

Stress block parameters for concrete flexural members reinforced with superelastic shape memory alloys

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Abstract The unique properties of superelastic shape memory alloys (SMAs) have motivated researchers to explore their use as reinforcing bars. The capacity of a steel reinforced concrete (RC) section is calculated by assuming a maximum concrete strain $\varepsilon_{c\text{-max}}$ and utilizing stress block parameters, α_1 and β_1 , to simplify the non-linear stress–strain curve of concrete. Recommended values for $\varepsilon_{c\text{-max}}$, α_1 , and β_1 are given in different design standards. However, these values are expected to be different for SMA RC sections. In this paper, the suitability of using sectional analysis to evaluate the monotonic moment–curvature relationship for SMA RC sections is investigated. A parametric study is then conducted to identify the characteristics of this relationship for steel and SMA RC sections. Specific mechanical properties are assumed for both steel and SMA. Results were used to evaluate $\varepsilon_{c\text{-max}}$, α_1 , and β_1 values given in the Canadian standards and to propose equations to estimate their recommended values for steel and SMA RC sections.

Keywords Reinforced concrete · Shape memory alloys · Moment–curvature relationship · Stress block · Ultimate concrete strain · Moment capacity · Axial load capacity

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List of notations

\bar{Y}	Distance between point of action of the concrete compressive force and the extreme compression fibre of the concrete section
A'_s	Compressive reinforcement area
A_g	Gross area of concrete section
ALI	Axial load index which represents the ratio between the applied axial load to the axial capacity of the cross-section
A_s	Tensile reinforcement area
b	Cross-section width
C	Compression zone height
C_c	Compressive force in concrete
cc	Point at which concrete reaches its crushing strain
$E_{\text{cr-SMA}}$	SMA modulus of elasticity before the start of martensite variant reorientation (austenite phase)
E_{p1}	SMA modulus of elasticity before the start of the stress induced martensite phase
E_{p2}	SMA modulus of elasticity after the start of the stress induced martensite phase (martensite phase)
$E_{\text{u-s}}$	Steel plastic modulus of elasticity
$E_{\text{u-SMA}}$	SMA post-yielding modulus of elasticity
$E_{\text{y-s}}$	Steel elastic modulus of elasticity
f'_c	Concrete compressive strength
f_c	Concrete compressive stress

f_{cr-SMA}	SMA critical stress (start of martensite variant reorientation)
f_{p1}	Martensite stress induced stress
f_s	Steel stress
f_{u-s}	Steel ultimate stress
f_{u-SMA}	SMA ultimate stress
f_{y-s}	Steel yielding stress
f_{y-SMA}	SMA yielding stress
h	Cross-section height
H	Point at which strain in the SMA bars exceeds ε_{p1}
M	Moment
M_{code}	Moment obtained using A23.3 recommended values (Eq. 1)
M_f	The failure moment
M_r	Moment obtained using the proposed equations for α_1 and β_1
M_u	Ultimate moment
M_y	Yielding moment
NSC	Normal strength concrete
P	Axial load
R	Coefficient of determination
r	Point at which rupture of reinforcing bars occurs
T_s	Tensile force in the reinforcing bars
y	Point at which bars reach f_{y-s} for steel or for f_{cr-SMA} SMA
Z	Slope of compressive strain softening branch
α_1, β_1	Stress block parameters
ε_c	Concrete compressive strain
ε_{c-max}	Concrete maximum strain corresponding to the peak moment
ε_{cr-SMA}	SMA critical strain
ε_{cu}	Ultimate concrete compressive strain
ε_{end}	End part of the bar strain
ε_{mid}	Middle part of the bar strain
ε_{p1}	Martensite stress induced strain
ε_{SMA}	SMA strain
$\varepsilon_{SMA-avg}$	SMA average bar strain
ε_{top}	Concrete top compressive strain
ε_{u-s}	Steel strain at failure
ε_{u-SMA}	SMA strain at failure
ε_{y-s}	Steel yielding strain
ε_{y-SMA}	SMA yielding strain
ρ	Tensile reinforcement ratios
ρ'	Compressive reinforcement ratios
Φ	Curvature

Φ_{cr-SMA}	Curvature corresponding to the SMA critical stress
Φ_{max}	Curvature corresponding to the peak moment
Φ_u	Ultimate curvature
Φ_{y-s}	Curvature corresponding to the steel yielding stress

1 Introduction

Although shape memory alloys (SMAs) have many applications in different fields, they are considered relatively new to the civil engineering field. The shape memory effect, superelasticity, and performance under cyclic loading are unique properties that distinguish SMAs from other metals and alloys and make them attractive for various civil engineering applications. Some of these applications have been discussed by Wilson and Wesolowsky [1], DesRoches and Smith [2], Song et al. [3], Alam et al. [4], Janke et al. [5], and Li et al. [6–8]. This paper focuses particularly on one of these applications in which superelastic SMA bars are used to reinforce concrete structures.

While the constitutive relationship for SMA is a function of three parameters: stress, strain, and temperature, most of the widely used SMA models [9–11] are developed for quasi-static loading and are a function of two parameters: stress and strain. The characteristic properties of the constitutive relationship of SMA are greatly affected by the strain rate [12–15]. This effect is not considered in the scope of this paper since a monotonic behaviour is assumed. However, the strain rate effect needs to be considered when dynamic loads are involved.

Generally, SMAs exhibit two distinct phases or crystal structures [15], Martensite (M-phase) and Austenite (A-phase). At the martensite phase, an SMA has the ability to completely recover residual strains by heating (shape memory effect), while at the austenite phase, it recovers residual strains upon unloading (superelasticity) [16].

The unique stress–strain relationship of superelastic SMA bars is expected to affect the moment–curvature (M – Φ) relationship of concrete sections. Thus values of α_1 , and β_1 corresponding to a specific ε_{c-max} and used to evaluate the average concrete

compressive stress and the location of the centroid of the compressive force are expected to be different for SMA RC sections. The Canadian standard A23.3 [17] specifies a value of 0.0035 for ϵ_{c-max} and provides Eq. 1a and 1b to calculate α_1 and β_1 (f'_c is the concrete compressive strength). The moment capacity of a concrete section supporting an axial load P can be evaluated using the plane sections assumption and utilizing equilibrium as shown in Fig. 1a.

$$\alpha_1 = 0.85 - 0.0015f'_c \geq 0.67 \tag{1a}$$

$$\beta_1 = 0.97 - 0.0025f'_c \geq 0.67 \tag{1b}$$

$M-\Phi$ analysis utilizing non-linear material constitutive models, Fig. 1b, can accurately determine the moment M_f and the ultimate curvature Φ_u corresponding to the ultimate concrete strain ϵ_{cu} , and ϵ_{c-max} and Φ_{max} corresponding to the ultimate moment capacity M_u for a steel RC section [18]. In this paper, the validity of this method for SMA RC sections is investigated. A parametric study is then conducted on concrete sections reinforced with either SMA or steel bars. The sections have different reinforcement ratio, dimensions, axial load level [$ALI = \frac{P}{f'_c \times A_g}$ where A_g is the area of the concrete section], and f'_c . The main features of the $M-\Phi$ relationship and normal force–moment interaction diagram for the studied SMA RC sections are identified considering monotonic loading. Based on the results of this study, the A23.3 [17] values for ϵ_{c-max} , α_1 , and β_1 are evaluated and new adequate values are proposed.

2 Research significance

The use of superelastic SMA as reinforcing bars in concrete structures is expected to be implemented in the construction industry in the near future owing to the benefits provided by this smart material and the ongoing trend of reduction in its cost. The behaviour of SMA RC sections is currently not well understood, which can hinder the use of SMA in concrete structures. This study examines the monotonic behaviour of SMA RC concrete sections and provides key design parameters for steel and SMA RC sections. These parameters are only valid for cases where the mechanical properties of steel and SMA are similar to those used in the present paper.

3 Material properties

The model of Scott et al. [19] given by Eq. 2 and shown in Fig. 2a, is used to model the stress–strain behaviour of concrete in compression. Concrete is assumed to crush when ϵ_{cu} reaches 0.0035 [17]. This value lies within the known range for unconfined concrete [20]. The tensile resistance of concrete is ignored.

$$f_c = f'_c \left[2.0 \left(\frac{\epsilon_c}{0.002} \right) - \left(\frac{\epsilon_c}{0.002} \right)^2 \right] \quad 0 \leq \epsilon_c \leq 0.002 \tag{2a}$$

$$f_c = f'_c [1 - Z(\epsilon_c - 0.002)] \quad \epsilon_c \geq 0.002 \text{ and } f_c \geq 0.2f'_c \tag{2b}$$

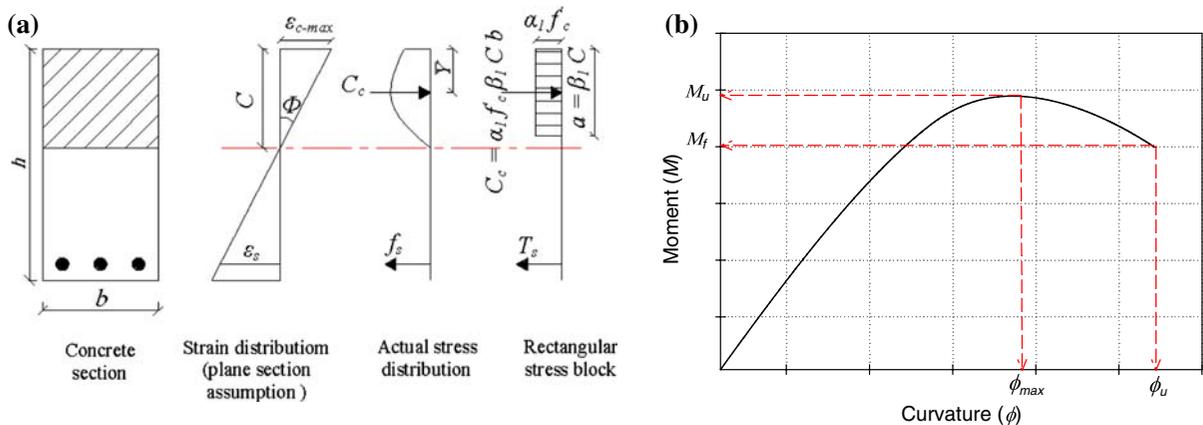


Fig. 1 a Stress block parameters for rectangular sections. b Typical $M-\Phi$ relationship

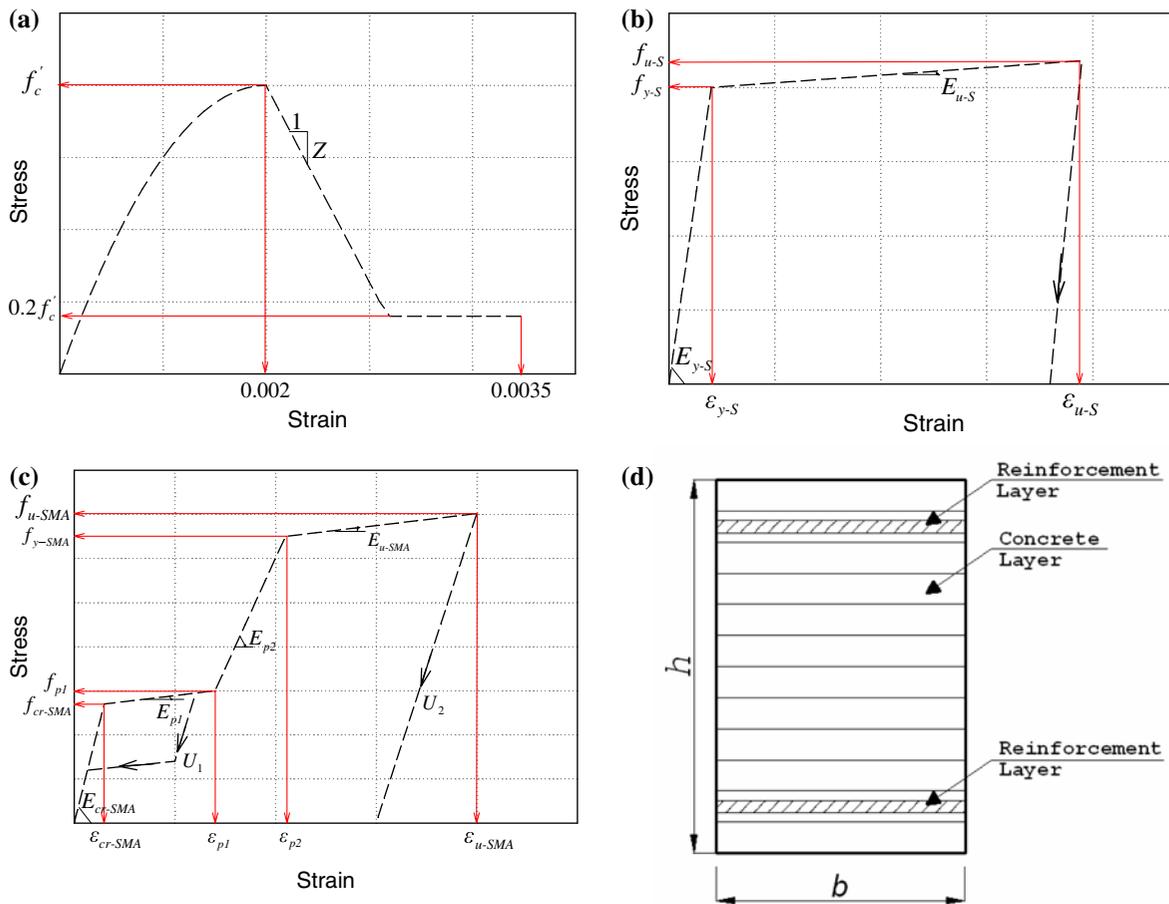


Fig. 2 a Stress–strain model for concrete in compression. b Stress–strain model for steel. c Stress–strain model for SMA. d Fibre model for a concrete section

$$Z = \frac{0.5}{\frac{3 + 0.29f'_c(\text{MPa})}{145f'_c(\text{MPa}) - 1000} - 0.002} \quad (2c)$$

where f'_c = concrete compressive stress, Z = slope of compressive strain softening branch, ϵ_c = concrete compressive strain.

The stress–strain relationship for steel is assumed to be bilinear as shown in Fig. 2b. The material behaves elastically with a modulus of elasticity E_{y-s} until the strain reaches ϵ_{y-s} . As the strain exceeds ϵ_{y-s} , the modulus of elasticity E_{u-s} is significantly reduced to about 1% to 2% of E_{y-s} . Unloading at strains greater than ϵ_{y-s} results in permanent deformations [20] as shown in Fig. 2b.

Superelastic Ni–Ti alloys are the most suitable SMA for structural applications because of their high recoverable strain, durability, and being stable

at the austenite phase at ambient temperature [5]. Ni–Ti stress–strain relationship consists of four linear branches that are connected by smooth curves. As a simplification, the smooth curves are ignored and the linear branches are assumed to intersect as shown in Fig. 2c [4, 15, 21–25]. The alloy behaves elastically with a modulus of elasticity E_{cr-SMA} until reaching the SMA critical stress f_{cr-SMA} , which represents the start of the martensite variant reorientation. As the strain ϵ_{SMA} exceeds the SMA critical strain ϵ_{cr-SMA} , the modulus of elasticity E_{p1} becomes 10–15% of E_{cr-SMA} . For strains above the martensite stress induced strain ϵ_{p1} , the material becomes stiffer and the modulus of elasticity E_{p2} reaches about 50–60% of E_{cr-SMA} . The final linear branch starts at the SMA yield point with a modulus of elasticity E_{u-SMA} that is 3–8% of E_{cr-SMA} .

As shown in Fig. 2c, the SMA bar can recover its full deformations upon unloading if the strain ε_{SMA} is less than the martensite stress induced strain ε_{p1} (superelasticity). If the strain exceeds ε_{p1} , permanent deformations will be obtained upon unloading. Full recovery of these deformations can be achieved through heating the SMA (shape memory effect) [15, 21–25]. Reaching the yielding stress $f_{y\text{-SMA}}$ results in losing the material superelasticity. For structural applications, it is recommended to design SMA RC sections to behave within the superelastic range [26]. Sections considered in this study will be designed such that it does not reach the real yielding ($f_{y\text{-SMA}}$). To simplify discussions and comparison with steel RC sections, the SMA critical stress ($f_{\text{cr-SMA}}$) is referred to as yielding in this paper.

4 Sectional analysis

The methodology adopted for sectional analysis is similar to that used by Youssef and Rahman [27]. Assumptions for this method are plane sections remain plane and perfect bond exists between the concrete and reinforcing bars. The method is based on using a fibre model. A section with width b and height h is divided into a finite number of layers as shown in Fig. 2d. Using the defined stress–strain models for steel, concrete, and SMA and taking into consideration section equilibrium and kinematics, the mechanical behaviour of the section can be analyzed for a given axial load and an increasing value of the applied moment.

5 Experimental validation

To validate the applicability of using sectional analysis for SMA RC sections, the behaviour of three small-scale superelastic SMA reinforced concrete (RC) beams tested by Saiidi et al. [28] was predicted numerically. The beams have the same dimensions but differ in their reinforcement ratio. All the beams have a span of 1,270 mm, b of 127 mm, midspan h of 152 mm, and end h of 305 mm. The chemical composition of the used SMA bars as reported by Special Metals Corporation, USA is shown in Table 1. Since the SMA rods had very low austenite phase starting temperature, the bars were

Table 1 Chemical composition of the SMA rods

Element	Weight %
Nickel	55.90
Titanium	44.01%
Oxygen	257 ppm
Carbon	374 ppm
Cu, Cr, Co, Mn, Mo, W, V	<0.01
Nb, Al, Zr, Si, Ta, Hf	<0.01
Ag, Pb, Bi, Ca, Mg, Sn, Cd	<0.01
Zn, Sb, Sr, Na, As, Be, Ba	<0.01
Fe	<0.05
B	<0.001
Hydrogen	14 ppm

Table 2 Properties of tested beams

Specimen	Midspan SMA reinforcement (mm)	$\varepsilon_{y\text{-SMA}}$ (mm/mm)	$f_{y\text{-SMA}}$ (MPa)	$E_{y\text{-SMA}}$ (MPa)
BNL2	2 Φ 6.40	0.013	400	34,078
BNH1	1 Φ 9.50	0.013	510	39,245
BNH2	2 Φ 9.50	0.013	510	39,245

cold drawn, thermally straightened to the superelastic condition, grit blasted, and heated at their ends. Details of the SMA reinforcement are given in Table 2.

Figure 3a and b shows the test setup and the cross-section of the tested beams. The beams were externally reinforced with SMA bars between the loading points and with regular steel reinforcing bars elsewhere. Figure 3c shows a typical SMA bar used in the beams. Saiidi et al. [28] indicated that their analytical predictions deviated from the experimental results because of the lack of bond between the concrete and the reinforcing bars and due to the variation of the diameter of the SMA bars. In this paper, the sectional analysis explained earlier is modified to account for the actual test setup.

The procedure used in the analysis in this section is similar to the procedure being widely used with unbonded tendons in prestressed concrete (e.g. [29]). The procedure starts by assuming an average SMA bar strain $\varepsilon_{\text{SMA-avg}}$. Based on the length and cross-sectional area of the middle and two end parts of the SMA bar, their strains (ε_{mid} and ε_{end}) are calculated. The force in the SMA bar is constant along its length

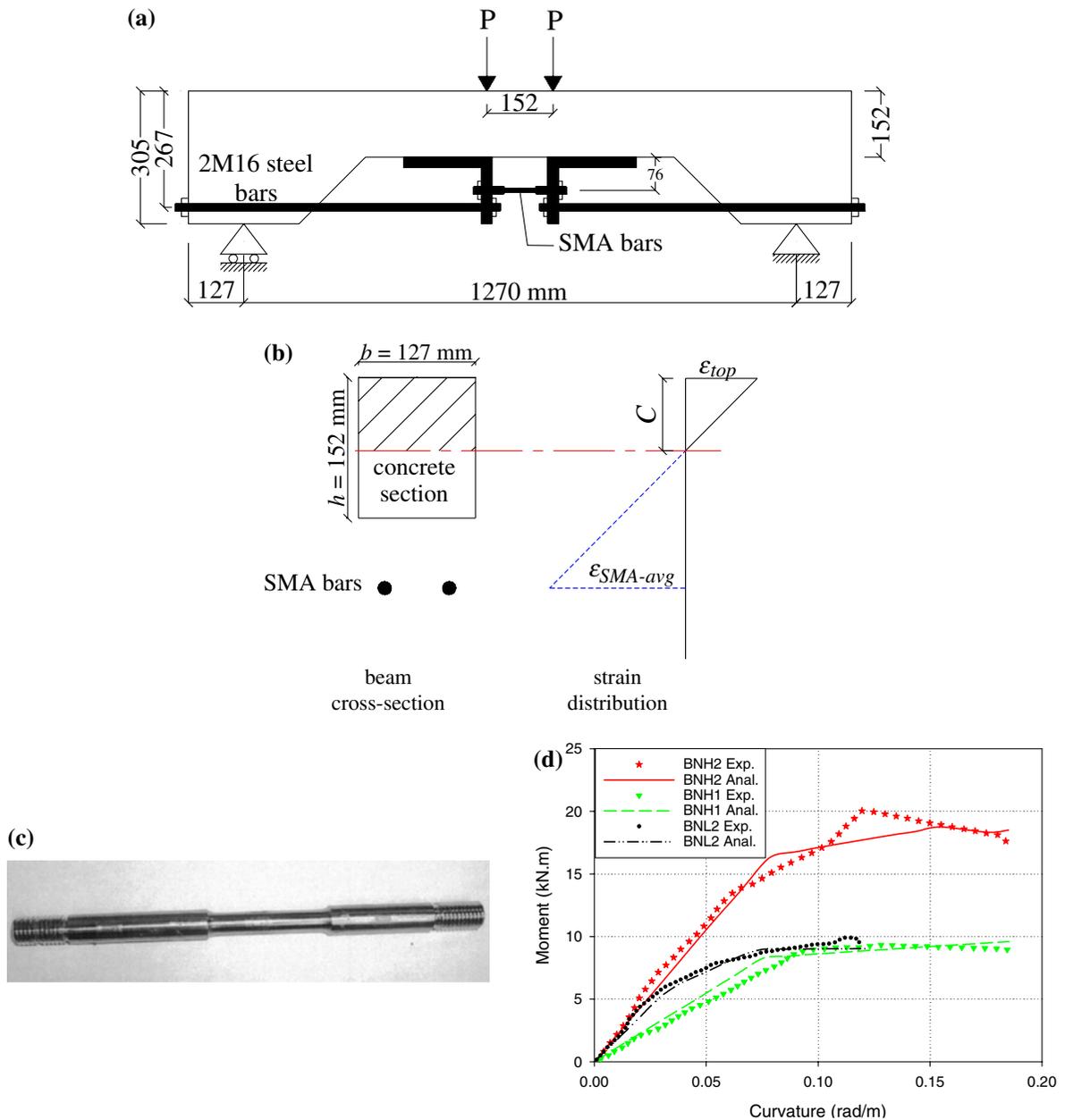


Fig. 3 a Details of the tested beam. b Beam mid-span cross-section. c SMA typical bar (Saiidi et al. [27] after permission). d Experimental versus analytical moment-curvature for SMA reinforced beams

and can be evaluated using ϵ_{mid} or ϵ_{end} . To satisfy section equilibrium, the compressive force in the concrete should be equal to the assumed tensile force. For a specific top compressive strain ϵ_{top} , the curvature Φ is iterated until equilibrium is satisfied. The corresponding moment is then calculated. The analysis is repeated for a range of top compressive

strains ϵ_{top} . The relationship between the moment and the concrete strain at the location of the bar is established. The moment corresponding to the assumed $\epsilon_{SMA-avg}$ is then obtained. This procedure is repeated for different values of $\epsilon_{SMA-avg}$, which allows defining the $M-\Phi$ relationship. The analysis is terminated when ϵ_{top} reaches 0.0035.

It was clear from the load–average bar strain relationship reported by Ayoub et al. [30] that the strain in BNH2 exceeded ε_{p1} . As the stress–strain relationship for the Ni–Ti bars provided by Saiidi et al. [28] was bilinear, ε_{p1} and E_{p2} were assumed to be equal to 0.05 mm/mm, 22,463 MPa, respectively. Moreover, it was observed that the neutral axis lies within the concrete section height at all load intervals. The relatively low modulus of elasticity of SMA bars is the main factor that controlled the location of the neutral axis. Figure 3d shows the comparison between the analytical and experimental M – Φ diagrams for SMA reinforced beams. Very good agreement between experimental and analytical results was observed for all specimens.

6 Parametric study

A parametric study was conducted for typical concrete sections with different h (500, 700, and 900 mm), b (200, 300, and 400 mm), tensile reinforcement ratios ρ (0.25%, 0.50%, and 0.75%), compressive reinforcement ratios ρ' (0%, 0.125%, and 0.25%), f'_c (20, 40, and 60 MPa), and axial load levels (ALI ranges from 0 to 1). Table 3 shows details of the analyzed sections. Each section was analyzed twice assuming that reinforcing bars are either SMA or steel with the mechanical properties given in Table 4. The mechanical properties of the used superelastic Ni–Ti are within the ranges provided by Alam et al. [4].

Because of the high value of ε_{cr-SMA} (0.015 mm/mm), SMA bars did not exhibit tensile yielding at ALI higher than 0.2. In this paper, ALI = 0 and 0.3 were chosen to present the behaviour of SMA RC sections in details. The results obtained for other ALI values were used to develop normal force–moment interaction diagrams. These diagrams were developed for both types of reinforcement, i.e. steel and SMA.

7 Moment–curvature response

Due to the difference in the modulus of elasticity of steel and SMA, the curvature Φ_{cr-SMA} corresponding to f_{cr-SMA} for SMA RC sections was found to be higher than Φ_{y-s} for similar steel RC sections. The failure of SMA RC sections was initiated by crushing of concrete. Rupture of SMA bars did not govern failure because of the high ultimate tensile strain of the SMA bars (0.2 mm/mm). For steel RC sections, the failure type varied between concrete crushing and rupture of steel bars depending on section dimensions, reinforcement ratio, and axial load level. The effect of the different parameters on the M – Φ relationship is shown in Figs. 4–8. In these figures, the point at which the reinforcing bars reach f_{y-s} for steel or f_{cr-SMA} for SMA is marked by (y) and referred to as yielding in the following paragraphs. The point at which the strain in the SMA bars exceeds ε_{p1} is defined by an (H). The two types of failure are defined by (cc) for concrete crushing and (r) for rupture of reinforcing bars.

Table 3 Details of analyzed sections

Section	Studied variables	h (mm)	b (mm)	A_s (mm ²)	A'_s (mm ²)	f'_c (MPa)
C_1	h	500	300	655	0	40
C_2	b, h	700	300	655	0	40
C_3	h	900	300	655	0	40
C_4	b	700	200	655	0	40
C_5	b	700	400	655	0	40
C_6	ρ, ρ', f'_c	700	300	525	0	40
C_7	ρ	700	300	1,050	0	40
C_8	ρ	700	300	1,575	0	40
C_9	ρ'	700	300	525	262.5	40
C_{10}	ρ'	700	300	525	525.0	40
C_{11}	f'_c	700	300	525	0	20
C_{12}	f'_c	700	300	525	0	60

Table 4 Mechanical properties of SMA and steel bars

Material	Property	E_y (GPa)	f_y (MPa)	f_{p1} (MPa)	f_u (MPa)	ϵ_{p1} (%)	ϵ_u (%)
SMA	Tension	36	540	600	1,400	7.0	20
	Compression	60	650	735	1,500	4.5	20
Steel	Tension or compression	200	438	NA	615	NA	3.5

7.1 Effect of cross-section height h

Figure 4a and b shows the effect of varying h on the $M-\Phi$ relationship at two levels of axial load (ALI = 0 and 0.3). At ALI = 0, yielding of SMA RC sections occurred at higher curvature values (400–500%) than that for the steel RC sections. The ultimate curvature Φ_u of steel RC sections was found to decrease by 50% as h increased by 80%. This decrease is attributed to the failure type as it occurred by rupture of steel rather than crushing of concrete. The section ultimate curvature, Φ_u for SMA RC sections was not significantly affected by a similar increase in h since failure is governed by crushing of concrete.

Increasing the axial load level from 0 to 0.3 resulted in a significant increase (315%) in the cracking moment for both steel and SMA RC sections. Although the yielding moment M_y for steel RC sections increased with axial load increase, SMA RC sections did not exhibit yielding. The amount of strain energy calculated by integrating the area under the $M-\Phi$ relationship increased with the increase in h for both cases of reinforcement. At ALI of 0.3, SMA RC

sections had similar initial stiffness, and their strain energy was comparable to that of steel RC sections.

7.2 Effect of cross-section width b

The effect of varying b on the $M-\Phi$ analysis is illustrated in Fig. 5a and b. At ALI = 0, increasing b had a minor effect on M_y and M_u for both steel and SMA reinforcement. Although Φ_u was not affected for steel RC sections, Φ_u for SMA RC sections increased by 90% as b increased by 100%. This increase in Φ_u resulted in 125% increase in the strain energy.

At higher axial load level (ALI = 0.3), Fig. 5b, the curves for SMA and steel RC sections coincided prior to cracking. For both types of RC sections, Φ_u was not affected by changing b and a significant increase, about 80%, in section capacity was achieved by increasing b by 100%. Although SMA bars did not exhibit any yielding for the studied sections, their strain energy reached values as high as 9,665 N rad. M_y for steel RC sections increased by 200–400% due to the increase in the axial load level.

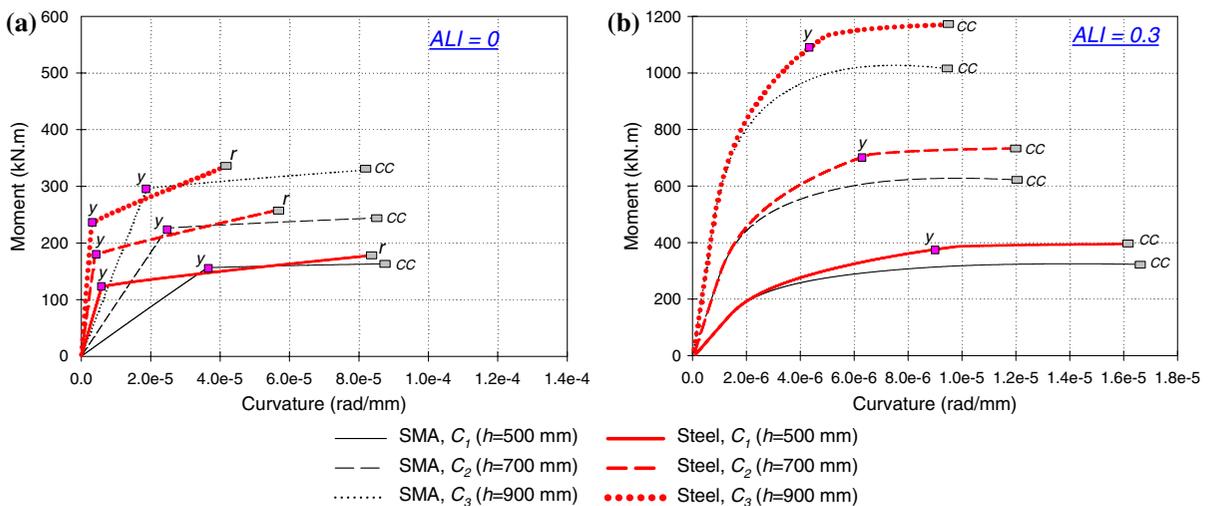


Fig. 4 Effect of varying h on the $M-\Phi$ relationship. **a** ALI = 0. **b** ALI = 0.3



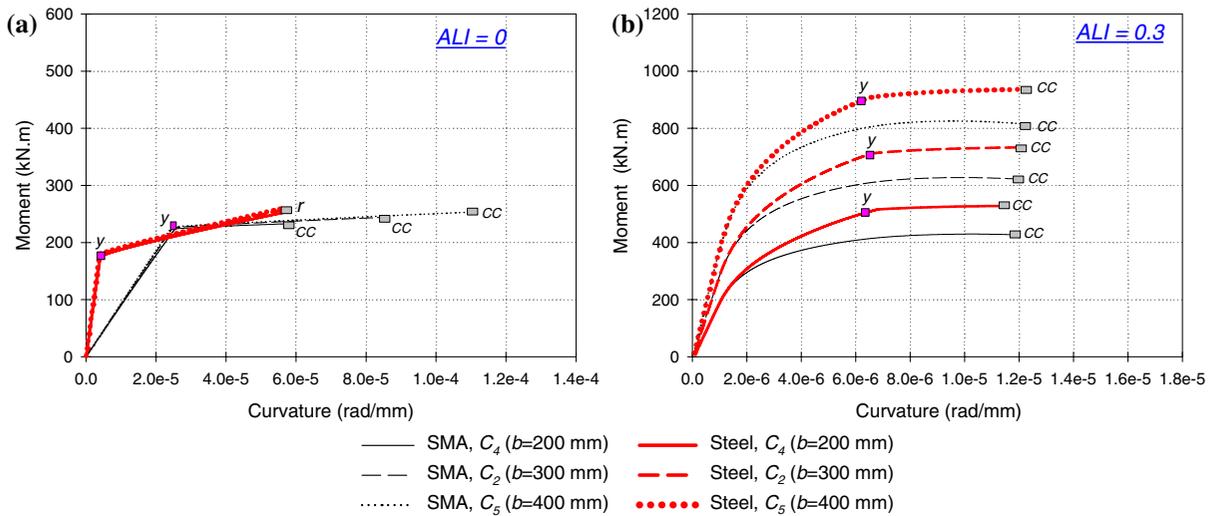


Fig. 5 Effect of varying b on the $M-\Phi$ relationship. a $ALI = 0$. b $ALI = 0.3$

The increase in the axial load level decreased the strain energy by about 60% for both types of reinforcement.

7.3 Effect of tensile reinforcement ratio ρ

As shown in Fig. 6a ($ALI = 0$), a 200% increase in ρ increased the section capacity by 160% for both types of reinforcement. Although M_y increased with increasing ρ , the yielding curvature was slightly affected. The increase in ρ resulted in decreasing Φ_u . For lower reinforcement ratio ($\rho = 0.25\%$), the strain

energy for SMA RC sections was 45% higher than that of steel RC sections since SMA bars exhibited extensive yielding. As ρ increased, the strain energy became higher for steel RC sections.

At higher levels of axial load, $ALI = 0.3$, the effect of increasing ρ on increasing the section capacity was higher for steel RC sections, Fig. 6b. Increasing ALI from 0 to 0.3 increased the cracking moment by 320%. Failure occurred by crushing of concrete, and thus Φ_u was not affected. The strain energy for steel RC sections was 11% higher than that of SMA RC sections.

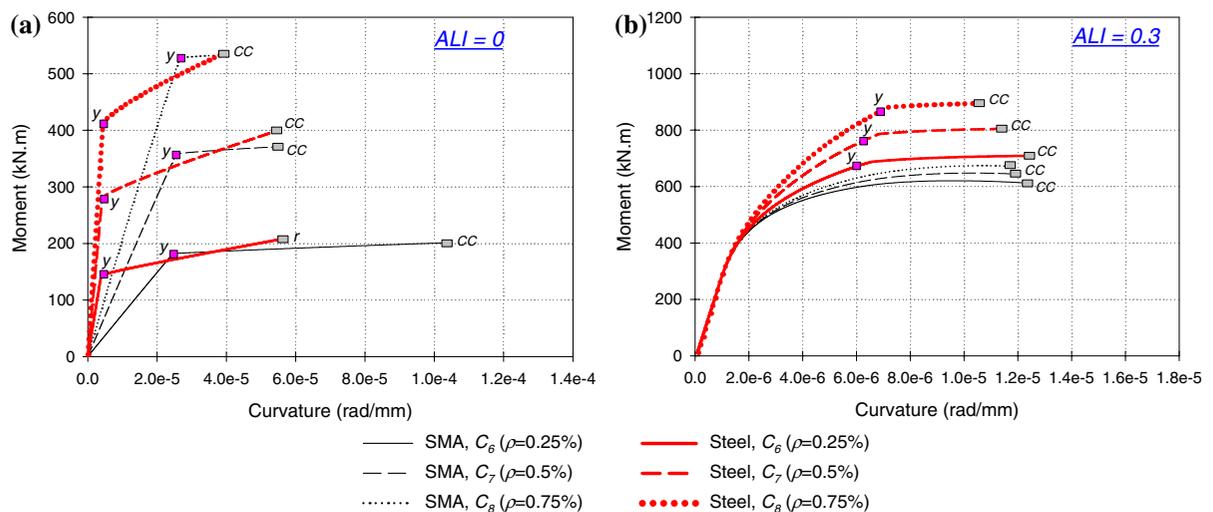


Fig. 6 Effect of varying ρ on the $M-\Phi$ relationship. a $ALI = 0$. b $ALI = 0.3$



7.4 Effect of compressive reinforcement ratio ρ'

Figure 7a represents the $M-\Phi$ relationship for SMA and steel RC sections for different values of ρ' . At $ALI = 0$, ρ' has no effect on M_y , Φ_u , and M_u . Failure for steel RC sections occurred by rupture of steel. As the SMA bars exhibited higher yielding than that of the steel bars, the strain energy and section ultimate curvature Φ_u were higher for SMA RC sections than for steel RC sections.

Increasing ALI from 0 to 0.3 resulted in higher section capacity for both types of reinforcement, Fig. 7b. ρ' was found to slightly affect the section capacity. M_y was also slightly affected by varying ρ' for steel RC sections. SMA RC sections did not exhibit yielding at this level of axial load. Failure occurred by crushing of concrete for both SMA and steel RC sections. This type of failure resulted in almost equal Φ_u for the analyzed sections. The strain energy for steel RC sections was 12–23% higher than that for SMA RC sections.

7.5 Effect of concrete compressive strength f'_c

At $ALI = 0$, Fig. 8a, increasing f'_c from 20 to 40 MPa did not notably affect M_y or M_u for both types of reinforcement. For steel RC sections, failure occurred by rupture of reinforcing bars, and thus Φ_u was almost constant. However, for SMA RC sections, Φ_u increased by 90% with the increase of f'_c . At

$f'_c = 60$ MPa, the yielding plateau of SMA bars was followed by a strain hardening behaviour resulting in a substantial increase in section capacity and ductility.

At $ALI = 0.3$, Fig. 8b, the cracking and ultimate moments for both types of reinforcement increased by 160–180%. Increasing f'_c from 20 to 60 MPa resulted in an increase of 155% in the yielding moment for the steel RC sections. SMA bars did not yield at this level of axial load. Φ_u was comparable for SMA and steel RC sections since failure occurred by crushing of concrete.

8 Normalized interaction diagrams

As mentioned earlier, the $M-\Phi$ analysis was conducted at different ALI. The M_u values obtained for different ALI were used to develop the normal force–moment interaction diagrams that are shown in Figs. 9–11. For each analyzed section, ε_{c-max} and Φ_{max} corresponding to the peak moment M_u were identified.

The point at which the interaction diagrams of steel RC sections change the sign of their slope is known as the balance point. It is the point at which steel yields ($\varepsilon_{y-s} = 0.0022$) simultaneously with concrete reaching its crushing strain ($\varepsilon_{cu} = 0.0035$). For the analyzed sections, the balance point occurred at an axial load level ALI ranging from 0.3 to 0.5.

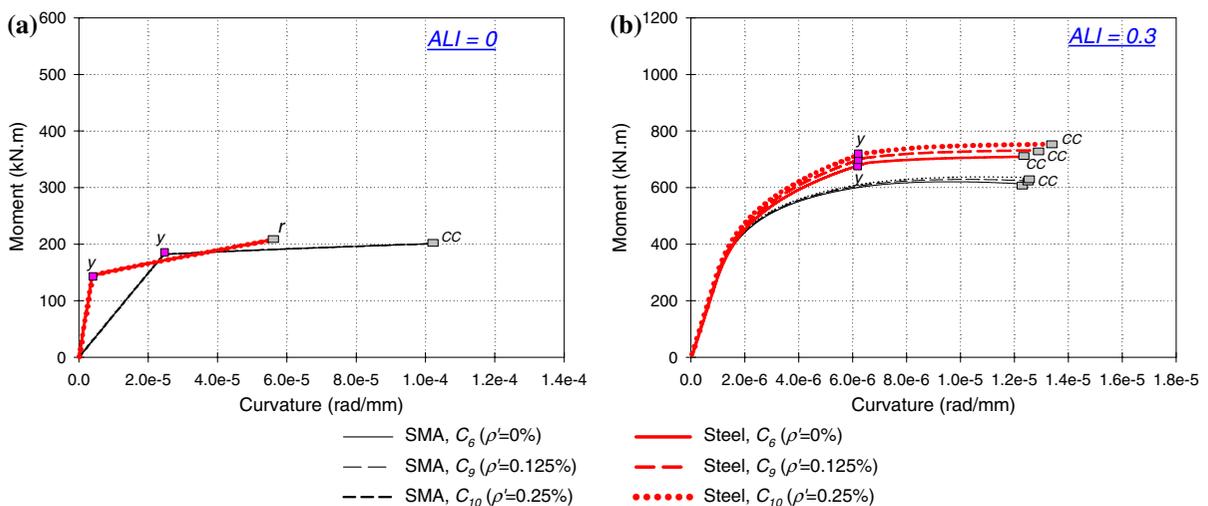


Fig. 7 Effect of varying ρ' on the $M-\Phi$ relationship. a $ALI = 0$. b $ALI = 0.3$



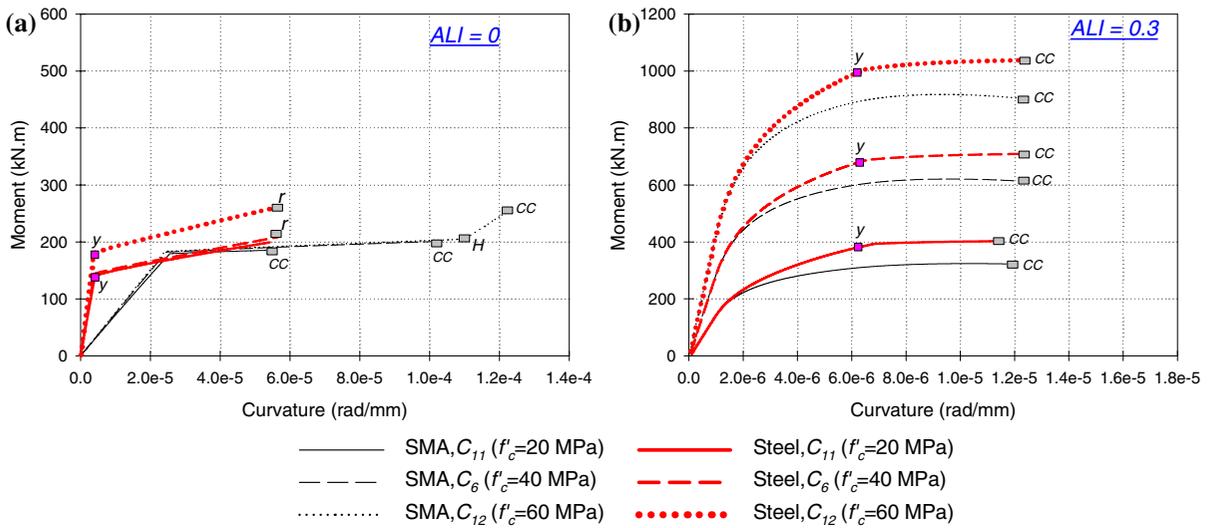


Fig. 8 Effect of varying f'_c on the $M-\Phi$ relationship. a $ALI = 0$. b $ALI = 0.3$

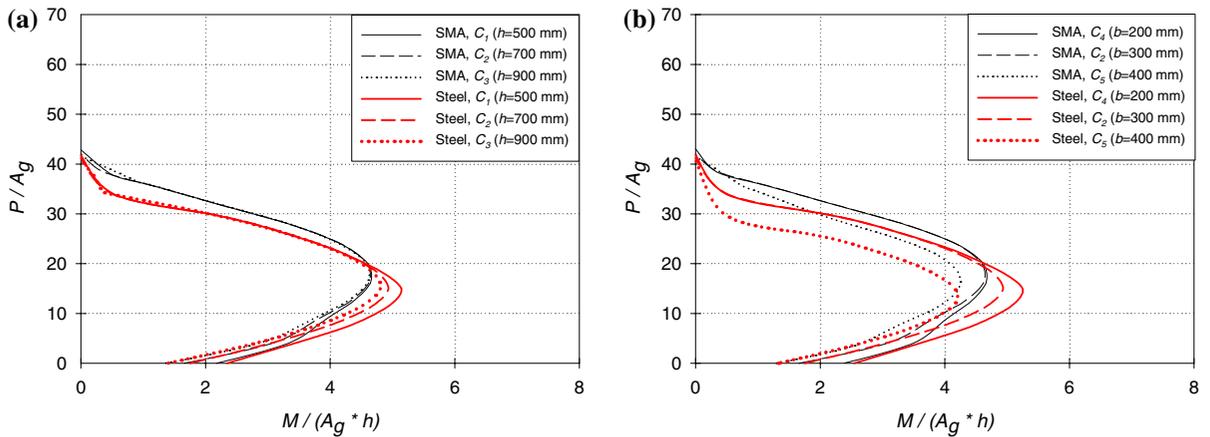


Fig. 9 Effect of varying h and b on the normalized interaction diagram. a h . b b

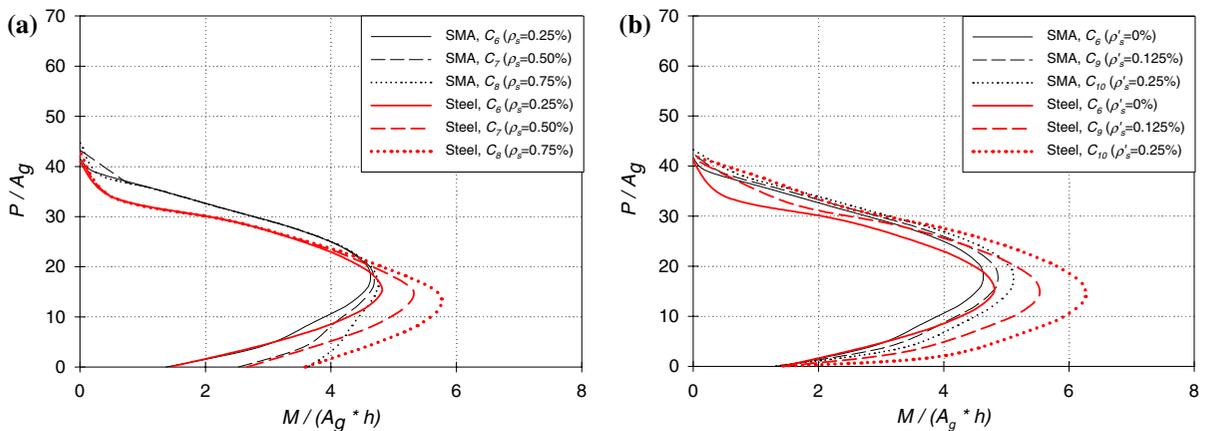


Fig. 10 Effect of varying ρ and ρ' on the normalized interaction diagram. a ρ . b ρ'



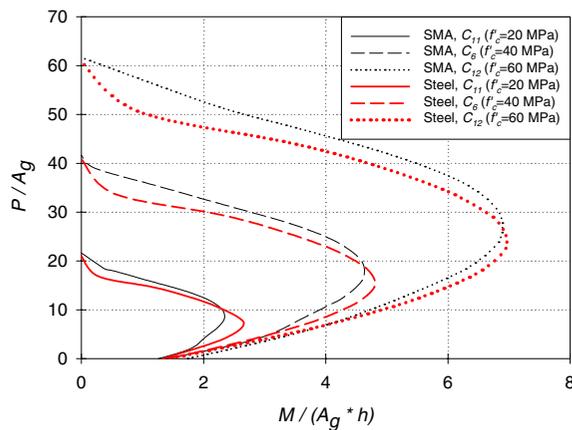


Fig. 11 Effect of varying f'_c on the normalized interaction diagram

The difference in the stress–strain relationship between steel and SMA resulted in a different behaviour for SMA RC sections. The point at which the curve changed the sign of its slope was not related to yielding of SMA bars. It occurred at an axial load level close to that for steel RC sections ($ALI = 0.3$ to 0.5). At this point, SMA bars did not yield and ε_{c-max} varied from 0.0026 to 0.0031.

8.1 Effect of cross-section height h

Figure 9a illustrates the effect of varying h on the interaction diagrams. The pure flexural capacity $\left[\frac{P}{A_g} = 0\right]$ was the most affected point. As the axial load level increased on the section, the effect of varying h on section capacity decreased. The pure axial capacity was slightly higher (3%) for SMA RC sections than that for the steel sections because of the higher yielding stress of the SMA bars. This increase was noticed for all other parameters.

8.2 Effect of cross-section width b

As shown in Fig. 9b, varying b had a clear effect on the interaction diagram for both the steel and SMA reinforcement. The pure flexural capacity, where the axial load is zero, changed by about 50% when b increased from 200 to 400 mm. Varying b from 200 to 300 mm did not affect the interaction diagrams at high levels of axial load ($ALI > 0.5$). However, increasing b from 300 to 400 mm resulted in a clear

effect on the section capacity at all levels of axial load for both steel and SMA RC sections.

8.3 Effect of tensile reinforcement ratio ρ

The interaction diagrams shown in Fig. 10a represent the effect of varying ρ on the section capacity. At low levels of axial loads ($ALI < 0.4$), increasing the reinforcement ratio ρ from 0.25% to 0.75% resulted in a significant increase in section capacity (140% for steel–165% for SMA). For $ALI > 0.4$, the effect of increasing ρ decreased since failure was governed by crushing of concrete rather than rupture of steel.

8.4 Effect of compressive reinforcement ratio ρ'

As shown in Fig. 10b, the interaction diagrams for steel RC sections were more affected by varying ρ' than that for SMA RC sections. The pure flexural capacity was not affected significantly by varying the reinforcement ratio ρ' for both the steel and SMA reinforcement. At $ALI = 0.4$, the capacity of steel RC sections increased by 11% as ρ' increased from 0 to 0.25%.

8.5 Effect of concrete compressive strength f'_c

It can be observed from Fig. 11 that increasing f'_c significantly increased the section capacity. The pure flexural capacity increased by 35% by changing f'_c from 20 to 60 MPa, and the section capacity at higher axial load levels (i.e. $ALI = 0.4$) drastically increased (195%) for both steel and SMA RC sections. The pure axial capacity ($M = 0$) also increased significantly (185%) with an increase in f'_c of 200%.

9 Rectangular stress block parameters

Building codes provide engineers with equivalent stress block parameters α_1 and β_1 to simplify the design process. The use of α_1 and β_1 allows calculating the concrete compressive force and its location. The A23.3 [17] procedure for calculating α_1 and β_1 , Eq. 1, is dependent on f'_c to account for the difference in behaviour of high strength concrete ($f'_c > 60$ MPa).



In this section, and for each of the analyzed sections, α_1 and β_1 were calculated from the known strain distribution at the peak moments. The compressive force in concrete C_c and its point of application were evaluated by calculating the area under the stress–strain relationship of concrete corresponding to the known ϵ_{c-max} , and Φ_{max} , and its centroid. The stress block parameters were then found such that they result in the same area and same location of the centroid. In addition, the Canadian code recommended α_1 and β_1 values were evaluated for steel RC sections.

9.1 Steel RC sections

Figure 12a shows the variation of ϵ_{c-max} with the axial load level for steel RC sections. At $ALI = 0$, failure occurred in some of the considered sections by rupture of the reinforcement, before ϵ_{c-max} reached its limit of

0.0035. As a result, ϵ_{c-max} varied between 0.0020 and 0.0035. For $0 \leq ALI \leq 0.1$, failure occurred at $\epsilon_{c-max} = 0.0035$. ϵ_{c-max} decreased as ALI increased approaching a value of 0.002 at $ALI = 1.0$. This behaviour is similar to the recommendation of the Eurocode [31] where the value for the limiting concrete compressive strain is a function of the load eccentricity. A value of 0.0035 is recommended for flexural and for combined bending and axial load where the neutral axis remains within the section. For other sections (neutral axis outside the section), a value between 0.0035 and 0.002 is to be used.

From the parametric study conducted in this paper, it is recommended to calculate ϵ_{c-max} as a function of ALI. ϵ_{c-max} can be assumed to be equal to 0.0035 for $ALI \leq 0.1$, 0.0028 for $0.2 \leq ALI \leq 0.5$, and 0.002 for $ALI = 1.0$. Linear interpolation can be used for different ALI values. The recommended values for ϵ_{c-max} are shown in Fig. 12a.

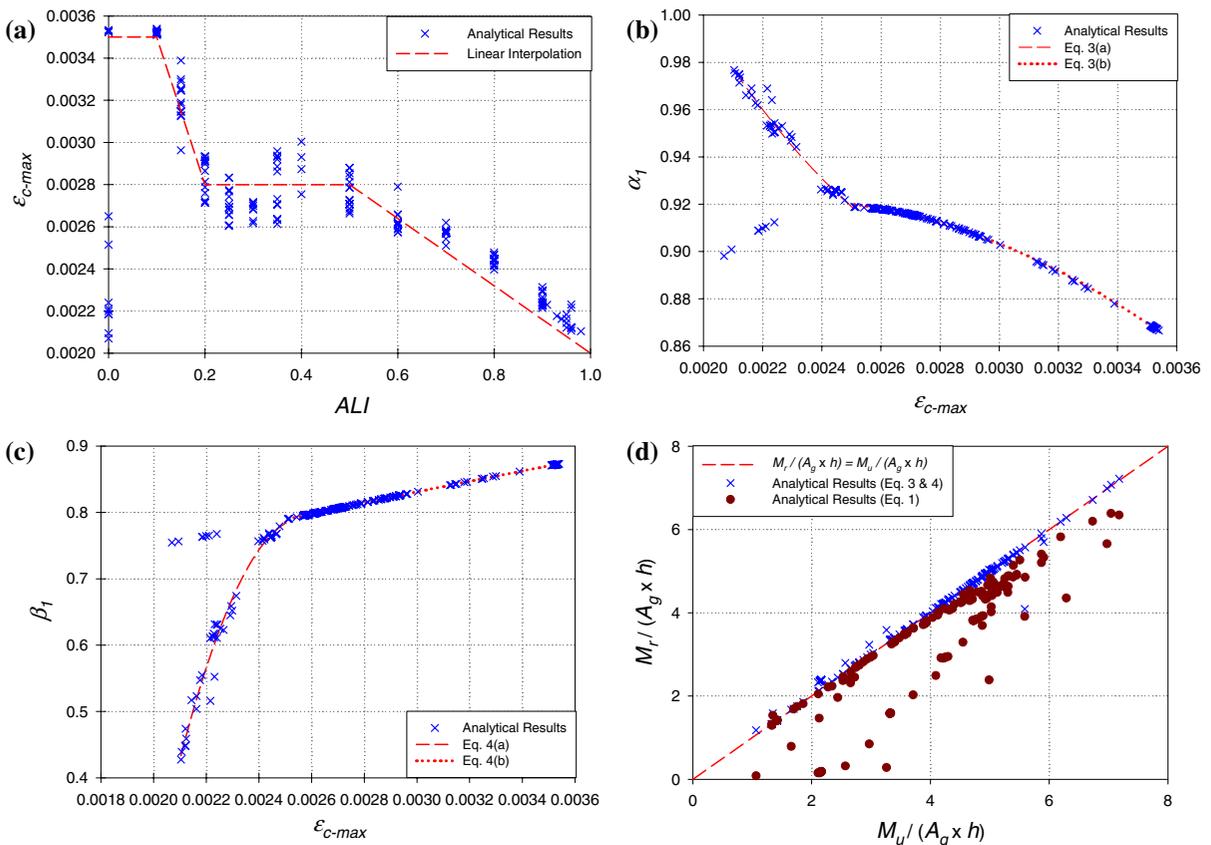


Fig. 12 Steel RC sections. a ϵ_{c-max} –ALI relationship. b α_1 – ϵ_{c-max} relationship. c β_1 – ϵ_{c-max} relationship. d $M_u / (A_g \times h)$ – $M_u / (A_g \times h)$ relationship



Figure 12b and c shows the variation of α_1 and β_1 with ϵ_{c-max} . α_1 was found to approach a value of 1.0 at ϵ_{c-max} of 0.002 (pure axial load). Based on the analytical results, Eqs. 3 and 4 were developed to calculate α_1 , and β_1 based on ϵ_{c-max} . The predictions of these equations are shown in Fig. 12b and c.

$$\alpha_1 = 88.36 \times 10^3 \epsilon_{c-max}^2 - 552.4 \epsilon_{c-max} + 1.750 \quad 0.002 \leq \epsilon_{c-max} \leq 0.00275 \quad (3a)$$

$$\alpha_1 = -33.54 \times 10^3 \epsilon_{c-max}^2 + 150.7 \epsilon_{c-max} + 750.0 \times 10^{-3} \quad 0.00275 \leq \epsilon_{c-max} \leq 0.0035 \quad (3b)$$

$$\beta_1 = -1630 \times 10^3 \epsilon_{c-max}^2 + 8388 \epsilon_{c-max} - 10.00 \quad 0.002 \leq \epsilon_{c-max} \leq 0.00275 \quad (4a)$$

$$\beta_1 = -5513 \epsilon_{c-max}^2 + 114.1 \epsilon_{c-max} + 540.0 \times 10^{-3} \quad 0.00275 \leq \epsilon_{c-max} \leq 0.0035 \quad (4b)$$

The capacity of the analyzed sections was calculated based on the values of ϵ_{c-max} , α_1 , and β_1 recommended

in this section and based on A23.3 [17] recommended values. Figure 12d shows a comparison of the calculated values and the exact values obtained using the $M-\Phi$ analysis. The capacity of the analyzed sections calculated using the proposed α_1 and β_1 values was found to be in a very good agreement with those obtained using $M-\Phi$ analysis; maximum error was equal to 5%. The recommended values by A23.3 [17] were found to significantly underestimate the section capacity at high levels of axial load.

9.2 SMA RC sections

For SMA RC sections, ϵ_{c-max} was found to be dependent on the axial load level ALI as shown in Fig. 13a. It is recommended to assume ϵ_{c-max} to be equal to 0.0035 for $ALI \leq 0.2$, 0.00275 for $ALI = 0.4$, 0.00255 for $ALI = 0.6$, and 0.002 for $ALI = 1.0$. Linear interpolation can be used for different ALI values.

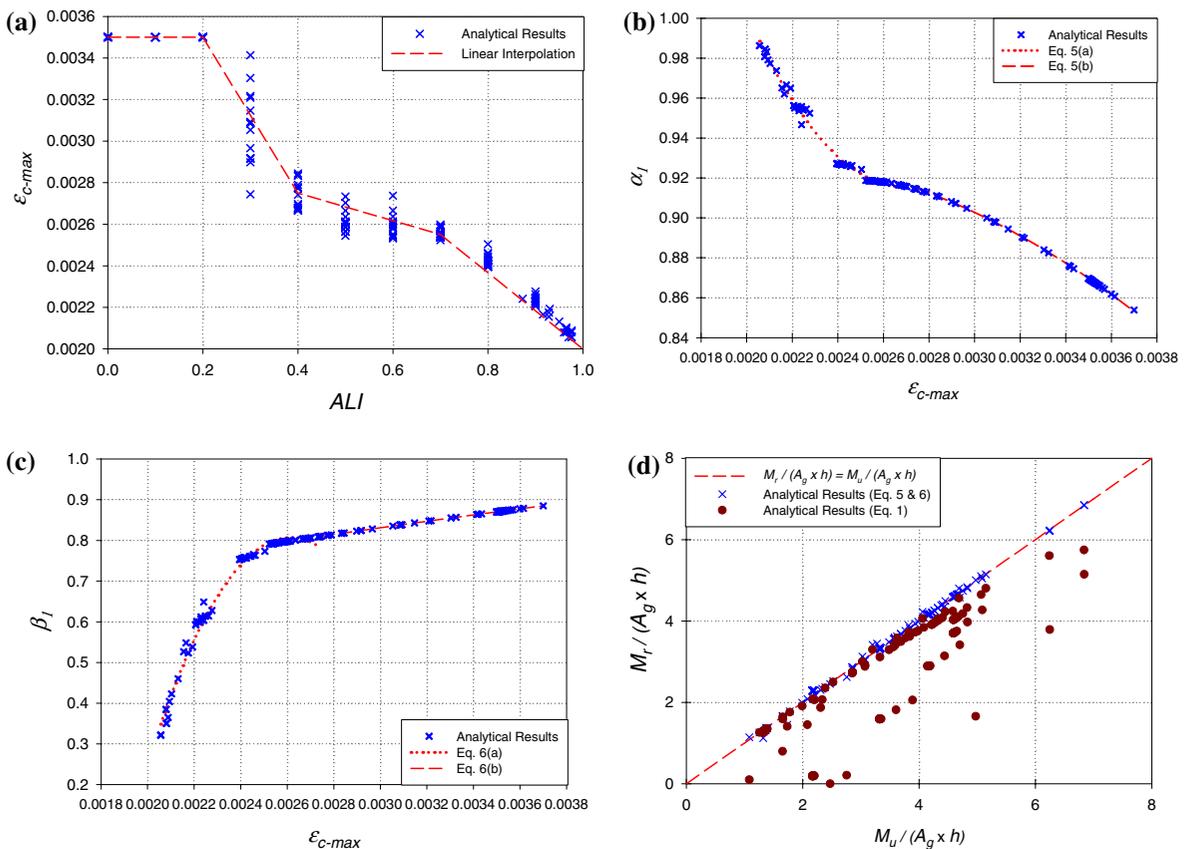


Fig. 13 SMA RC sections. **a** ϵ_{c-max} -ALI relationship. **b** α_1 - ϵ_{c-max} relationship. **c** β_1 - ϵ_{c-max} relationship. **d** $M_r / (A_g \times h)$ - $M_u / (A_g \times h)$ relationship



Figure 13b and c shows the variation of α_1 and β_1 with $\varepsilon_{c-\max}$. Based on these figures, Eqs. 5 and 6 were developed to calculate α_1 and β_1 . The square value of the coefficient of determination R^2 corresponding to these equations ranged between 0.9922 and 0.9999.

$$\alpha_1 = 182.7 \times 10^3 \varepsilon_{c-\max}^2 - 982.1 \varepsilon_{c-\max} + 2.240$$

$$0.002 \leq \varepsilon_{c-\max} \leq 0.00275 \quad (5a)$$

$$\alpha_1 = -24.62 \times 10^3 \varepsilon_{c-\max}^2 + 94.05 \varepsilon_{c-\max} + 840.0$$

$$\times 10^{-3} \quad 0.00275 \leq \varepsilon_{c-\max} \leq 0.0035 \quad (5b)$$

$$\beta_1 = -1477 \times 10^3 \varepsilon_{c-\max}^2 + 7719 \varepsilon_{c-\max} - 9.280$$

$$0.002 \leq \varepsilon_{c-\max} \leq 0.00275 \quad (6a)$$

$$\beta_1 = -5867 \varepsilon_{c-\max}^2 + 116.4 \varepsilon_{c-\max} + 540.0 \times 10^{-3}$$

$$0.00275 \leq \varepsilon_{c-\max} \leq 0.0035 \quad (6b)$$

The accuracy of the estimated values for $\varepsilon_{c-\max}$, α_1 and β_1 was checked by calculating the section capacity based on the proposed values (Eqs. 5 and 6). Figure 13d shows the relationship between the normalized ultimate moment M_u obtained from the $M-\Phi$ analysis and the normalized moment M_r obtained based on the recommended values of $\varepsilon_{c-\max}$, α_1 and β_1 . The maximum error in M_r was 2% for $ALI < 0.5$, and 6% for ALI ranging from 0.6 to 0.9. The error for sections with compression reinforcement was slightly higher. The normalized moment M_{code} calculated based on the recommended values by A23.3 [17] were also plotted versus the normalized ultimate moment M_u obtained from the $M-\Phi$ analysis, Fig. 13d. A23.3 [17] recommended values were found to be conservative in calculating the section capacity at all levels of axial load. At high ALI , the A23.3 [17] recommended values were found to significantly underestimate the section capacity.

10 Summary and conclusions

This study investigates the flexural behaviour of SMA RC sections as compared to that of steel RC sections. The accuracy of using sectional analysis for SMA RC sections was validated by comparing analytical predictions and experimental results for three simply supported beams. Sectional analysis was modified to account for the test setup that included using external unbonded superelastic SMA bars.

A number of steel and SMA RC sections were then chosen to carry out a parametric study. The studied variables were the cross-section height and width, tensile and compressive reinforcement ratios, concrete compressive strength, and axial load level. For each section, the $M-\Phi$ relationship was established and used to evaluate the moment capacity M_u , the corresponding curvature Φ_{max} , and maximum concrete strain $\varepsilon_{c-\max}$. Based on the results of this parametric study, the following conclusions can be drawn.

10.1 Moment–curvature relationship

At $ALI = 0$, SMA RC sections have lower initial stiffness than that of steel RC sections. The difference in the initial stiffness vanishes for higher ALI as the axial load delays cracking of the section. Although SMA bars did not yield at $ALI > 0.2$, the strain energy of SMA RC sections had values comparable to that of steel RC sections.

Steel RC sections failed either by rupture of steel bars or concrete crushing at low axial load levels and by concrete crushing at high ALI . SMA RC sections failed by concrete crushing rather than rupture of SMA bars because of the high tensile strain of SMA. For higher concrete compressive strength, sections with low SMA reinforcement ratio exhibited strain hardening following the initial yielding. This behaviour may not be acceptable as the strain in the SMA bars exceeded their recovery strain, which defeats the purpose of using SMA.

10.2 Normal force–moment interaction diagrams

The change in the sign of the slope of steel RC interaction diagrams occurs at the balanced moment. This point is defined as the point at which the steel yields in tension and the concrete crushes in compression. For SMA RC sections, the point at which the sign changes is not related to yielding of SMA bars. It rather occurs as a result of the change in the maximum concrete strain and the compression zone height. It was also observed that the pure axial capacity of SMA RC sections is higher than that of similar steel RC sections due to the higher yield stress of SMA bars.

10.3 Stress block parameters

The maximum concrete strain ε_{c-max} for steel RC sections was found to be equal to 0.0035 for ALI between 0 and 0.1. This correlates well with the Canadian standards guidelines. However, for ALI = 0, ε_{c-max} was found to deviate from this value because of the change in the failure mode from a compression failure of concrete to reinforcement rupture. This change was discussed by other researchers and found to have minor effect on the calculations of the moment capacity. Another deviation was observed at ALI exceeding 0.1. It is proposed to assume ε_{c-max} equal to 0.0035 for ALI \leq 0.1, 0.0028 for $0.2 \leq$ ALI \leq 0.5, and 0.002 for ALI = 1.0. Linear interpolation can be used for different ALI values. The corresponding values for α_1 and β_1 were proposed in the present study.

For SMA RC sections, it was found that ε_{c-max} can be assumed to be 0.0035 for ALI \leq 0.2, 0.00275 for ALI = 0.4, 0.00255 for ALI = 0.6, and 0.002 for ALI = 1.0. For other ALI values, ε_{c-max} is proposed to be evaluated by linear interpolation. Two equations were developed to calculate α_1 and β_1 for SMA RC sections.

The accuracy of the proposed values of ε_{c-max} , α_1 , and β_1 for steel and SMA RC sections was validated by comparing the moment capacity calculated based on these parameters and that obtained from the moment–curvature relationships. The proposed equations provided good estimates of the moment capacity and were found to be superior to the equations proposed by the Canadian code.

The conclusions reached in this paper are based on the specific properties assumed for the steel and SMA bars used in the present study. For materials with different properties, the validity of the proposed values and the proposed equations needs to be explored.

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