SIMPLIFIED METHOD FOR PREDICTING DEFORMATIONS OF RC FRAMES DURING FIRE EXPOSURE

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Abstract

Structural engineers are in need of analytical methods to assess the performance of Reinforced Concrete (RC) frames during fire events. Existing numerical methods require extensive knowledge of heat transfer calculations and the finite element method. This paper proposes a practical approach to track the fire performance of indeterminate RC frames during ASTM-E119 and ISO 834 standard fires exposure. The proposed method utilizes a finite difference method to predict the temperature distribution within the section of the RC frame. The predicted elevated temperatures are then used to conduct a sectional analysis. The effective flexural and axial stiffnesses are evaluated and used to predict the overall behavior of the structure during fire. The proposed approach is validated by comparing its predictions with analytical results by others.

Keywords: concrete, fire, elevated temperatures, sectional analysis, indeterminate structures, thermal restraint.

INTRODUCTION

Fire initiates when combustible materials ignite. Then, it spreads horizontally and/or vertically depending on the compartment boundaries. A temperature gradient is generated through exposed RC elements. These elevated temperatures cause the element's stiffness to degrade and produce thermal deformations. Structural fire safety of RC structures is currently evaluated based on the fire ratings of single elements, i.e. columns, beams, walls, and slabs. However, the overall behavior of the structure during a fire should be assessed to ensure the safety of the occupants and the fire fighters during evacuation.

Fire testing is the most reliable approach to assess the fire endurance of a structure but its use for concrete frames is very limited. This is mainly because of its cost, which makes it unsuitable for regular design. Finite Element (FE) tools are very powerful and capable of analyzing RC structures during fire events. Drawbacks of using the FE method including: the need for a comprehensive computer program, the difficulty to comprehend its results and to identify potential modeling errors, and the long running time make it impractical for design engineers. This paper provides engineers with a practical approach to predict the fire response of statically determinate or indeterminate RC frames. The proposed method extends the work done by El-Fitiany and Youssef, 2009 that proposed a one-dimensional (1D) sectional analysis method to predict the flexural behavior of the heated section at different axial load levels (λ).

1 PROPOSED METHOD

For a given fire duration, the proposed method can be applied using the following steps:

1. determining of an equivalent one-dimensional average temperature distribution for the cross-section of the heated elements.
2. identifying the needed constitutive models for the heated elements.
3. predicting the unrestrained thermal deformations for the heated elements.

4. evaluating the flexural and axial stiffnesses of the heated elements based on their axial forces and moments.
5. Analyzing the fire-exposed frame under the effect of the applied loads while accounting for the thermal deformations using linear elastic analysis. The flexural and axial stiffnesses obtained in step 4 are utilized in this step. The moments and axial forces are redistributed based on the assigned stiffness values.
6. Recalculating the flexural and axial stiffnesses in step 4 for the revised moments and axial forces obtained in step 5.

Steps 4, 5, and 6 are repeated until the change in the obtained axial forces and moments is less than an assumed tolerance. The following sections explain these steps.

2 AVERAGE TEMPERATURE CALCULATION

At specific fire duration, a heat transfer analysis is conducted to predict the temperature distribution using the Finite Difference Method (FDM) (Lie et al, 1992). The cross-section is then divided into horizontal layers and an average temperature T_{avg} is calculated for each layer. T_{avg} represents the algebraic average temperature, in °C, of the elements within each layer and is suitable for calculating the thermal and transient creep strains as they are temperature dependent (El-Fitiany and Youssef, 2009).

3 CONCRETE AND STEEL CONSTITUTIVE RELATIONSHIPS

Fire temperature reduces the mechanical properties of concrete and steel. It also induces new strains, i.e. thermal and transient creep strains. The following sub-sections provide a brief summary of the concrete and steel models used in this paper.

3.1 Concrete Strains

The total concrete strain at elevated temperatures (ε) is composed of three terms: unrestrained thermal strain (ε_{th}), instantaneous stress related strain (ε_c), and transient creep strain (ε_{tr}). The total strain is given by Eq. (1).

$$\varepsilon = \varepsilon_{th} + \varepsilon_c + \varepsilon_{tr} \tag{1}$$

The free thermal strain, ε_{th}, is a strain resulting from fire temperature and can be predicted using the Eurocode 2 model for siliceous and carbonate concretes.
The value of the instantaneous stress-related strain (ε_c) at the peak compressive stress (f'_{cT}), i.e. ε_{oT}, defines the stress-strain relationship during fire exposure. For loaded concrete, the effect of elevated temperatures on ε_{oT} is negligible. The variation of $\varepsilon_{oT} + \varepsilon_{tr}$ with fire temperature is proposed by Eurocode 2. A linear relationship, Eq. (2), is chosen to represent the Eurocode 2 recommendation. Such a relationship simplifies the sectional analysis and results in good prediction of the flexural and axial stiffnesses.

$$\varepsilon_{oT} + \varepsilon_{tr} = 2.52 \times 10^{-5}\, T_{avg} \qquad 80\text{ °C} < T_{avg} \leq 1200\text{ °C} \tag{2}$$

3.2 Concrete Ultimate Strain

Concrete ultimate strain is the strain at which concrete crushing occurs. Elevated temperatures increase this strain. Eurocode 2 proposes a linear relationship between ε_{uT} and T_{avg}. This relationship can be represented by Eq. (3). ε_{uT} is defined in Eurocode 2 as the strain corresponding to zero compression stress. The difference between ε_{uT} and $\varepsilon_{oT} + \varepsilon_{tr}$ is constant and equal to 0.02.

$$\varepsilon_{uT} = 2.52 \times 10^{-5}\, T_{avg} + \Delta\varepsilon = \varepsilon_{oT} + \varepsilon_{tr} + 0.02 \tag{3}$$

3.3 Concrete Compressive Strength

Concrete compressive strength experiences significant degradation at elevated temperatures. Eurocode 2 predicts the reduced compressive strength (f'_{cT}) for siliceous and carbonate concretes as a ratio from its ambient value (f'_{c}). The reduction in f'_{cT} for siliceous concrete is represented by Eq. (4), as it allows reaching closed form solution for flexural stiffness.

$$f'_{cT}/f'_{c} = 1.76 \times 10^{-9}\, T_{avg}{}^{3} - 3 \times 10^{-6}\, T_{avg}{}^{2} + 2.5 \times 10^{-4}\, T_{avg} + 1.00 \qquad (4)$$

3.4 Concrete Stress-Strain Relationships

A general and simple approach to estimate the $f_{cT} - \varepsilon_{cT}$ descending branch is proposed by Eurocode 2 and represented by Eq. (5). Eqs. (2) and (4) are used to calculate ($\varepsilon_{oT} + \varepsilon_{tr}$) and f'_{cT}/f'_{c}, respectively.

$$f_{cT} = f'_{cT}\left[2\left(\frac{\varepsilon_{cT}}{\varepsilon_{oT}+\varepsilon_{tr}}\right) - \left(\frac{\varepsilon_{cT}}{\varepsilon_{oT}+\varepsilon_{tr}}\right)^{2}\right] \qquad \varepsilon_{cT} \leq (\varepsilon_{oT} + \varepsilon_{tr}) \qquad (5a)$$

$$= f'_{cT}\left[\frac{\varepsilon_{uT}-\varepsilon_{cT}}{0.02}\right] \qquad (\varepsilon_{oT} + \varepsilon_{tr}) < \varepsilon_{cT} \leq \varepsilon_{uT} \qquad (5b)$$

3.5 Steel Stress-Strain Relationships

The model proposed by Lie et al (1992), is used to predict the reduced yield strength of reinforcing bars (f_{yT}) and the stress-strain ($f_{sT} - \varepsilon_{sT}$) relationship.

4 PREDICTION OF THE UNRESTRAINED DEFORMATION

A sectional analysis approach suitable for the analysis of rectangular RC beams at elevated temperatures was proposed by El-Fitiany and Youssef, 2009. Fig. 1a shows the fiber model of a typical RC cross-section subjected to fire from three faces. The average temperature (T_{avg}) distribution, Fig. 1b, can be calculated using the Finite Difference Method (FDM). T_{avg} induces thermal strains that can be evaluated using the following method:

1. The nonlinear thermal strain (ε_{th}) distribution, Fig. 1g, is calculated using T_{avg}. The thermal strain of steel bars is calculated based on the elevated temperature at their locations.

2. ε_{th} is then converted to an equivalent linear thermal strain ($\overline{\varepsilon_{th}}$), Fig. 1d, by considering self-equilibrium of internal thermal forces in concrete and steel layers. $\overline{\varepsilon_{th}}$ is represented by the value of the center axial strain (ε_i) and the curvature (ψ_i). ε_i and ψ_i define the unrestrained thermal deformation of a heated section.

3. Fig. 1f shows the differences between the equivalent linear and nonlinear thermal strains, which represent the self-induced thermal strains (ε_{st}). These strains are assigned as initial strains for the concrete and steel when calculating the flexural and axial stiffnesses. The following sections present a simplified approach to calculate ε_{st} using the predicted T_{avg} distribution and material models presented earlier in this paper.

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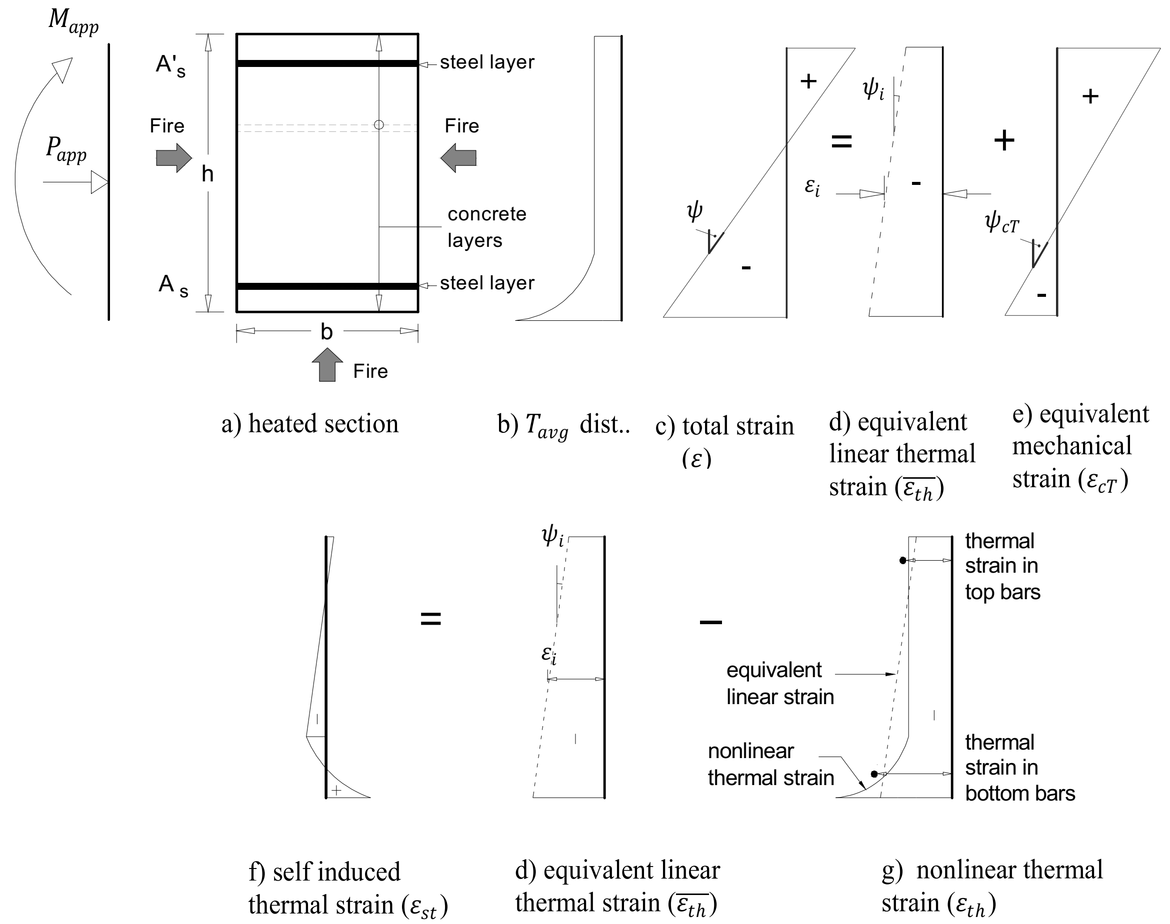


Fig. 1 Sectional analysis approach for heated RC sections

5 EVALUATION OF THE FLEXURAL AND AXIAL STIFFNESSES

Fig. 1a shows the applied axial force (P_{app}) and moment (M_{app}) on a RC section exposed to fire from three faces. The use of sectional analysis to evaluate the flexural and axial stiffnesses for this section involves the following steps:

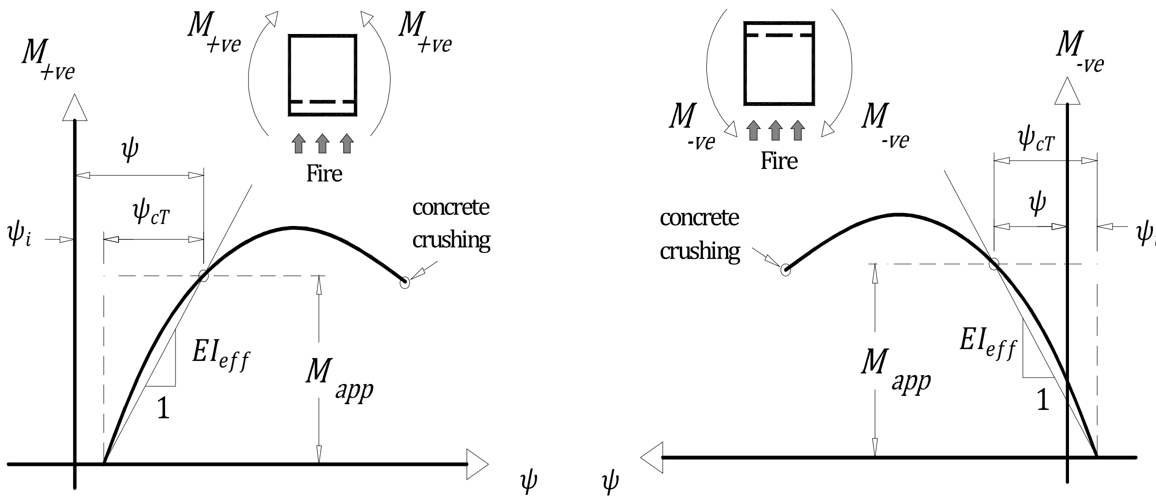
1. The self-induced thermal strains (ε_{st}), calculated in the previous section, are assigned as initial strains for the concrete and steel to model the corresponding self-induced self-equilibrating thermal stresses. The terms ε_{st}, ε_c, and ε_{tr} are lumped into an equivalent mechanical strain ε_{cT}, Eq. (6).

$$\varepsilon = \overline{\varepsilon_{th}} + \left(\varepsilon_{st} + \varepsilon_c + \varepsilon_{tr}\right) = \overline{\varepsilon_{th}} + \varepsilon_{cT} \quad (6)$$

2. For assumed values of ε_{cT} at the center of the section and ψ_{cT}, the internal stresses in the concrete and steel are evaluated. The corresponding internal axial forces are then calculated. To satisfy equilibrium between the calculated internal forces and the external loads, i.e. P_{app} and M_{app}, iterations are executed by changing the values of center ε_{cT} and ψ_{cT}. This process can repeated for different values of M_{app}. A typical relationship between ψ_{cT} and M_{app}, for a constant P_{app}, is sketched in Fig. 2.

3. The secant slope of the $M-\psi_{cT}$ diagram represents the section's effective flexural stiffness (EI_{eff}) at M_{app}(El-Fitainy and Youssef, 2012). The corresponding effective axial stiffness (EA_{eff}) equals to P_{app} divided by the center axial strain (ε_{cT}).

As shown in Fig. 2, heating RC sections from the bottom face and the two sides cause the bottom concrete fibers to thermally expand more than the top concrete fibers and results in ψ_i. The acting moment induces a mechanical curvature (ψ_{cT}), which is either added to or deducted from ψ_i. As shown in Fig. 2a, a positive (sagging) moment induces a curvature that adds to the initial curvature. For negative (hogging) moments, compression stresses are applied on the bottom fibers. Curvature caused by these stresses opposes the initial curvature, Fig. 2b.



(a) sagging moments (M_{+ve}) (b) hogging moments (M_{-ve})

Fig. 2 (M)–(ψ) diagrams for RC sections during fire

For a specific fire duration, the effect of thermal strain on the M–ψ relationship is not governed by M_{app}. It represents the unrestrained/free thermal expansion of the unloaded concrete element and results in shifting the M–ψ and diagram by a value ψ_i, Fig. 2. Consequently, the total curvature (ψ) is the sum of the unrestrained thermal curvature (ψ_i) and the mechanical curvature (ψ_{cT}) and can be expressed in terms of the effective stiffness (EI_{eff}) as follows.

$$\psi = \psi_i + M_{app}/EI_{eff} \tag{7a}$$

Similarly, the total center axial strain (ε) is the sum of the unrestrained thermal strain (ε_i) and the center mechanical strain (ε_{cT}) and can be expressed in terms of the effective axial stiffness (EA_{eff}) as follows.

$$\varepsilon = \varepsilon_i + P_{app}/EA_{eff} \tag{7b}$$

The following sections present a simplified approach to calculate EI_{eff} and EA_{eff} using the predicted T_{avg} distribution and material models presented earlier in this paper.

6 VALIDATION CASE (IDING ET AL., 1977)

Iding et al, 1977 has analytically investigated the behavior of RC frames during fire exposure. Fig. 3 shows a schematic of a single bay RC frame analyzed using FIRES-RC II, a comprehensive FE software developed at University of California, Berkeley. The beam and column dimensions are $355\ mm \times 711\ mm$. The frame was exposed to a $1.0\ hr$ of ASTM-E119 standard fire over its entire length while supporting the vertical loads shown in Fig. 3. The frame was analyzed assuming siliceous concrete and a compressive strength of $27.6\ MPa$. The yield strength of the reinforcing bars was $275.8\ MPa$.

This frame is analyzed using an elastic FE software SAP2000, and analyzed using the degraded flexural and axial stiffnesses. The effect of thermal expansion is considered by modeling ε_i and ψ_i as an induced deformation for all fire-exposed RC members. The predicted deformed shape is shown in Fig. 3. A good match is found between the proposed method and the nonlinear FIRES-RC II FE software. The difference in deformations can be due to using different material models.

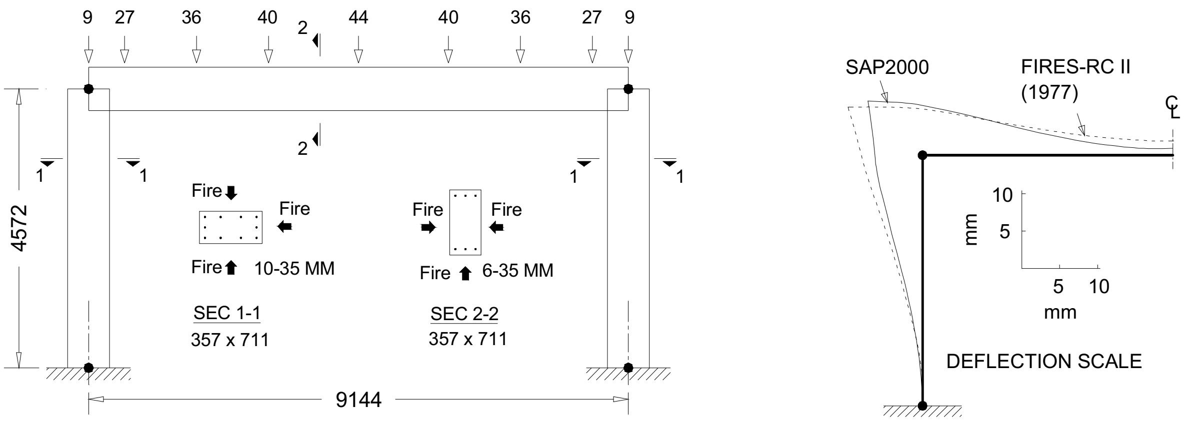


Fig. 3 Layout for a RC frame exposed to ASTM-E119 fire [Dims in mm, loads in kN]

7 SUMMARY AND ACKNOWLEDGMENT

The overall behavior of RC framed structures is studied in this paper. A practical approach based on superimposing the effects of thermal expansion and material degradation is introduced. The nonlinear thermal expansion is converted to an equivalent uniform thermal distribution, which can be represented by the unrestrained thermal axial strain ε_i and curvature ψ_i. The degradation effect in material strength is considered by accounting for the reduction in the effective flexural and axial stiffnesses, EI_{eff} and EA_{eff}, respectively. The proposed method is validated by comparing its results with a case study for a single storey RC frame analyzed by Iding et al, 1977. A good agreement is found between the finite element method predictions and the results of the proposed method for both case studies.

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