OPTIMUM NONCOHERENT DIGITAL MULTI-h SIGNALING

K. R. Raveendra
CASP Laboratory
E & C E Department
Delhi Institute of Technology
Delhi 110 006 INDIA

S. Thakur
CASP Laboratory
E & C E Department
Delhi Institute of Technology
Delhi 110 006 INDIA

A. K. Sinha
E & C Department
D. C. E., Kashmere Gate
Delhi 110 006
INDIA

ABSTRACT

The problem of noncoherent detection of multi-h CPM signals received over AWGN channel is considered. Receiver structures for slow- and fast-fading cases are presented. Employing the well known union and Average Matched Filter (AMF) bounds, error rate performance of the optimum noncoherent receiver is estimated. Through minimization of theses bounds optimum multi-h schemes are determined. The results reveal that there seems to be no apparent advantage in power in going for multi-h schemes relative to single-h schemes for the case of noncoherent detection.

1. INTRODUCTION

Digital transmission employing Continuous Phase-Modulated (CPM) signals is an important signaling technique that find applications over high speed digital mobile radio links [1-3]. In recent years several constructions of CPM have been found which offer significant bandwidth and power savings [4-5] relative to more conventional digital signaling that are in vogue. Some of the well known constructions of CPM are MSK, CPFSK, CORPSK etc., including M-ary versions. Among the class of CPM signals multi-h phase coded signals is a subclass of signals that have attractive spectral and power saving properties [6-10]. In multi-h coding time-varying modulation indices are employed in a cyclic fashion in order to achieve impressive tradeoff in power and bandwidth. In [6], some specific multi-h coded CPM constructions, together with Viterbi Algorithm (VA) decoder, were shown to permit transmitter savings of 2-4 dB over binary PSK in narrower bandwidth. Subsequently [7-10], detailed analyses of various types of multi-h signals have been carried our for their power and spectral properties. All analyses, however, are for coherent demodulation of multi-h signals over AWGN channel.

In order to meet the stringent requirements in the utilization of bandwidth and yet reliably transmit data over mobile communication channels [13], it is important to look for digital modulation techniques that are power as well as bandwidth efficient. Thus, over such channels multi-h signaling is likely to be employed. Furthermore, over mobile radio channels signal fading is a major problem accompanied with difficulties in maintaining phase synchronization. It is, therefore, the objective of the paper wherein we consider the problem of optimum noncoherent detection of multi-h CPM signals buried in AWGN. In the process we arrive at optimum multi-h schemes and estimate the associated error rates. Two receiver structures for slow- and fast-fading cases are also presented for detection of multi-h CPM signals. In the light of the results presented the tradeoff available between power and bandwidth is discussed. Throughout the paper we consider several important input data pulse shapings.

The paper is organized as follows: In Section 2 the multi-h CPM signaling is briefly discussed. In Section 3 the receiver structures are presented. The error probability analysis in terms of bounds is presented in Section 4. The optimum multi-h schemes and a discussion about their performance is given in Section 5. The paper is concluded in Section 6.

2. MULTI-h SIGNALING FORMAT

The general expression for a binary multi-h CPM signal over an n-bit interval is given by

$$S(t,\tilde{a}) = \sqrt{2S}\,\mathrm{Cos}(2\pi f_c t + \phi(t,\tilde{a}) + \phi_o) \qquad 0 \le t \le nT, \qquad (1)$$

where

$$\phi(t,\tilde{a}) = 2\pi \int_0^t \sum_{i=1}^{n} h_{(i)} a_i g(\tau-(i-1)T)d\tau \qquad 0 \le t \le nT \qquad (2)$$

and $\tilde{a} = (a_1, a_2, \ldots, a_n)$ is an n-bit equally likely binary sequence with S the signal power per bit interval and f_c the carrier frequency. The modulation index employed for the ith bit interval is denoted by $h_{(i)}$, and ϕ_o is the starting phase at the beginning of the observation interval. In eqn. 2 $g(\tau)$ is the frequency pulse lasting T seconds. Defining the baseband phase function by

$$q(t) = \int_0^t g(\tau)d\tau \quad 0 \le t \le nT \qquad (3)$$

the information carrying phase in eqn. 2 can be written as

$$\phi(t,\tilde{a}) = 2\pi \sum_{i=1}^{n} h_{(i)} a_i q(\tau-(i-1)T)d\tau \qquad 0 \le t \le nT \quad (4)$$

As examples of phase functions we cite the linear phase (LP) case, half-cycle sinusoid (HCS) and raised-cosine (RC) pulses:

$$q(t) = \begin{cases} 0, & t \le 0; \text{ LP, HCS, RC} \\ t/2T, & 0 < t \le T; \text{ LP} \\ 1/4\,(1-\mathrm{Cos}\pi t/T), & 0 < t \le T; \text{ HCS} \quad (5) \\ 1/2(\frac{t}{T} - \frac{1}{2\pi}\,\mathrm{Sin}2\pi\frac{t}{T}), & 0 \le t \le T \\ 1/2, & t > T; \text{ LP, HCS, RC} \end{cases}$$

In the LP case, the frequency is held constant throughout the data interval, whereas with HCS and RC the instantaneous frequency varies smoothly. While in standard digital FM $h_{(i)}$, $i=1,2,\ldots,n,\ldots$, is maintained constant, in multi-h CPM $h_{(i)}$ is chosen cyclically from a set of K modulation indices $\{h_1,h_2,\ldots,h_K\}$; i.e. $h_{(i+K)} = h_{(i)}$, $i=1,2,\ldots$.

When the modulation indices are restricted to be ratios of small integers having common denominator $h_i = l_i/D$, $i=1,2,\ldots,K$, the phase trellis associated with $\{h_i\}$ is composed only of transitions between 2D phase values $n\pi/D$, $n=0,1,\ldots,2D-1$. By choosing K modulation indices of which no two subsets have the same sum modulo 1, it is possible to achieve a maximum possible constraint length for a given K.

Binary multi-h CPFSK is a subclass of binary multi-h CPM defined by eqn. 1, for which the phase function is given by eqn. 5, which corresponds to having linear phase trajectories over each bit interval. For HCS and RC pulse functions the phase trajectories are smoothed over each bit interval. The phase over a given bit interval must change slowly for faster spectral roll-off.

3. RECEIVER STRUCTURES

In this Section we derive multiple-bit observation receivers for detecting binary CPM signals subject to slow- and fast-fading in AWGN. The slow-fading refers to the case wherein the amplitude and phase of the received signal are random but constant over the entire decision interval. In the fast-fading case the amplitude and phase are random but over sub-intervals of the decision interval. In both situations, the detection strategy is to observe the received signal over n bit intervals and to produce an estimate of a specific data bit transmitted a_δ, $1 \le \delta \le n$. It is noted that the derivation of the receiver structures is independent of the choice of the decision bit location.

3.1 Slow-fading case:

The detection problem at hand is a composite hypothesis statistical test which may be stated as

$$H_1: r(t) = b\,\mathrm{Cos}(2\pi f_c t + \phi(t, a_\delta = +1, A_k) + \theta) + n(t)$$
$$H_2: r(t) = b\,\mathrm{Cos}(2\pi f_c t + \phi(t, a_\delta = -1, A_k) + \theta) + n(t) \qquad (6)$$

where $0 \le t \le nT$, $j=1,2,\ldots,2^{n-1}$, b and θ are composite parameters with the latter uniformly distributed in $(-\pi,+\pi)$ and the former having an arbitrary fading distribution. Further, it is assumed that b and θ are independent. A_j is the (n-1)-tuple $(a_1,a_2,\ldots,a_{\delta-1},a_{\delta+1},\ldots,a_n)$ and represents another composite parameter whose distribution is easily determined. Setting up the likelihood ratio test (LRT) and simplifying it we get the receiver structure shown in Fig.1. The receiver structure is same as that derived in [14] for binary CPFSK. For the case of multi-h CPM the only modification is that the receiver has the implicit knowledge of the sequence in which the modulation indices are employed at the transmitter.

3.2 Fast-fading case:

The hypothesis testing problem for this case may be stated as:

$$H_1: r(t) = \sum_{i=1}^{n} b_i S(t, a_\delta = +1, A_j, \theta_i) + n(t)$$
$$H_2: r(t) = \sum_{i=1}^{n} b_i S(t, a_\delta = -1, A_j, \theta_i) + n(t) \qquad (7)$$

where $b_i S(t, a_\delta = \pm 1, A_j, \theta_i)$ is the signal waveform received during the ith bit interval, b_i and θ_i are random amplitude and phase of the ith bit signal waveform. These are assumed to be independent of each other. Furthermore, it is assumed that fading is independent from bit to bit. Employing similar steps used for slow-fading case, the receiver structure obtained is shown in Fig.2.

4. PERFORMANCE ANALYSIS OF THE NONCOHERENT (SLOW-FADING) RECEIVER

The error probability analysis of the receiver shown in Fig. 1 can be carried out in a similar manner [11,14] by using the union (upper and lower) and Average Matched Filter (AMF) receiver bounds. The high SNR suboptimum receiver is shown in Fig.1 itself and the low-SNR AMF receiver, both for multi-h signals, is shown in Fig.3. The performance of the high-SNR suboptimum receiver may be bounded using the union bound. The upper bound is given by:

$$P_e \le \sum_{p=1}^{K} \sum_{i=1}^{m} \sum_{j=1}^{m} 0.5[1 - Q_M(\sqrt{x_+}, \sqrt{x_-}) + Q_M(\sqrt{x_-}, \sqrt{x_+})] \quad (8)$$

where

$x_\mp = nE_b/N_o\,[1 \mp \sqrt{1 - |\rho_c^P(i,j)|^2}]$ and $\rho_c^P(i,j)$ is the normalized complex correlation between the envelopes of complex signals $S_c^p(t, a_\delta=+1, A_i)$ and $S_c^p(t, a_\delta=-1, A_j)$. Each of these signals is given by

$$S_c^p(t, a_\delta, A_i) = S^p(t, a_\delta, A_i, 0) + j\ S^p(t, a_\delta, A_i, \pi/2)$$

The superfix p has been used in the above eqns. to take into account all possible sequences of modulation indices over the observation interval.

The lower bound on the performance of the optimum receiver, at high values of SNR, may be determined by using the expression:

$$P_e \geq (Km)^{-1} \sum_{p=1}^{K} \sum_{i=1}^{m} 0.5[1 - Q_M(\sqrt{x_+}, \sqrt{x_-}) + Q_M(\sqrt{x_-}, \sqrt{x_+})] \tag{9}$$

where in $x_\mp$ the correlation used is given by:

$$|\rho_c^p|^* = \max_j \{|\rho_c^p(i, j)|\} \tag{10}$$

At low values of SNR the performance is upper bounded and is given by:

$$P_e = (Km)^{-1} \sum_{p=1}^{K} \sum_{j=1}^{m} 0.5[1 - Q_M(\sqrt{b_+}, \sqrt{b_-}) + Q_M(\sqrt{b_-}, \sqrt{b_+})] \tag{11}$$

$$b_\mp = \frac{1}{2\sigma^2} \left[\frac{|\mu^p_{+1j}|^2 + |\mu^p_{-1j}|^2 - 2\mathrm{Re}(\mu^p_{+1j}\mu^p_{-1j}\rho^p)}{1 - |\rho^p|^2} \mp \frac{|\mu^p_{+1j}|^2 - |\mu^p_{-1j}|^2}{\sqrt{1 - |\rho^p|^2}} \right] \tag{12}$$

where

$$\mu^p_{\pm 1j} = (2S)^{1/2} \int_0^{nT} \mathrm{Cos}(2\pi f_c t + \phi^p(t, a_\delta = \pm 1, A_j)) \times \bar{S}_c(t, a_\delta = \pm 1)\ dt \tag{13}$$

$$\sigma^2 = 0.5 N_o \int_0^{nT} |\bar{S}_c(t, a_\delta = \pm 1)|^2 dt \tag{14}$$

$$\rho^p \sigma^2 = 0.5 N_o \int_0^{nT} |\bar{S}_c(t, a_\delta = +1)|\,|\bar{S}_c^*(t, a_\delta = -1)|\, dt \tag{15}$$

with

$$\bar{S}_c(t, a_\delta = \pm 1) = \sum_{k=1}^{m} S_c(t, a_\delta = \pm 1, A_k) \tag{16}$$

In [15] highly simplified and very easy to compute expressions have been obtained for evaluation of eqns. 8-16, for the signaling schemes described in Section 2.

5. NUMERICAL RESULTS & DISCUSSION

The error rate bounds on the performance of the optimum noncoherent receiver for multi-h CPM are functions of: i) Signal-to-Noise Ratio, E_b/N_o; ii) the number of observation intervals, n; iii) the signal parameter set $\{h_i; i=1,2,\ldots,K\}$; iv) the phase function q(t); and v) the location of the decision bit δ. The optimum $\{h_i\}$s that minimize the error rate bounds, high- and low-SNR upper bounds, have been determined [15] as a function of E_b/N_o, n, and δ for REC, HCS, and RC phase functions. The modulation parameter space has been chosen to be $0 < h_i \leq 1$. For all three phase functions, for n=2,3,4 and $1 \leq \delta \leq n$, the sets $\{h_1, h_2\}$, $\{h_1, h_2, h_3\}$, and $\{h_1, h_2, h_3, h_4\}$ that minimize the high-SNR upper bound turn out to be those with $h_1=h_2$, $h_1=h_2=h_3$, $h_1=h_2=h_3=h_4$. On the other hand, for $n \geq 5$, for optimum decision decision bit location [δ = int(n/2)+1, for n odd and δ=(n/2) or (n/2)+1, for n even] we find that the modulation indices in the optimum sets are different. An investigation of the variation of optimum modulation indices as a function of SNR reveals that the variation is less than about 12% over the range $6 \leq SNR \leq 15$ dB. For high SNR (error rates less than 10^{-6}) the upper and lower bounds are essentially the same. For 5T observation length, the optimum REC, HCS, and RC multi-h systems are {0.68, 0.73}, {0.57, 0.72}, and {0.55, 0.71}. These systems outperform coherent PSK by nearly 0.8 dB. In Figs. 4, 5, and 6 these results are illustrated.

The performance of the AMF receiver has been analyzed in the same fashion using the low-SNR upper bound. The optimum performance of this receiver is insensitive to the use of multi-modulation indices. For n=5 and δ=3, the optimum modulation indices are 0.75, 0.66, and 0.63 for REC, HCS, and RC systems, respectively. In fact this behavior can be analytically observed [15]. In Fig. 7, the performance of AMF receiver for RC systems with 5T observation length is shown.

6. CONCLUSIONS

In the paper optimum noncoherent multi-h CPM receiver for slow- and fast-fading cases have been determined. The performance of the optimum noncoherent receiver has been determined via high- and low-SNR bounds. Optimum noncoherent multi-h schemes have been determined. There seems no significant advantage, at least in terms of SNR gain, in the use of multi-modulation indices compared to the use of single modulation index in CPM systems.

REFERENCES

[1] C.E. Sundberg, "On continuous phase modulation in cellular digital mobile radio systems," Bell Syst. Tech. Jour., Vol.62, No. 7, 1983, pp. 2067-2097.

[2] M.K. Simon and C.C. Wang, "Differential detection of Gaussian MSK in mobile radio environment," IEEE Trans. on Vehicular Technology, Vol VT-33, No.4, 1984, pp.307-320.

[3] S. Asakawa and F. Sugiyama, "A compact spectrum constant envelope digital phase modulation," IEEE Trans. on Vehicular Technology, Vol. VT-30, No.3, 1983, pp.102-111.

[4] C.E. Sundberg, "Continuous Phase

Modulation," IEEE Commun. Mag., Vol. 24, No.4, 1986, pp. 25-38.
[5] T.Aulin, "Three papers on continuous phase modulation," Ph.D dissertation, Telecommunication Theory Dept., University of Lund, Sweden, 1979.
[6] J.B.Anderson and D.P.Taylor "A bandwidth-efficient class of signal space codes," IEEE Trans. on Inform. Theory, Vol. IT-24, No.6, 1978, pp. 703-712.
[7] T.Aulin and C.E.Sundberg, "On the minimum Euclidean distance for a class of signal space codes," IEEE Trans., on Inform. Theory, Vol. IT-28, No. 1, 1982, pp.43-45.
[8] K.R.Raveendra and R.Srinivasan, "Coherent detection of binary multi-h CPM," Proc. IEE, Pt. F, 131, (4), 1987, pp. 416-426.
[9] L.F.Lind and A.A.deAlbuquerque, "Special calculation of partial response multi-h phase codes," Electronics Letters, Vol. 17, 1981, pp. 713-717.
[10] D.P.Taylor and H.C.Chan, "A simulation study of two bandwidth-efficient modulation techniques," IEEE Trans. on Commun., Vol. COM-29, No.3, 1981, pp. 267-275.
[11] K.R.Raveendra and R.Srinivasan, "Noncoherent detection of binary multi-h CPM," Proc. Int. Symp. on Elec. Dev., Cir.,and Systems, Indian Institute of Tech., Kharagpur, India, 16-18 December, 1987, pp. 238-240.
[12] R.F.Pawula and R.Golden, "Simulations of convolutional coding/Viterbi Decoding with noncoherent CPFSK," IEEE Trans. on Commun., Vol. COM-29, No. 10, 1981, pp.1522-1526.
[13] W.C.Y.Lee, Mobile Communications Engineering, McGraw-Hill: NY, 1982.
[14] W.P.Osborne and M.B.Luntz, "Coherent and noncoherent detection of CPFSK," IEEE Trans, on Coomun., COM-22, 1974, pp.1023-1036.
[15] S.Thakur, Receivers and Performances for noncoherent digital multi-h signals, M.E.Dissertation, Faculty of Technology, University of Delhi, 1991.

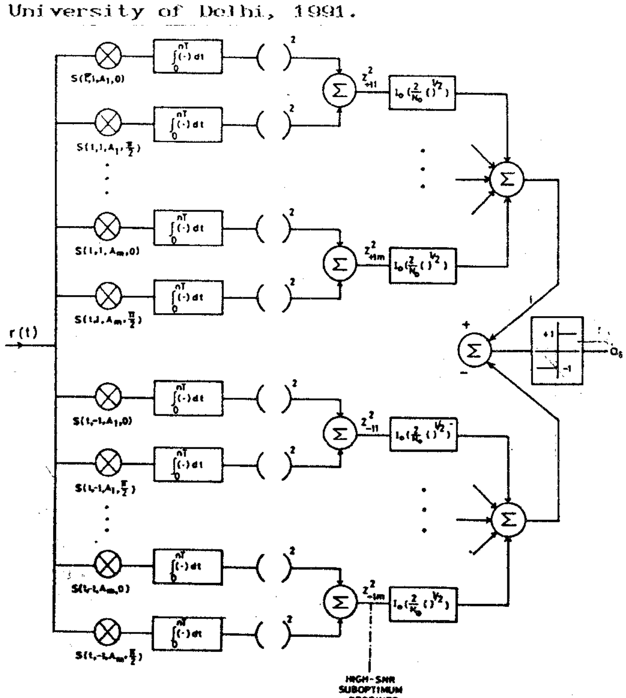

FIG. 1 : OPTIMUM AND HIGH-SNR SUBOPTIMUM NONCOHERENT STRUCTURES

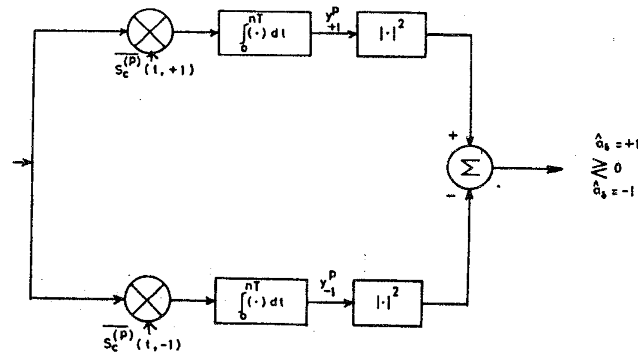

FIG.3 NONCOHERENT AVERAGE MATCHED FILTER RECEIVER FOR MULTI-h CPM

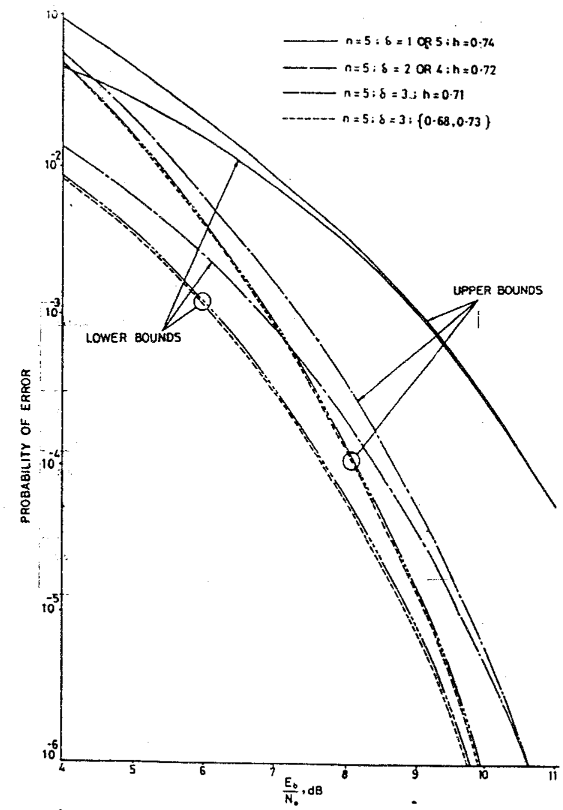

FIG. 4 : ERROR PROBABILITY BOUNDS FOR OPTIMUM NONCOHERENT MULTI-h CPM (REC) RECEIVER (n= 5).

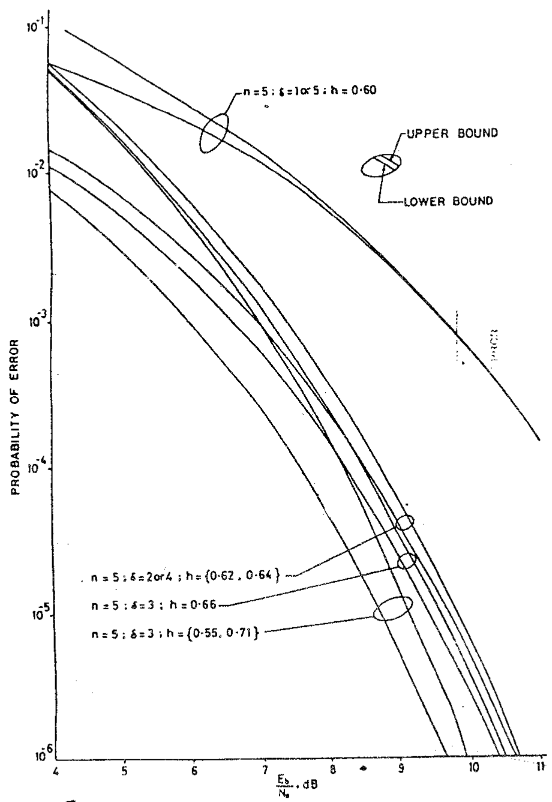

FIG. 5 : ERROR PROBABILITY BOUNDS FOR OPTIMUM NONCOHERENT MULTI-h CPM (RC) RECEIVER (n = 5)

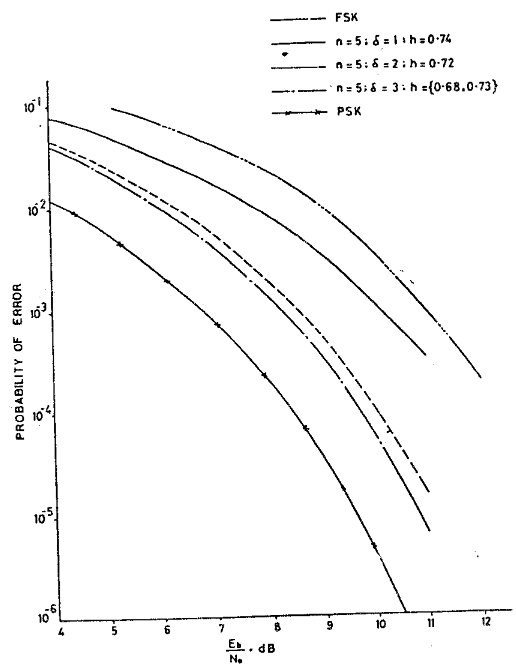

FIG. 7 : ERROR RATES FOR NONCOHERENT AMF-MULTI-h CPM (RC) RECEIVER (n = 5)

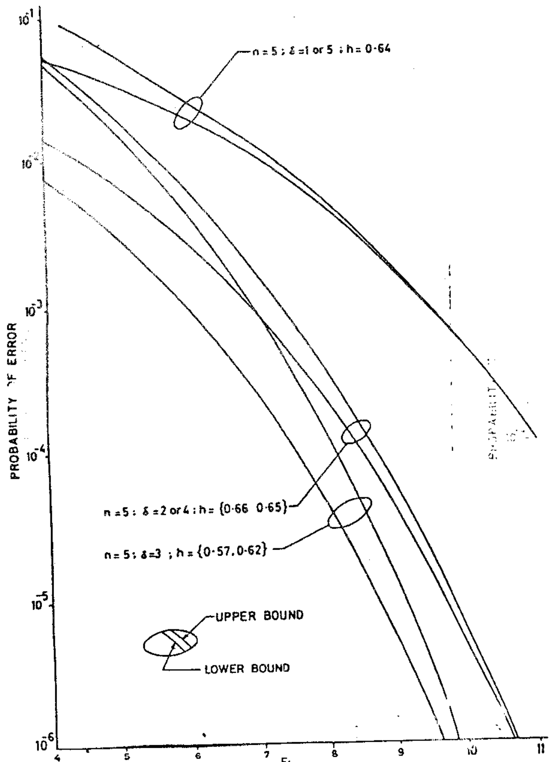

FIG. 6 ERROR PROBABILITY BOUNDS FOR OPTIMUM NONCOHERENT MULTI-h CPM (HCS) RECEIVER (n = 5)