB. However, when both coding and weighting are used the gain in PAPR reduces by nearly 0.5 dB. For 4-QAM with Gaussian weighting and even parity coding it is possible to achieve 1.5 dB improvement in PAPR.

In Tables VIII-XIV achievable gains in PAPR have been tabulated, as a function of the number of subcarriers, signal mapping, and weighting functions. For an 8 subcarrier BPSK system, the

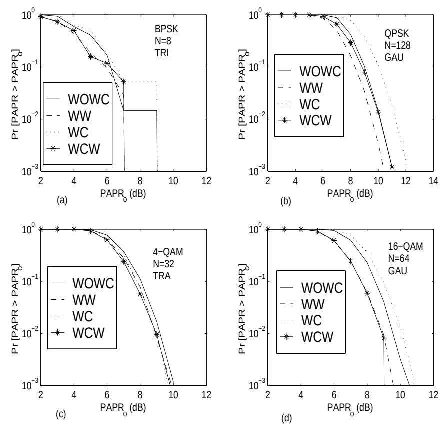

Fig. 11. CCDF for even parity coded OFDM system as a function mapper, subcarriers, and weightings

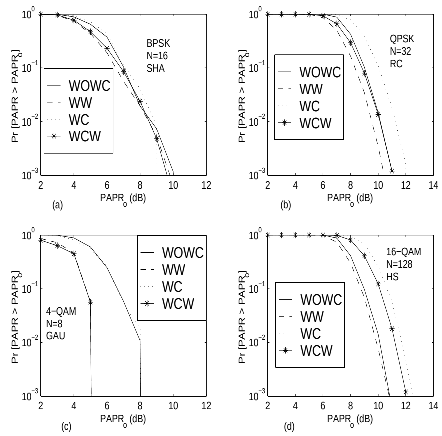

Fig. 12. CCDF for even parity coded OFDM system as a function mapper, subcarriers, and weightings

best weighting function is Gaussian and with this it is possible to achieve nearly 4.2 dB improvement in PAPR. The next best weighting function is that of Shannon followed by raised cosine. For an 8 subcarrier BPSK system, the achievable improvement in PAPR of 3.75 dB is attained when even parity coding and Gaussian weighting is employed. For a QAM signal mapper, the weighting function that provides best improvement in PAPR is again the Gaussian function. With Chebyshev weighting and even parity coding maximum achievable improvement in PAPR is 2 dB.

C. Horizontal/Vertical Parity Coding and Weightings

In Figs. 13 and 14 sample CCDFs for BPSK, QPSK and QAM signal mappers with horizontal/vertical parity coding are plotted for various weightings, such as Bartlett, Gaussian, Shannon,

TABLE VIII

Gains as a function of modulations/subcarriers for Bartlett weighting and even parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.70	0.71	1.70
16	0.40	1.00	1.00	0.71	-0.60	0.58
32	-0.50	0.21	0.52	0.58	-0.80	0.40
64	0.18	0.95	1.20	0.00	-2.00	-0.85
128	0.05	0.85	0.65	0.30	-1.00	0.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.00	0.00	1.00	1.00	0.00	1.00
16	1.00	0.85	1.00	0.40	0.00	0.61
32	0.52	0.42	0.52	0.38	0.10	0.62
64	0.75	0.74	1.15	0.78	-0.50	0.78
128	0.38	1.00	1.00	0.10	-1.80	-1.21

TABLE IX

Gains as a function of modulations/subcarriers for Gaussain weighting and even parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	4.21	0.00	4.21	3.75	0.75	3.75
16	2.00	1.00	2.00	2.61	-0.58	1.62
32	0.21	0.20	0.00	1.41	-0.80	1.41
64	0.20	0.82	1.15	0.20	-1.20	0.61
128	0.75	0.81	0.42	0.61	-0.95	0.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	3.00	0.00	3.00	2.32	0.00	3.10
16	2.78	0.61	2.78	1.51	0.00	1.50
32	1.00	0.35	1.00	1.00	0.15	1.20
64	1.51	0.75	2.15	1.20	-0.52	1.85
128	0.50	1.05	1.05	0.95	-1.45	0.95

TABLE X

Gains as a function of modulations/subcarriers for Shannon weighting and even parity bit coding..

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.10	0.00	2.10	1.72	0.72	1.72
16	0.21	1.00	0.51	0.61	-0.60	0.60
32	-0.52	0.31	1.10	0.60	-0.82	0.40
64	0.15	1.00	1.21	0.00	-2.00	-1.00
128	0.08	0.85	0.61	0.50	-1.00	0.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.00	0.00	1.00	1.00	0.00	1.00
16	0.60	0.60	0.60	0.42	0.00	0.51
32	0.61	0.52	0.21	0.25	0.07	0.50
64	0.52	0.60	1.10	0.52	-0.41	0.00
128	0.32	1.00	1.00	0.05	-1.80	1.20

TABLE XI

Gains as a function of modulations/Subcarriers for half-sine weighting and even parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	1.00	0.00	1.00	0.60	0.60	1.60
16	0.20	1.00	0.40	0.45	-0.45	0.05
32	-0.40	0.10	1.00	0.55	-0.78	-0.21
64	0.00	0.85	1.00	0.00	-2.00	-1.00
128	0.05	0.80	0.65	0.40	-1.10	-3.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	0.26	0.00	0.25	1.00	0.00	1.00
16	0.70	0.70	0.70	0.25	0.00	0.15
32	0.35	0.35	0.00	0.21	0.10	0.51
64	0.40	0.60	1.00	0.00	-0.40	-0.30
128	0.00	1.00	1.00	0.00	-1.50	-1.00

trapezoidal, raised cosine, Chebyshev and half-cycle sinusoid. In Fig. 13(b), QPSK with the number of subcarriers equal to 16 and Gaussian weighting is considered. It is observed that with Gaussian weighting alone it is possible to achieve nearly 1.5 dB improvement in PAPR relative to OFDM system WOWC. When coding is employed this gain reduces by about 0.25 dB only. BPSK with the number of subcarriers equal to 8 with Gaussian weighting provides nearly 4 dB improvement in PAPR relative to OFDM system that does not employ coding and weighting. This is evident from plots in Fig. 14(a). One of the other weighting functions worth considering is raised cosine which when used with QPSK mapper in a 16 subcarrier system provides a gain of nearly 1.75 dB. In Tables XV to XXI achievable gains relative to OFDM systems without coding and weighting are tabulated. Similar observations have been arrived at for horizontal parity coding. When 16-QAM with the number of subcarriers equal to 8 is to be employed, it is wise to employ horizontal/vertical coding instead of even parity coding, as the former pro-

TABLE XII

Gains as a function of modulations/subcarriers for raised-cosine weighting and even parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.70	.70	2.70
16	0.20	1.00	1.00	1.00	-0.60	0.50
32	-0.35	0.15	0.00	0.65	-0.85	0.35
64	0.00	0.91	1.10	0.00	-2.00	-1.00
128	0.05	0.90	0.40	0.40	-1.00	0.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.21	0.00	1.21	1.22	0.00	1.20
16	1.15	0.75	1.00	1.00	0.00	1.00
32	1.00	0.40	1.00	0.25	0.10	1.00
64	0.80	0.75	1.00	0.45	-0.40	0.45
128	0.40	1.00	1.00	0.05	-1.85	-0.65

TABLE XIII

Gains as a function of modulations/subcarriers for Chebyshev weighting and even parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.71	0.71	1.71
16	0.30	1.00	1.10	0.51	-0.62	0.51
32	-0.10	0.30	0.31	0.61	-0.82	0.38
64	0.10	1.00	1.00	1.00	-2.00	1.00
128	0.20	0.80	0.80	0.28	-1.10	-1.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.00	0.00	1.00	1.00	0.00	1.00
16	0.62	0.62	1.21	1.00	0.00	0.51
32	0.21	0.35	0.35	0.45	0.05	1.00
64	1.00	0.85	1.00	0.00	-1.00	0.60
128	0.00	1.00	1.00	0.05	-1.80	-.95

vides an improvement of 3 dB whereas the latter provides only 2.3 dB, when Gaussian weighting is employed. Similarly, 4-QAM , 8-subcarriers and Shannon weighting system is superior by 0.75 dB when horizontal/vertical coding is employed relative to when even parity is used.

TABLE XIV

Gains as a function of modulations/subcarriers for trapezoidal weighting and even parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.71	0.70	1.71
16	0.00	1.00	0.60	0.58	-0.60	0.57
32	-0.60	0.15	0.45	0.40	-0.80	-0.62
64	0.00	0.90	1.00	0.00	-2.00	-1.00
128	0.05	0.85	1.00	0.10	-1.00	-0.60
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.00	0.00	1.00	1.00	0.00	1.00
16	0.72	0.72	0.72	0.00	0.00	0.20
32	0.15	0.40	0.30	0.30	0.15	1.00
64	0.40	0.60	1.00	-0.10	-0.40	-0.35
128	0.00	1.00	1.00	0.02	-1.80	-1.20

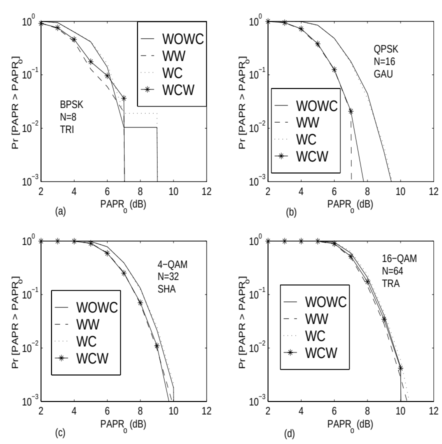

Fig. 13. CCDF for horizontal/vertical parity coded OFDM system as a function mapper, subcarriers, and weightings

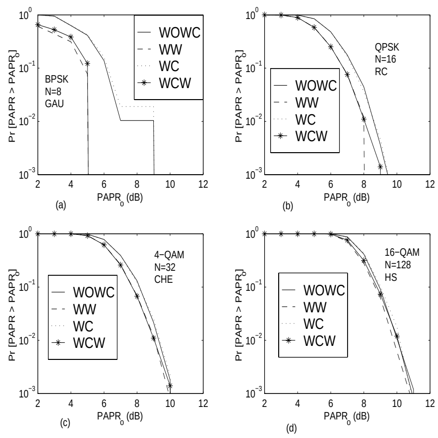

Fig. 14. CCDF for horizontal/vertical parity coded OFDM system as a function mapper, subcarriers, and weightings

TABLE XV

Gains as a function of modulations/subcarriers for Bartlett weighting and horizontal/vertical parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.80	-0.05	1.80
16	0.30	0.30	0.80	1.10	-0.05	0.60
32	0.40	0.00	0.21	0.20	-0.30	0.00
64	0.20	0.00	0.00	0.41	1.10	0.71
128	0.15	-0.15	0.65	0.40	0.60	0.80
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	2.00	0.10	2.00	1.00	0.00	1.00
16	0.70	-0.50	0.60	0.22	0.00	1.00
32	0.20	0.00	0.50	1.00	0.00	0.45
64	0.10	-0.50	0.20	0.00	-0.52	0.00
128	0.15	0.00	0.21	0.42	0.20	0.21

TABLE XVI

Gains as a function of modulations/subcarriers for Gaussain weighting and horizontal/vertical parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	4.00	0.00	4.00	3.80	-0.10	3.80
16	1.85	0.20	1.85	1.40	-0.10	1.20
32	0.60	0.00	0.00	1.00	-0.25	0.90
64	0.30	0.00	0.00	1.20	1.00	0.80
128	0.60	-0.15	0.25	1.00	0.60	0.60
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	3.75	0.10	3.75	3.00	0.00	3.00
16	1.65	-0.45	1.80	2.00	0.00	2.00
32	1.00	0.00	1.00	1.00	0.00	1.00
64	0.60	-0.60	0.80	0.45	-0.50	0.00
128	0.40	0.00	0.65	1.10	0.21	0.51

TABLE XVII

Gains as a function of modulations/subcarriers for Shannon weighting and horizontal/vertical parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.55	0.00	1.55
16	0.82	0.82	0.82	0.75	0.00	0.50
32	0.20	0.00	0.15	0.20	-0.20	0.00
64	0.00	0.00	0.00	0.20	1.00	0.70
128	0.00	-0.10	0.60	0.25	0.40	0.75
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.75	0.10	1.75	1.00	0.00	1.00
16	0.58	-0.50	0.30	0.22	0.00	0.51
32	0.15	0.00	0.30	1.00	0.00	0.25
64	0.15	-0.60	0.20	0.00	-0.60	0.00
128	0.00	0.00	0.20	0.45	0.20	0.15

TABLE XVIII

Gains as a function of modulations/subcarriers for half-sine weighting and horizontal/vertical parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	1.00	0.00	1.00	0.65	0.00	0.75
16	0.05	0.20	0.05	0.50	0.00	0.50
32	0.35	0.00	0.35	0.20	-0.25	0.00
64	0.00	0.00	0.00	0.20	1.10	0.85
128	-0.10	-0.10	0.60	0.30	0.60	1.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	0.75	0.10	0.75	1.00	0.00	1.00
16	0.60	-0.40	0.20	0.00	0.00	0.25
32	0.00	0.00	0.20	1.00	0.00	0.21
64	0.00	0.10	-0.60	0.00	0.00	-0.60
128	0.00	0.00	0.15	0.40	0.22	0.18

TABLE XIX

Gains as a function of modulations/subcarriers for raised-cosine weighting and horizontal/vertical parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.75	0.00	1.80
16	0.25	0.25	0.40	1.50	0.00	0.45
32	0.50	0.00	0.00	0.30	-0.20	0.25
64	0.10	0.00	0.00	0.50	1.00	0.52
128	0.15	-0.10	0.50	0.35	0.60	1.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.80	0.15	1.80	1.00	0.00	1.25
16	0.65	-0.35	0.60	1.00	0.00	1.00
32	0.20	0.00	0.40	1.00	0.00	0.51
64	0.50	-0.60	0.20	0.00	-0.51	0.00
128	0.15	0.00	0.30	0.45	0.21	0.42

TABLE XX

Gains as a function of modulations/subcarriers for Chebyshev weighting and horizontal/vertical parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.75	0.00	1.75
16	0.30	0.30	0.30	0.50	0.00	0.50
32	-0.40	0.00	0.20	0.45	-0.25	0.00
64	0.15	0.00	0.20	0.50	1.00	1.00
128	-0.10	-0.10	0.00	0.60	0.60	0.95
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.75	0.00	1.75	1.00	0.00	1.00
16	0.80	-0.55	0.60	1.00	0.00	0.41
32	0.10	0.00	0.00	0.35	0.00	0.40
64	0.60	-0.60	0.20	-0.21	-0.45	0.00
128	0.41	0.05	0.15	0.61	0.18	0.15

TABLE XXI

Gains as a function of modulations/subcarriers for trapezoidal weighting and horizontal/vertical parity bit coding.

N	Gain (dB)					
	BPSK			QPSK		
	WW	WC	WCW	WW	WC	WCW
8	2.00	0.00	2.00	1.75	0.00	1.75
16	0.00	0.00	0.20	0.50	0.00	0.00
32	0.20	0.00	0.00	-0.40	0.00	0.00
64	0.00	0.00	0.00	0.20	1.00	0.60
128	-0.15	-0.15	0.51	0.30	0.40	1.00
	4-QAM			16-QAM		
N	WW	WC	WCW	WW	WC	WCW
8	1.75	0.00	1.75	1.00	0.00	1.00
16	0.40	-0.60	0.20	0.00	0.00	0.00
32	0.15	0.00	0.30	1.00	0.00	0.10
64	0.05	-0.60	0.05	-0.35	-0.51	0.00
128	0.00	0.00	0.00	0.00	0.21	0.10

VI. Conclusions

In this paper we have considered the problem of reduction of PAPR in OFDM systems by jointly employing coding and weighting. The coding schemes that we have considered are simple and easy to implement. Several weighting functions have been considered of which Chebyshev and trapezoidal are new in the context of PAPR reduction in OFDM systems.

Thorough extensive numerical simulations we have determined the achievable gains in PAPR as a function of number of sub-carriers, signal mapping, and weighting function. For example, it is observed that for an 8 subcarrier OFDM-PSK system it is possible to achieve nearly 4.2 dB improvement in PAPR by employing Gaussian weighting. In general Gaussian weighting function provides the best improvements in PAPR relative to an OFDM system without any coding and weighting. While odd and even parity codings have the same distance properties, it is noted that odd parity coding can be gainfully employed for superior PAPR performance. Chebyshev, trapezoidal and Shannon weightings provide PAPR improvements of the order of 2 dB. It is noted that with the BPSK signal mapping, improvement from joint use of coding and weighting can be the order of 3 dB. We conclude that the best way to keep PAPR at a certain level and at the same time better the error rate performance is to jointly use coding and weighting in OFDM systems.

It is worthwhile considering coding techniques such as block codes, convolutional codes, turbo codes etc. jointly with weighting in order to obtain a detailed picture of the effects on PAPR. Also the problem is theoretically yet to be modelled to obtain analytical bounds on PAPR.

References

[1] G. L. Stuber, "Principles of mobile communication," Second Edition, Kluwer Academic Publishers, pp. 329–330, 2001.

[2] R. V. Nee and R. Prasad, "OFDM for wireless multimedia communications," Norwood, MA: Artech House Publishers, January 2000.

[3] ETSI, "Digital video broadcasting: Framing structure, channel coding, and modulation of digital terrestrial television," EN, pp. 300–744, August 1997.

[4] ETSI, "Radio broadcasting systems: digital audio broadcasting (DAB) to mobile, portable and fixed receivers," ETS, vol. ed.2, pp. 300–401, May 1997.

[5] J. A. C. Bingham, "ADSL, VDSL, and multicarrier modulation," New York: Wiley, 2000.

[6] P. S. Chow, J. C. Tu, and J. M. Cioffi, "Performence evaluations of a multichannel transceiver system for ADSL and VHDSL services," IEEE J. Sel. Areas in Communications, pp. 909–919, August 1991.

[7] H. Sari and G. Karam, "Orthogonal frequency-division multiple access and its application to CATV network," European Trans. Telecomm., vol. 9, no. 6, pp. 507–516, Nov.-Dec. 1998.

[8] IEEE 802.11, "IEEE standard for wireless LAN medium access control (MAC) and physical layer (PHY) specifications," Nov. 1997.

[9] ETSI, "Radio equipments and systems, high performence radio local area network (HIPERLAN) type I," ETS, pp. 300–652, Oct. 1996.

[10] L. J. Cimini Jr. and N. R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequence," IEEE Communications Letters, vol. 4, no. 3, pp. 86–87, March 2000.

[11] X. Li and Jr. L. J. Cimini, "Effects of clipping and filtering on the performence of OFDM," IEEE Communications Letters, vol. 2, no. 5, pp. 131–133, May 1998.

[12] H. Nikookar and R. Prasad, "Weighted OFDM for wireless multipath channels," IEICE TRANS. COMMUNICATIONS, vol. E83-B, no. 8, pp. 1864–1872, August 2000.

[13] I.A. Tasadduq and R.K. Rao, "Weighted OFDM with block codes for wireless communication," Proc. 2001 IEEE Pacific Rim Conf. on Commun., Computers, and Signal Proc. (PACRIM 2001),Victoria, B.C., 2001.

[14] P. Van Eetvelt, G. Wade, and M. Tomlinson, "Peak-to-average power reduction for OFDM schemes by selective scrambling," Electronics Letters, vol. 32, no. 21, pp. 1963–1964, October 1996.

[15] R. V. Nee and A. de Wild, "Reducing the peak-to-average power ratio of OFDM," IEEE Vehicular Technology Conference, vol. 3, pp. 2072–2076, 1998.

[16] T. May and H. Rohling, "Reducing the peak-to-average power ratio in OFDM Radio transmission systems," 3, vol. 3, pp. 2474–2478, 1998.

[17] V. Tarokh and H. Jafarkhani, "An algorithm for reducing the peak-to-average power ratio in a multicarrier communications system," IEEE Vehicular Technology Conference, vol. 1, pp. 680–684, 1999.

[18] M. Friese, "Multicarrier modulation with low peak-to-average power ratio," Electronics Letters, vol. 32, no. 8, pp. 713–714, 11th April 1996.

[19] R. W. Baumal, R. F. H. Fisher, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," Electronics Letters, vol. 32, no. 22, pp. 2056–2057, 24th October 1996.

[20] S.H. Muller and J.B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequence," Electronics Letters, vol. 33, no. 5, pp. 368–369, 27th February 1997.

[21] C. Tellambura, "Phase optimisation criterion for reducing peak-to-average power ratio in OFDM," Electronics Letters, vol. 34, no. 2, pp. 169–170, 22nd January 1998.

[22] A. D. S. Jayalath and C. Tellambura, "Reducing peak-to-average power ratio of orthogonal frequency division multiplexing signal through bit or symbol interleaving," Electronics Letters, vol. 36, no. 13, pp. 1161–1163, 22nd June 2000.

[23] G. R. Hill, M. Faulkner, and J. Singh, "Reducing peak-to-average power ratio in OFDM by cyclically shifting partial transmit sequences," Electronics Letters, vol. 36, no. 6, pp. 560–561, 16th March 2000.

[24] B. Sklar, "Digital communications: Fundamentals & applications," 2nd Edition, PHI, NJ, USA, 2001.

[25] J. G. Proakis, "Digital communications," McGraw-Hill, NY, 2001.