Cover page

Title: A Simplified Sectional Analysis Approach for RC Elements during Fire Events

Authors: S.F. El-Fitiany
M.A. Youssef

ABSTRACT

Sectional analysis is widely used to assess the design of Reinforced Concrete (RC) members at ambient temperature. During fire exposure, a heat gradient is created within the concrete mass that induces non-uniform thermal and mechanical strains. These strains complicate using the same approach to predict the fire resistance of RC members. A simplified sectional analysis that tracks the axial and flexural behavior of RC elements during fire events is presented in this paper. The proposed method is based on using the Finite Difference (FD) analysis to estimate the temperature distribution within a concrete section. A rational approach is then proposed to convert the FD two-dimensional temperature distribution to one-dimensional distribution. This modification converts a complex problem to a simplified one and thus enables engineers to better understand the behavior and have higher confidence in the results. This paper covers the use of the simplified method for square columns subjected to fire from four sides and for rectangular beams exposed to fire from three sides. The validation of the proposed method is presented by comparing its predictions with other experimental and analytical results.

INTRODUCTION

Concrete as well as steel reinforcement experience significant deterioration when subjected to elevated temperatures. The strength reduction is accompanied with induced thermal and transient strains [1, 2].

S.F. El-Fitiany (corresponding author), Ph.D. Candidate, Department of Civil and Environmental Engineering, The University of Western Ontario, London, Ontario, Canada N6A 5B9
M.A. Youssef, Associate Professor, Department of Civil and Environmental Engineering, The University of Western Ontario, London, Ontario, Canada N6A 5B9

Concrete structures are currently being designed for fire using prescribed methods that are based on experimental tests. These methods specify minimum cross-section dimensions and minimum clear cover to the reinforcing bars. As new codes are moving towards performance-based design, engineers are in-need of new design tools to achieve specific performance criteria for a defined fire scenario. These tools must be analytical as conducting experimental tests to satisfy different fire scenarios would be an expensive solution. The Finite Element Method (FEM) has proven to be a powerful method to predict the behaviour concrete structures during exposure to fire events [3, 4]. Drawbacks of using the FEM, including the need to have a coupled thermal-stress analysis computer program and difficulty to comprehend its results and identify potential modelling errors, make it impractical for design engineers.

An approach that relies on using both Finite Difference Method (FDM) and a modified sectional analysis was proposed by the authors in a recent publication [5]. FDM is considered a simple method for evaluating the temperature variation within a concrete cross-section [6]. Sectional analysis allows evaluating the axial and/or flexural behaviour of concrete elements and is based on simple equilibrium and compatibility equations that can be easily applied by design engineers [5]. The modified sectional analysis was validated for concrete square columns exposed to fire from four faces [5]. To study RC beams at elevated temperatures, simple modifications to the proposed approach are introduced and validated in this paper..

The research conducted in this paper is limited to rectangular concrete sections exposed to a standard ASTM-E119 fire from three or four sides. Normal strength concrete is assumed and, thus explosive spalling is not considered [7].

SECTIONAL ANALYSIS AT AMBIENT TEMPERATURE

At ambient temperature, RC sections can be analyzed using the well-known sectional analysis approach [8]. For cases of single curvature, i.e. bending about horizontal axis, the concrete section is divided into horizontal discrete fibers as shown in Figure 1. Utilizing the uniaxial stress-strain relationship for each fiber and taking into account equilibrium and kinematics, the mechanical behavior of the section can be predicted. To simplify the analysis, two variables are assumed: incremental centroidal axial strain ($\Delta\varepsilon_c$) and incremental curvature ($\Delta\psi$). Assuming a linear strain distribution, the incremental moment and axial force are obtained using Eq. (1).

$$\begin{pmatrix} \Delta M \\ \\ \Delta P \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} E_i \times A_i \times y_i^2 & -\sum_{i=1}^{n} E_i \times A_i \times y_i \\ \\ -\sum_{i=1}^{n} E_i \times A_i \times y_i & \sum_{i=1}^{n} E_i \times A_i \end{pmatrix} \times \begin{pmatrix} \Delta\psi \\ \\ \Delta\varepsilon_c \end{pmatrix} \qquad (1)$$

Where E_i is the modulus of elasticity of layer i, A_i is the area of layer i, y_i is the distance between the centroid of layer i and the centroid of the cross-section.

For a given axial load, the moment-curvature behavior is obtained in two stages. In the first stage, the axial strain is increased incrementally while curvature is kept equal to zero until reaching the given axial load. In the second stage, the axial load is kept constant and the applied curvature is increased. The corresponding changes in the axial strain and the moment are calculated using Eq. (1). This process is repeated until reaching a predefined curvature value.

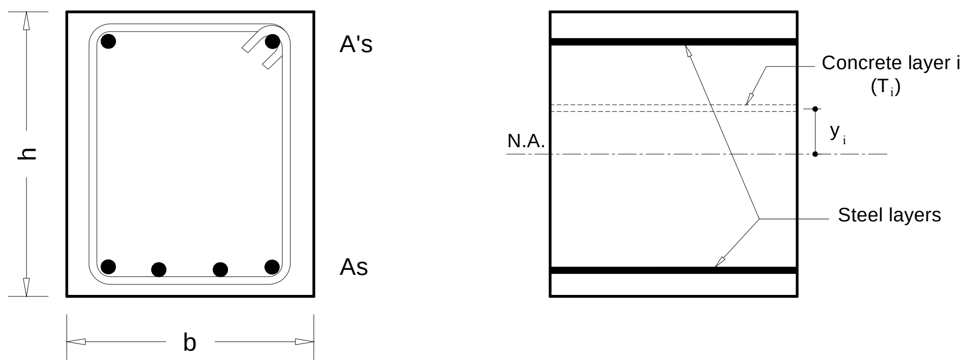


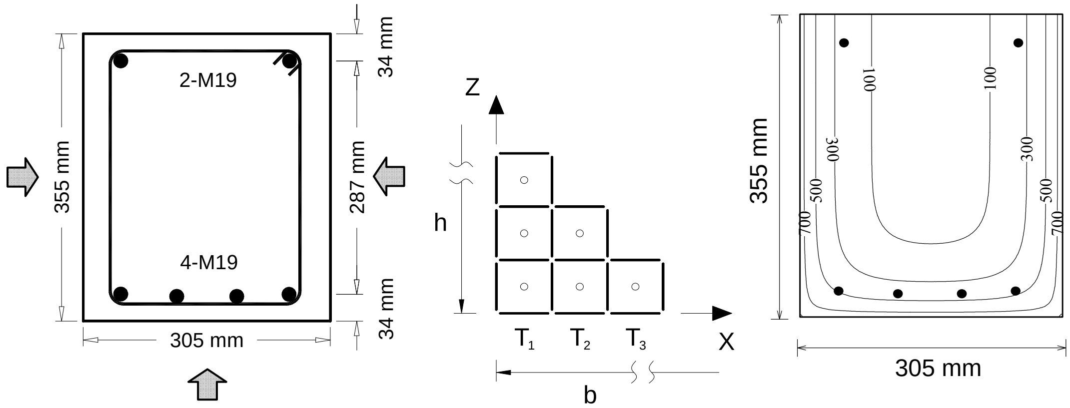
Figure 1. Cross-section and fiber model of the analyzed RC beam.

SECTIONAL ANALYSIS AT ELEVATED TEMEPRATURES

To apply the same sectional analysis approach to RC columns at elevated temperatures (fire from four sides), a number of modifications were proposed and validated by El-Fitiany and Youssef [5]. Additional modifications are presented in the following sections to analyze RC beams at elevated temperatures (fire from three sides). The 6.1 m simply supported beam tested by Lin et al. [9] is considered for illustration, Figure 2a.

Heat Transfer Model

The first step to analyze a RC section subjected to elevated temperature is to estimate the heat gradient within the concrete mass. The Finite Difference Method (FDM) has the advantage of accounting for irregular shapes with good accuracy in addition to the ease of implementation in any programming code [6]. The implementation of this method in the proposed sectional analysis approach was discussed in-details by El-Fitiany and Youssef [15]. Figure 2b shows the FDM square mesh, where the temperature of each element is represented by the temperature at its center of area. A contour map for the heat gradient within the studied beam, after 1hr ASTM-E119 fire exposure, is shown in Figure 2c.



a) concrete section b) square mesh c) temperature distribution

Figure 2. Heat transfer analysis of a concrete beam exposed to fire from three sides.

Concrete and Steel Constitutive Models

Based on the temperature reached in each square element, the mechanical properties of concrete and/or steel can be estimated. Youssef and Moftah [1] provided an assessment of available constitutive models for concrete and steel at elevated temperatures. The authors have utilized the material models recommended by Youssef and Moftah [1] in this study.

Average Layer Temperature

The square mesh elements are grouped into horizontal fibers to conduct sectional analysis. Therefore, an equivalent temperature T_i has to be assigned for each fiber. El-Fitiany and Youssef [5] suggested utilizing two temperature distributions, Figure 3.

The first temperature distribution is based on the fiber temperature producing the same average compressive strength (f'_{cT}) for the square mesh elements composing this fiber, Figure 2b. As f'_{cT} is proportional to the modulus of elasticity, this temperature distribution is representative of the average stiffness of the fibers.

Thermal and transient strains of a given fiber are proportional to its average elevated temperature. Thus, a second temperature distribution can be estimated using the algebraic average temperature of the square elements within the fiber.

Figure 3 shows the distribution of the two proposed average temperatures for the analyzed beam after one hour of ASTM-E119 standard fire exposure. The temperature of the steel bars can be assumed to be the same as the temperature of the square mesh element within which they are located [6].

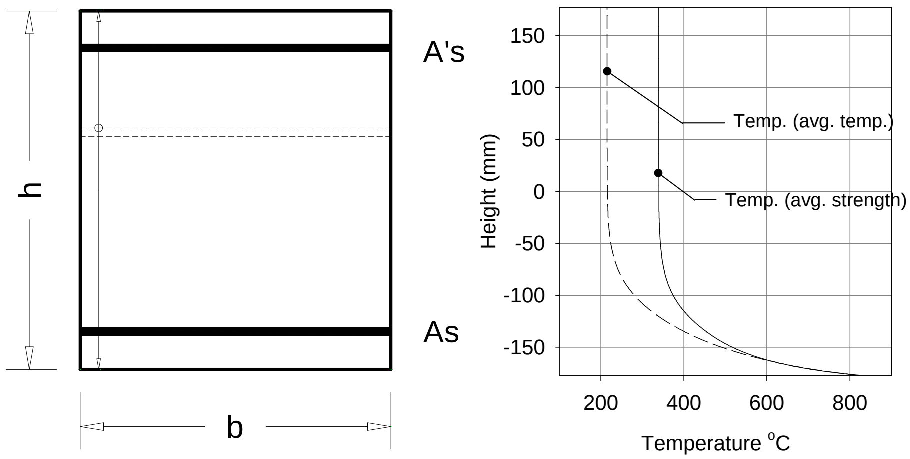
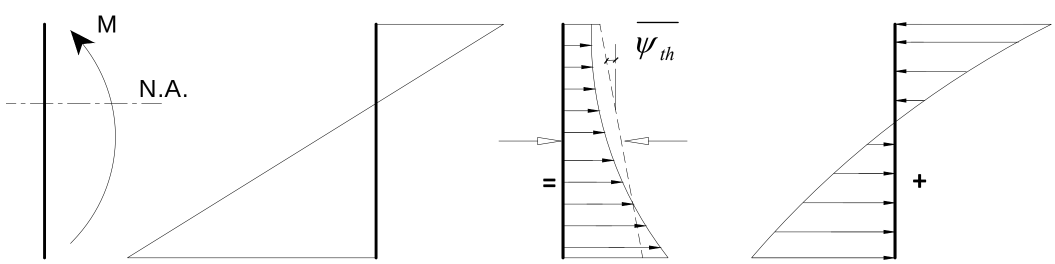


Figure 3. Average temperature distributions for beams during fire exposure.

Thermal and effective strains

Total concrete strain at elevated temperatures (ε_{tot}) is composed of three terms: instantaneous stress related strain (ε_{fT}), unrestrained thermal strain (ε_{th}), and transient creep strain (ε_{tr}). Figure 4 shows the expected linear distribution of the total strain (ε_{tot}) under a pure bending moment (M). This linear shape is based on the fact that plane sections remain plane after loading, which is still valid at elevated temperatures [10,**Error! Reference source not found.** 11]. Knowing the average temperature for each layer, concrete and steel thermal strains (ε_{th}) can be evaluated. A schematic distribution of (ε_{th}) for typical RC beams exposed to fire temperature from three sides is shown in Figure 4. For the previously reported case of interior columns subjected to fire from four faces, the thermal strain distribution is symmetric about the section mid-height [5].



Total strain (ε_{tot})

Thermal strain (ε_{th})

Mechanical strain (ε_{cT})

Figure 4. Components of total strain at elevated temperatures.

Youssef and Moftah [1] proposed concrete material model accounts for the transient creep strain (ε_{tr}). For unrestrained concrete sections, the effective strain (ε_{cT}) can be calculated by subtracting concrete and steel thermal strains from the total strain. The nonlinear distribution of thermal strains results in a nonlinear effective (mechanical) strain distribution, Figure 4. As Eq. (1) is only applicable for linear strain distributions, the following sub-section present a method to convert the nonlinear thermal strain distribution into an equivalent linear strain distribution.

Isolation of Thermal Strain Component

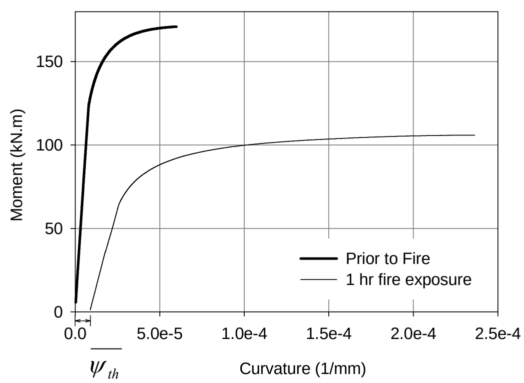
The equivalent thermal strain distribution is defined by the mid-height axial strain ($\overline{\varepsilon_{th}}$) and the curvature ($\overline{\psi_{th}}$), Figure 4. The values of $\overline{\varepsilon_{th}}$ and $\overline{\psi_{th}}$ are evaluated such that the forces in the concrete and steel fibers resulting from the difference between the actual thermal strain and $\overline{\varepsilon_{th}}$-$\overline{\psi_{th}}$ distribution are in self-equilibrium. An iterative procedure is used to calculate the values of $\overline{\varepsilon_{th}}$ and $\overline{\psi_{th}}$. Concrete tensile strength is neglected. The $\overline{\varepsilon_{th}}$-$\overline{\psi_{th}}$ distribution reflects the actual deformation of the concrete section under zero external loads and moments. Differences between the non-linear and the $\overline{\varepsilon_{th}}$-$\overline{\psi_{th}}$ distribution represent internal stresses in the concrete and steel materials that are in-equilibrium.

PERFORMANCE OF RC ELEMENTS DURING FIRE EXPOSURE

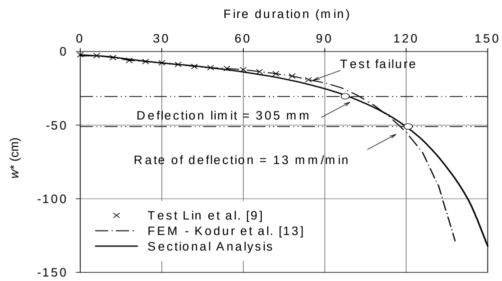
Sectional analysis can be conducted to construct the axial load-axial strain and moment-curvature diagrams. Figure 5a shows the effect of 1 hr standard ASTM-E119 fire exposure on the studied unrestrained beam section (load level, λ, equals to zero). As shown in Figure 5a, elevated temperatures have increased the ductility and reduced the capacity of the RC beam. The initial point of the moment-curvature diagram defines the equilibrium curvature $\overline{\psi_{th}}$. This curvature value represents the initial rotation of the fire-damaged beam due to the non-linear thermal distribution.

VALIDATION

The moment-curvature diagrams for the beam tested by Lin et al. [12] were constructed at different fire durations. The vertical deflection (w*) at each duration was evaluated using the moment-area method. The obtained results were compared with the experimental results, Figure 5b. An excellent matching is found between the sectional analysis and the FEM conducted by Kodur and Dwaikat [13], and the experimental results (up to 80 minutes). Failure criteria proposed by BS 476 [14] and adopted by Kodur and Dwaikat [13] are used. These criteria are setting limits for the maximum allowable deflection and maximum rate of deflection. As shown in Figure 5b, the proposed sectional analysis results in about 17 minutes difference in predicting failure compared with the test failure and about 5 min difference with Kdour et al. [13]. Additional analytical and experimental work is required for further validation of the proposed methodology and the BS 476 failure criteria.



a) moment–curvature relationship



b) deflection of the studied beam [9]

Figure 5. Flexural and axial behavior of RC sections at elevate temperatures.

CONCLUSIONS

The sectional analysis approach, proposed by the authors in a previous publication, is extended in this paper to cover RC beams subjected to fire from three sides. The proposed approach is found to be a simple yet accurate method to track the behavior of rectangular RC beams at elevated temperatures. Analysis of RC section during fire exposure can be concluded into three main steps:

1. The heat transfer model is applied and the heat gradient through the cross section is predicted. The average temperatures for each layer are then obtained.
2. The equivalent uniform thermal strain $\overline{\varepsilon_{th}}$ and curvature $\overline{\psi_{th}}$ are then calculated by equilibrating the forces in the concrete and steel layers resulting from the actual thermal strain distribution. The difference between the actual and uniform strain distributions represents the induced strains $\varepsilon_{\sigma th}$ in concrete and steel layers to satisfy the section geometry. These strains are considered as initial strains in the following step.
3. Sectional analysis is conducted to construct the axial load-axial deformation and/or moment-curvature diagrams.

Additional tests are needed to further validate and improve the method for different concrete mixtures, aggregate types, fire scenarios, cross-section dimensions, number of exposed surfaces, and loading conditions. Extensive research is also needed to utilize this method at the structure level which requires accounting for fire-induced deformations.

ACKNOWLEDGMENTS

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