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Performance of OFDM-CPM Signals Over Wireless Fading Channels

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ABSTRACT

A class of Orthogonal Frequency Division Multiplexing - Continuous Phase Modulation (OFDM-CPM) signals is introduced in which binary data sequence is mapped to complex symbols using the concept of correlated phase states of a CPM signal. Multiple-symbol-observation receiver is used to decode the received sequence and an investigation of Bit-Error-Rate (BER) over typical wireless multipath channels with AWGN is presented. The performance of a variety of OFDM-CPM signals is presented and analysed. It is a function of parameter h and observation interval, both of which are at the disposal of system designer. It is shown that OFDM-CPM is a promising signaling scheme in multipath fading channels. Results for multi- h and asymmetric OFDM-CPM signals are also presented.

KEY WORDS

OFDM, CPM, BER, Fading Channels

1. Introduction

OFDM is a good candidate for wireless multimedia communication by virtue of its excellent properties in frequency-selective fading environment [1, 2]. In OFDM, we transmit data over several parallel low data rate channels. This provides data integrity due to fading, relative to methods that employ single channel for high data rate transmission. Among other benefits of OFDM is that it fully exploits the advantages of digital signal processing concepts [3].

A typical OFDM transmitter is shown in Fig. 1(a) and works as follows. A serial-to-parallel converter serially takes in the data stream and forms a parallel stream which is then sent to a mapper that outputs complex numbers. The mapper could be PSK, QAM, DPSK or DAPSK. Inverse Fast Fourier Transform (IFFT) is then applied to the parallel stream of complex numbers that results in orthogonal signals on the subchannels. In order to mitigate the effects of ISI, a guard interval is inserted at the transmitter that is later removed at the receiver. The orthogonal signals are then converted back into a serial stream and af-

ter up converting the signal to desired carrier frequency the signal is transmitted.

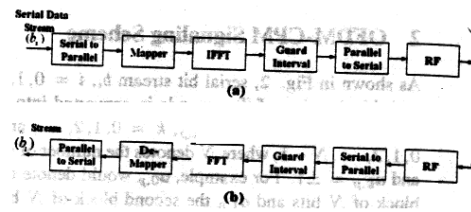


Figure 1. Typical OFDM transmitter and receiver

At the receiver (Fig. 1(b)), the process described above is reversed. It starts with down converting the received signal. The resulting signal is then passed through a serial-to-parallel converter, a guard interval remover, an FFT block, a de-mapper and finally a parallel-to-serial converter to eventually obtain the transmitted data sequence. In the absence of noise and fading, transmitted data is recovered without errors.

While in the literature OFDM-PSK, -QAM, -DPSK and -DAPSK have been considered [4]-[7], OFDM-CPM signals that use the concept of correlated phase states of a CPM signal has recently been introduced [8, 9]. One of the advantages of OFDM-CPM signals is that we can systematically introduce correlation amongst adjacent OFDM symbols by an appropriate choice of parameter h (in typical CPM signals h is modulation index). Furthermore, this correlation can be exploited in order to reduce the BER in such a system. In this paper, we analyse the BER performance of OFDM-CPM signals using multiple-symbol-observation receiver. In such a receiver, we observe n symbols and arrive at an optimum decision on one of the symbols. The channel is modeled as multipath and AWGN.

The paper is organized as follows. In Section 2 we introduce OFDM-CPM signaling scheme. The multiple-symbol-observation receiver is described in Section 3. In Section 4 we present BER simulation results over fading

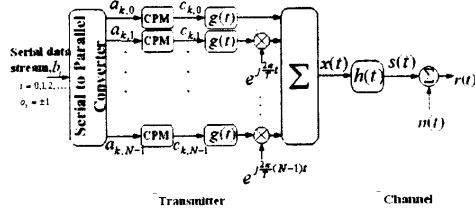


Figure 2. OFDM-CPM Transmitter and Channel

channels for a number of scenarios. The paper is concluded in Section 5.

2 OFDM-CPM Signaling Scheme

As shown in Fig. 2, serial bit stream b_i , $i = 0, 1, 2, \dots$, with bit duration of T_b seconds is converted into blocks of N bits represented by $a_{k,p}$, $k = 0, 1, 2, \dots$, and $p = 0, 1, 2, \dots, N-1$, where N denotes the number of carriers and $a_{k,p} = \pm 1$. For example, $a_{0,p}$ would denote the first block of N bits and $a_{1,p}$ the second block of N bits and so on. The CPM mappers transform the incoming $a_{k,p}$ into appropriate complex numbers $c_{k,p}$ given by

$$c_{k,p} = \cos(\theta_{k,p}) + j \sin(\theta_{k,p}), \quad (1)$$

with

$$\theta_{k,p} = \begin{cases} a_{k,p}\pi h + \pi h \sum_{q=0}^{k-1} a_{q,p} + \phi; & k \geq 1 \\ a_{0,p}\pi h + \phi; & k = 0 \end{cases} \quad (2)$$

where parameter h defines the CPM mapper and ϕ represents the initial mapping point that is assumed zero without loss of generality. The angles $\theta_{k,p}$ depend not only on the current data but also on the past data. Fig. 3 shows values of $\theta_{k,p}$ as a function of time when $h = \frac{1}{2}$. Current value of θ is determined by adding $+\pi h$ (if data bit is a +1) or $-\pi h$ (if data bit is a -1) to the previous value of θ . The corresponding complex numbers lie on a circle.

The complex numbers from the output of CPM mappers are passed through pulse shaping filters $g(t)$, then modulated by orthogonal carriers and finally summed to give the transmitted OFDM symbol which is mathematically represented as

$$x(t) = \sum_k \sum_p c_{k,p} g(t - kT) e^{j\frac{2\pi}{T}kt}, \quad 0 \leq t < \infty \quad (3)$$

where

$$g(t) = \begin{cases} \frac{1}{\sqrt{T}}; & -T_g \leq t \leq LT \\ 0; & \text{elsewhere.} \end{cases} \quad (4)$$

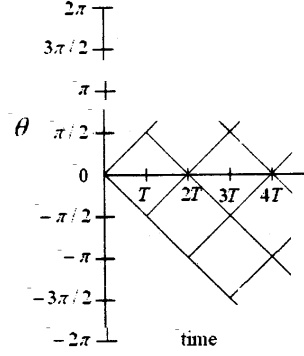


Figure 3. Phase trellis for OFDM-CPM signaling

In (3) and (4) $T (= NT_b)$ is the OFDM symbol duration and T_g is the guard interval. The sampling time T_b of the signal should be decreased to $T'_b = \left(\frac{LT}{LT+T_g}\right) T_b$ to transmit the data at the same information rate. In (4) $L = 1$ for full response signaling. The parameters h and L can be chosen in various ways giving rise to some of the following possible OFDM-CPM signals.

2.1 Single- h OFDM-CPM Signals

In this case, the value of h remains constant for all OFDM symbols. By choosing h to be rational and $0 < h < 1$ it is possible to have finite number of points in the CPM constellation to reduce receiver complexity. If h could be written as $2k/p$, where k and p are integers then p denotes the number of points in CPM constellation. In Fig. 4 is shown the constellation diagram of CPM mapper for $h = \frac{1}{2}$ (four constellation points) and $h = \frac{1}{4}$ (eight constellation points).

2.2 Multi- h OFDM-CPM Signals

The value of h is cyclically chosen from a set of K values, $\{h_1, h_2, \dots, h_K\}$. The value of h employed during the i th symbol is given by h_l , $l = i \text{ modulo } K$. By restricting h_l to include only multiples of $1/q$, q an integer, one obtains a property that all phases at times nT (n being observation interval) are some multiple of $2\pi/q$. A demodulator/decoder need only deal with transitions to these q phases [10].

For example, the complex numbers of a 4-carrier OFDM-CPM signal with $H_2 = \{\frac{2}{4}, \frac{1}{4}\}$ for first two blocks of data sequences are shown below (assuming initial map-

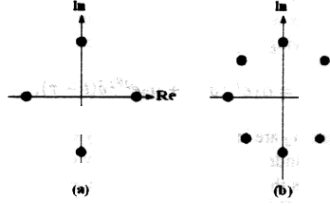


Figure 4. Constellation diagram of CPM mapper for (a) $h = 0.5$ and (b) $h = 0.25$

ping points to be $1 + j0$):

$$\begin{bmatrix} a_{k,p}, a_{k+1,p} \\ +1 & -1 \\ +1 & +1 \\ -1 & +1 \\ +1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} c_{k,p}, c_{k+1,p} \\ +j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ +j & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ -j & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \\ +j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \end{bmatrix}$$

Asymmetric OFDM-CPM Signals

While in multi- h OFDM-CPM signals h values are independent of data bits $a_{k,p}$ ($= \pm 1$), in asymmetric multi- h signals h is made a function of $a_{k,p}$. That is, the value of h during the i th symbol interval is chosen h_{+i} or h_{-i} accordingly as data is a $+1$ or -1 respectively. This gives additional flexibility to the designers to optimize system performance. For example, let $\{h_a, h_b, h_c\}$ be the set for a 3- h scheme. One way of implementing asymmetric signaling is to shift h_{-i} with respect to h_{+i} by one symbol interval as shown below:

$$\begin{array}{cccccccc} i: & 1 & 2 & 3 & 4 & 5 & 6 & \dots \\ h_{+i}: & h_a & h_b & h_c & h_a & h_b & h_c & \dots \\ h_{-i}: & h_b & h_c & h_a & h_b & h_c & h_a & \dots \end{array}$$

i.e., $h_{-i} = h_{+(i+1)}$ [11].

Partial Response OFDM-CPM Signals

In (4), by making $L > 1$ the pulse duration can be extended to more than one OFDM symbol. Using a value of $L = 2, 3, \dots$ systematic correlation can be furthered amongst OFDM symbols which in turn can be exploited for improvement in system performance.

3 Detection of OFDM-CPM Signals

With reference to Fig. 2, we can model the received signal as

$$\begin{aligned} r(t) &= x(t) * h(t) + n(t); \quad 0 \leq t \leq nT \\ &= s(t) + n(t) \end{aligned} \quad (5)$$

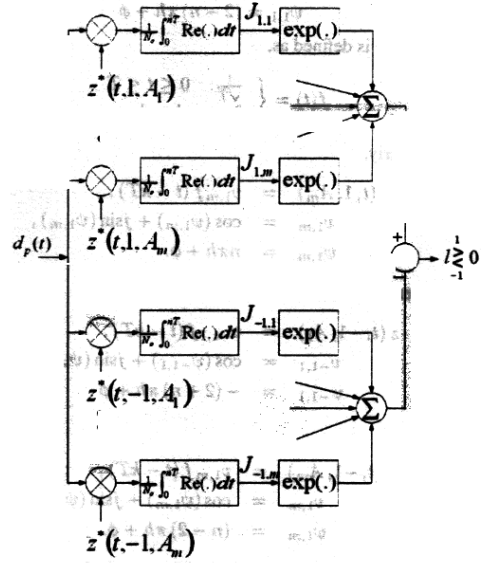


Figure 5. CPM De-mapper

where $h(t)$ is the channel impulse response, $n(t)$ is AWGN with a double sided power spectral density of $\frac{N_0}{2}$ and $*$ denotes convolution.

As shown in Fig. 1(b), the received signal $r(t)$ passes through several blocks before arriving at the de-mapper. The de-mapper is merely the optimum or suboptimum receiver that is used to demodulate PSK, DPSK or QAM signals in single-carrier systems corrupted by Gaussian noise. Using the detection theory presented in [12], we arrived at the optimum receiver (de-mapper) for OFDM-CPM signals which is shown in Fig. 5. This receiver is very similar to the optimum receiver for CPFSK signals presented in [13].

Let $d_p(t)$ be the input to this receiver from p th carrier available at the output of FFT. The receiver observes the signal $d_p(t)$ over nT seconds and arrives at an estimate of data bit b_i transmitted during $0 \leq t \leq T$ seconds. In this figure, $z(t, \pm 1, A)$ represent known complex signals assuming the first bit was a $+1$ or -1 and A represents the rest $(n-1)$ bits out of the possible $m (= 2^{n-1})$ sequences. For example, A_1 will be a sequence of $n-1$ bits where each bit is a -1 while A_m will be a sequence with all the bits equal to $+1$. The signals $z(t, \pm 1, A)$ are given by:

$$z(t, \pm 1, A) = v_{1,1} f(t - kT),$$

where,

$$v_{1,1} = \cos(\psi_{1,1}) + j\sin(\psi_{1,1}),$$

using (2),

$$\psi_{1,1} = (2-n)\pi h + \phi$$

and $f(t)$ is defined as,

$$f(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t \leq T \\ 0 & \text{elsewhere.} \end{cases} \quad (6)$$

Similarly,

$$\begin{aligned} z(t, 1, A_m) &= v_{1,m} f(t - kT), \\ v_{1,m} &= \cos(\psi_{1,m}) + j\sin(\psi_{1,m}), \\ \psi_{1,m} &= n\pi h + \phi. \end{aligned}$$

When the first bit is a -1,

$$\begin{aligned} z(t, -1, A_1) &= v_{-1,1} f(t - kT), \\ v_{-1,1} &= \cos(\psi_{-1,1}) + j\sin(\psi_{-1,1}), \\ \psi_{-1,1} &= -(2+n)\pi h + \phi \end{aligned}$$

and

$$\begin{aligned} z(t, -1, A_m) &= v_{-1,m} f(t - kT), \\ v_{-1,m} &= \cos(\psi_{-1,m}) + j\sin(\psi_{-1,m}), \\ \psi_{-1,m} &= (n-2)\pi h + \phi. \end{aligned}$$

Rest of the signals $z(t, 1, A_2)$ to $z(t, 1, A_{m-1})$ and $z(t, -1, A_2)$ to $z(t, -1, A_{m-1})$ will depend upon the length of observation interval n .

The receiver correlates the incoming $d_p(t)$ with all $z(t, \pm 1, A)$, integrates the real part over nT seconds, multiplies the result with $1/N_o$, takes the exponential, adds all the resulting signals that come from $z(t, +1, A)$ and the ones that come from $z(t, -1, A)$ and takes the difference. If the difference is positive, it decides a +1 was transmitted else it decides in favor of -1. For high SNR approximation of this receiver [13, 14], the performance can be upper bounded. This high SNR receiver is thus given by,

$$\sum_{j=1}^m \exp\left(\frac{1}{N_o} J_{\pm 1, j}\right) \approx \exp\left(\frac{1}{N_o} \bar{J}_{\pm 1}\right) \quad (7)$$

where,

$$J_{\pm 1, j} = \frac{1}{N_o} \int_0^{nT} \text{Re}[d_p(t) z^*(t, \pm 1, A_j)] dt \quad (8)$$

and

$$\bar{J}_{\pm 1} = \max\{J_{\pm 1, j}; j = 0, 1, \dots, m-1\} \quad (9)$$

Since the function $\exp(\cdot)$ is monotonic, $\bar{J}_{\pm 1}$ is an equivalent test parameter suggesting that the corresponding sub-optimum receiver should compute all $J_{\pm 1, j}$ and produce a decision depending on the largest of these. In all our simulations we have used the high SNR approximation of CPM receiver.

4 Simulation Results

The simulations were performed assuming a two path fading channel having impulse response given by [15]

$$h(t) = \alpha_1 e^{j\theta_1} \delta(t) + \alpha_2 e^{j\theta_2} \delta(t - \tau) \quad (10)$$

where α_1 and α_2 are independent and Rayleigh distributed, θ_1 and θ_2 are independent and uniformly distributed over $(0, 2\pi]$ and τ is the time delay between the two rays. The power sum of $E\{\alpha_1^2\}$ and $E\{\alpha_2^2\}$ is set to unity in the simulation, so the channel has unity average gain over a simulation run. The ratio of $E\{\alpha_1^2\}$ to $E\{\alpha_2^2\}$ is the power ratio of the desired ray (D) to the undesired ray (U) and is denoted as D/U .

Other parameters of the system are, information rate (R_b) = 800 kbps, carrier frequency (f_c) = 910 MHz and Doppler frequency (f_D) = 5 Hz [5]. A guard interval of $8 \mu\text{s}$ is assumed and $L = 1$ in (4). Also, we have assumed that no channel information is available to the receiver.

In this work, we have concentrated on the investigation of single- h OFDM-CPM signals and have presented an example each of multi- h and asymmetric OFDM-CPM signals. Multi- h and asymmetric encompass a wide class of OFDM-CPM signals that need separate treatment in order to be thoroughly explored.

4.1 Simulation of Single- h OFDM-CPM Signals

We investigated some of the rational values of h and observed that those values of h that are closer to 0 or 1 tend to give a high bit-error-rate while those in the vicinity of 0.4 and 0.6 perform better. This is to be expected because a value of $h = 4/5$ will result in constellation points that are closer to each other as compared to the ones that result for $h = 4/7$. In the latter case, it will be easier for the receiver to distinguish between adjacent signal points. Therefore, we chose some representative values of h to demonstrate the potential of single- h OFDM-CPM signals.

Fig. 6 shows a plot of probability of bit error (P_b) versus SNR as a function of h for a 32-carrier OFDM-CPM system with $\tau = 6 \mu\text{s}$ and $D/U = 10 \text{ dB}$. It is noted that $h = 4/7$ performs better than $h = \frac{1}{2}, \frac{4}{5}$ for two symbol intervals. The performance for $h = 4/7$ improves further when observation length is increased to 3. This is because the receiver now has more information to take a decision. However, we observe no appreciable improvement in performance when observation length is increased for other values of h and hence is not shown in the figure.

Fig. 7 shows a plot of P_b versus number of carriers (N) with SNR = 30 dB, $\tau = 6 \mu\text{s}$ and $D/U = 10 \text{ dB}$ for the same set of h values used in Fig. 6. In this case the information rate changes with N due to guard interval and can be calculated as $R_b = \left(\frac{NT_b}{NT_b + T_g}\right) \frac{1}{T_b}$. The performance does not change with N for a fixed h with $n = 2$.

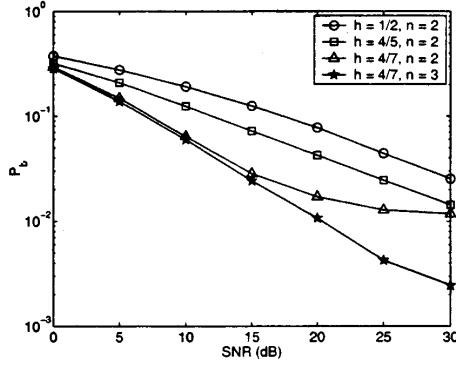


Figure 6. BER as a function of SNR for various values of h with $N = 32$, $\tau = 6 \mu\text{s}$ and $D/U = 10 \text{ dB}$

However, when n is increased to 3 with $h = 4/7$, the overall performance improves but deteriorates with N .

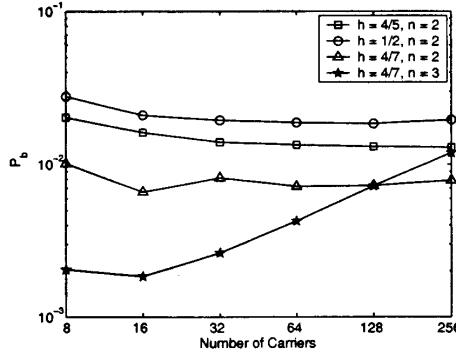


Figure 7. BER as a function of number of carriers for various values of h with SNR = 30 dB, $\tau = 6 \mu\text{s}$ and $D/U = 10 \text{ dB}$

P_b versus τ is plotted in Fig. 8 for three values of h for a 32-carrier system with SNR= 30 dB and $D/U = 10 \text{ dB}$. Performance remains somewhat stable until τ does not exceed the guard interval as there is no ISI. As τ crosses the $8 \mu\text{s}$ boundary, ISI comes into play and severely deteriorates the performance. In this case as well, $h = 4/7$ with $n = 3$ gives the best performance.

In Fig. 9 is shown the performance of a 32-carrier OFDM-CPM system as a function of τ and D/U when h is fixed at $4/7$, $n = 3$ and SNR= 30 dB. Until τ is less than the guard interval there is no ISI and P_b remains almost stable for various values of D/U . Once the guard interval is not able to remove ISI completely, performance deteriorates. With ISI, system performance keeps on improving with D/U because the ISI introduced by weaker

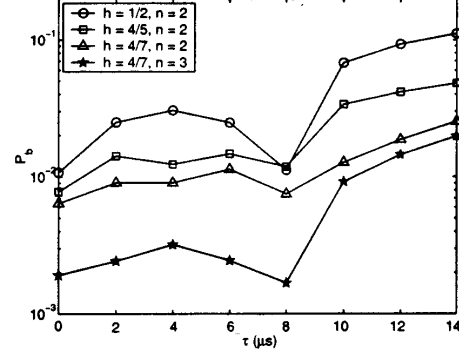


Figure 8. BER as a function of τ for various values of h with $N = 32$, SNR = 30 dB and $D/U = 10 \text{ dB}$

delayed wave keeps on reducing.

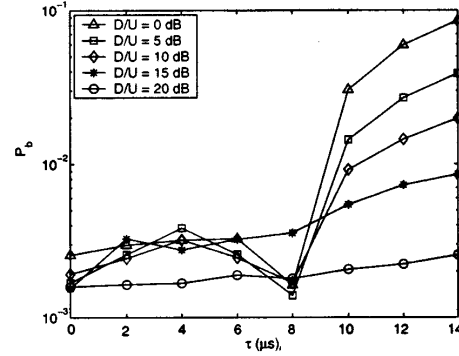


Figure 9. BER as a function of τ and D/U with $N = 32$, SNR = 30 dB, $h = 4/7$ and $n = 3$

4.2 Simulation of Multi- h and Asymmetric OFDM-CPM Signals

In this section we present an example each from multi- h and asymmetric OFDM-CPM signals and show that these are also promising signaling schemes that could be used over fading channels.

For multi- h , we show the results of a 2- h scheme with $H_2 = \{\frac{8}{16}, \frac{9}{16}\}$ and $n = 2$. For asymmetric, we use a 2- h scheme with $H_{+i} = \{\frac{3}{7}, \frac{5}{7}\}$, $H_{-i} = \{\frac{5}{7}, \frac{3}{7}\}$ and $n = 3$. Fig. 10 shows a plot of P_b versus SNR for a 32-carrier OFDM-CPM system with $\tau = 6 \mu\text{s}$ and $D/U = 10 \text{ dB}$. It is noted that multi- h scheme with the chosen parameters achieves almost the same performance as the single- h scheme but with a shorter observation length which is 2.

Asymmetric signaling with the chosen parameters does not depict any advantage over single- h or multi- h schemes and gives almost the same performance.

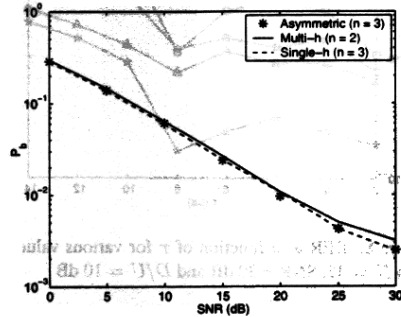


Figure 10. BER as a function of SNR for various signaling schemes with $N = 32$, $\tau = 6 \mu\text{s}$ and $D/U = 10 \text{ dB}$

5 Conclusions

We have presented a new class of OFDM signals called OFDM-CPM that can be coherently demodulated and show promising performance in multipath fading channels without channel estimation. By choosing the parameter h , a class of OFDM-CPM signals can be designed. Simulation results show that the performance of an OFDM-CPM system depends upon the type of signaling used at the transmitter and the observation interval employed at the receiver. Analytical expressions need to be developed in order to find optimum values of h and observation interval that will minimize the probability of error. Multi- h and asymmetric are also promising signaling schemes that need further investigation.

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